" Exact value ": in fraction form (ex. $\frac{1}{81}$ NOT 0. ....)

10/10

HW Mark:	10	9	8	7	6	RE-Submit

175

Exponents: Integral & Rational

Quiz : pg 1-16

(Exponents may be integers or fractions.)

152

This booklet belongs to: Marissa Period 4

LESSON #	DATE	QUESTIONS FROM NOTES	Questions that I find difficult
1.	Oct. 20/14	Pg. 4-7	11 ✓
2.	Oct. 22/14	Pg. 8-11 → hand out	50, 58, 59 ✓
3.	Oct. 23/14	Pg. 12-16	71, 92 ✓
4.	Oct. 29/14	Pg. 17-19	
5.	Oct. 30/14	Pg. QUIZ + 20-23	140, 162, 152 ✓
6.	NOV. 3/14	Pg. 24-27	173, 175, 181, 182, 188, 190, 191, 192, 193 ✓
7.		Pg.	
8.		Pg.	
9.		Pg.	
10.		Pg.	
11.		REVIEW	
12.		TEST	

Your teacher has important instructions for you to write down below.

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Exponents: Integral & Rational

IRP Organizer Algebra and Number	#	Daily Topic	Key Ideas
Warmup		Explore the nature of exponents that are positive or negative integers, or zero.	Evaluate the following $-5^2, (-4)^{-2}, \left(\frac{2}{3}\right)^2$
3. Demonstrate an understanding of powers with integral and rational exponents. [C, CN, PS, R]	1.	Explain, using patterns, why $a^{-n} = \frac{1}{a^n}$	Show using powers of 2 that $2^{-3} = \frac{1}{8}$.
	2.	Explain, using patterns, why $a^{\frac{1}{n}} = \sqrt[n]{a}$ and why $a^{-\frac{1}{n}} = \frac{1}{\sqrt[n]{a}}$.	Evaluate $16^{\frac{1}{2}}$. Evaluate $8^{-\frac{1}{3}}$. Simplify $x^{-\frac{2}{3}}$
	3.	Apply the exponent laws: $a^m \times a^n = a^{m+n}$ $a^m \div a^n = a^{m-n}$ (remember $a \neq 0$) $(a^m)^n = a^{m \times n}$ $(ab)^m = a^m b^m$ $\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}$ (remember $b \neq 0$)	Simplify $x^3 \times x^7$. Simplify $x^{13} \div x^{10}$. Simplify $(x^2)^5$. Simplify $(3x)^3$. Simplify $\left(\frac{4}{a}\right)^3$
	4.	Express powers with rational exponents as radicals and vice versa.	Write in radical form $x^{\frac{4}{5}}$. Write in exponential form $(\sqrt[3]{x})^5$.
	5.	Solve a problem that involves exponent laws or radicals.	Simplify $\sqrt[3]{x^2} \times \sqrt[4]{x^5}$. Find the exact area of a square with a side length of $5^{\frac{3}{2}}$.
	6.	Identify and correct errors in a simplification of an expression that involves powers.	Jack simplified the following expression. In which step did he make his first error? $\frac{a^2 + \sqrt[4]{a}}{a^{\frac{1}{2}}}$ Step 1: $= \frac{a^{\frac{8}{4}} + a^{\frac{1}{4}}}{a^{\frac{1}{2}}}$ Step 2: $= \frac{a^{\frac{9}{4}}}{a^{\frac{1}{2}}}$ Step 3: $= a^{\frac{9}{4} - \frac{2}{4}}$ Step 4: $= a^{\frac{7}{4}}$

[C] Communication [PS] Problem Solving, [CN] Connections [R] Reasoning, [ME] Mental Mathematics

[T] Technology, and Estimation, [V] Visualization

Term	Definition	Example
Power	$2^1, 2^2, 2^3, 2^4, \ldots$ are powers of 2. A power is made up of a base and an exponent.	
Exponent	The smaller number written to the upper right of the base that tells you how many times to multiply the base by itself.	$2^4 = 2 \times 2 \times 2 \times 2$ 4 is the exponent.
Base	The "larger" number that the exponent is applied to. (The bottom number in a power)	$2^4 = 2 \times 2 \times 2 \times 2$ 2 is the base.
Rational number	Numbers that can be written as fractions.	
Rational Exponent	The exponent on a power is a rational number (fraction). $x^{\frac{2}{3}} = \left(\sqrt[3]{x}\right)^2$	$27^{\frac{2}{3}} = \left(\sqrt[3]{27}\right)^2 = (3)^2 = 9$
Integral number	An integer {...-3,-2,-1,0,1,2,3,...}.	
Integral Exponent	The exponent on a power is an integer.	Such as x^2, x^{-3}.
Coefficient	The numbers in front of the letters in mathematical expressions.	In $3x^2$, 3 is the coefficient.
Variable	The letters in mathematical expressions.	In $3x^2$,'x' is the variable.
Undefined	If there is no good way to describe something, we say it is undefined.	$\frac{3}{0}$ is undefined because we cannot divide by zero.
Radical form	$\left(\sqrt[3]{8}\right)^2$ is in radical form.	
Exponential Form	$8^{\frac{2}{3}}$ is in exponential form.	
Zero Exponent	Any expression to the power of 0 will equal 1.	$(2xyz)^0 = 1$
Negative Exponent	Reciprocate the base and perform repeated multiplication OR use repeated division.	$5^{-3} = \left(\frac{1}{5}\right)^3 = \frac{1}{5} \times \frac{1}{5} \times \frac{1}{5} = \frac{1}{125}$
Multiply Powers with the Same base	Add the exponents.	$m^5 \times m^2 = m^7$
Dividing Powers with the same base.	Subtract the exponents.	$q^6 \div q^4 = q^2$
Power of a Power	Multiply the exponents.	$(x^2)^4 = x^8$
Power of a Product	Apply the exponent to all factors.	$(3x^2)^3 = 27x^6$
Power of a Quotient	Apply the exponent to both numerator AND denominator	$\left(\frac{a}{b}\right)^3 = \frac{a^3}{b^3}$

Introduction to Exponents

Challenge #1: Solve each riddle using any strategy that works.

1. Evaluate.
$3^2 \times 3^2$

$2+2=4$ $3^4 = \boxed{81}$

Rate the riddle:
Easy, Medium, Hard

Easy

2. Evaluate.
$2^2 \times 2^2 \div 2^3$

$= 2^4 \div 2^3$

$= 2^1 = \boxed{2}$

Rate the riddle:
Easy, Medium, Hard

Easy

3. Evaluate.
$x^3 \times x^5$

$\boxed{x^8}$

Rate the riddle:
Easy, Medium, Hard

Easy

4. Evaluate.
$8x^4 \div 4x^3$

$\frac{{}^{2}\cancel{8}x^4}{{}_{1}\cancel{4}x^3} = 2x^1 = \boxed{2x}$

Rate the riddle:
Easy, Medium, Hard

Easy

5. Find a strategy that is different from the one you used in Question 1 and solve the question again.

$3 \times 3 \times 3 \times 3 = 3^4 = \boxed{81}$
(Expand)

6. Find a strategy that is different from the one you used in Question 4 and solve the question again.

$8 \div 4 = 2$
$x^4 \div x^3 = xxxx \div xxx = x$
$= \boxed{2x}$

What is an Exponent?

Exponents are symbols that indicate an operation to be performed on the base.

positive exponents → Repeated Multiplication
negative exponents → Repeated Division

b^e b is the base, and e is the exponent. Together, we call them a *power*.

Some examples...

$2^1, 2^2, 2^3, 2^4, 2^5$ are the first five ***powers of 2***. x^1, x^2, x^3, x^4, x^5 are the first five ***powers of x***.

Your Notes Here...

Positive Integral Exponent (multiplication)	Zero Exponent	Negative Integral Exponent (repeated division)
$a^n = 1 \times a \times a \times a \times \ldots \times a$ (n factors) Eg. $3^4 = 1 \times 3 \times 3 \times 3 \times 3 = 81$	$a^0 = 1, \quad (a \neq 0)$ Eg. $5^0 = 1, \quad \left(\frac{3}{2}\right)^0 = 1$	$a^{-n} = 1 \div a^n$ $= \frac{1}{a^n}$ Eg. $5^{-2} = \frac{1}{5^2} = \frac{1}{25}$

Challenge #2

7. Evaluate each of the following and examine the pattern:

$2^4 = 16$

$2^3 = 8$

$2^2 = 4$

$2^1 = 2$

$2^0 = 1$

$2^{-1} = \frac{1}{2}$

$2^{-2} = \frac{1}{4}$

$2^{-3} = \frac{1}{8}$

$2^{-4} = \frac{1}{16}$

8. What patterns do you notice in the list you created to the left?

If you divide each by 2 (when going down) or multiply each by 2 (when going up) you will get the answer

9. Does the value of 2^0 make sense when put into this list?

yes, b/c if you use the pattern I mentioned above it makes sense.

$2 \div 2 = \boxed{1}$

10. Do negative exponents make sense in this list?

yes you just have to change the negative to a positive and put it under 1. ($2^{-2} = \frac{1}{2^2} = \boxed{\frac{1}{4}}$)

11. Why might people say negative exponents mean "repeated division?"

Because going from a negative exponent to an even more negative exponent just means divide by 2 here. (you divide by 2 over and over = "repeatedly")

12. Identify the base in the following equation.

$4^3 = 64$

$\boxed{4}$

13. Identify the power in the following equation.

$2^5 = 32$

$\boxed{2^5}$

14. Identify the exponent in the following equation.

$-3^2 = -9$

$\boxed{2}$

15. Which of the following is equivalent to -16?

$-4^2 = -16$

$(-4)^2 = +16$

$4^{-2} = \frac{1}{16}$

$-4^{-2} = \frac{1}{-4^2} = \frac{1}{-16}$

16. Which of the following is equivalent to -81?

$-9^2 = -81$

$(-3)^4 = +81$

$9^{-2} = \frac{1}{81}$

$-3^{-4} = \frac{1}{-3^4} = \frac{1}{-81}$

17. Which of the following are equivalent to 1.

-3^0 → $-1\times1=-1$

$\frac{2x^3}{2x^3}$ → $1x^0 = 1\times1=1$

$(5x)^0$ → $+1$

correction

All

18. Which of the following is equivalent to 9?

$-3^2 = -9$

$(-3)^2 = +9$

$3^{-2} = \frac{1}{9}$

$(-3)^{-2} = \frac{1}{(-3)^2} = \frac{1}{9}$

19. Evaluate.

-2^6

$$= -1 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2$$
$$= -64$$

$-2^6 = -1 \times 2^6$

$-1 \times 64 = \boxed{-64}$

20. Evaluate.

$(-3)^3$

$-3 \times -3 \times -3 = \boxed{-27}$

21. -4^2

$-1 \times 16 = \boxed{-16}$

22. $(-4)^{-2}$

$\frac{1}{(-4)^2} \rightarrow \boxed{\frac{1}{16}}$

23. -4^{-2}

$\frac{1}{-4^2} = \frac{1}{-1\times16} = \boxed{\frac{1}{-16}}$

☆BOOK SAYS $-\frac{1}{16}$☆

24. 3^{-4} $= \frac{1}{3^4} = \boxed{\frac{1}{81}}$

$$= \frac{1}{3^4} \qquad = 1 \div 3 \div 3 \div 3 \div 3$$

$$= 1 \times \frac{1}{3} \times \frac{1}{3} \times \frac{1}{3} \times \frac{1}{3}$$

$$= \frac{1}{81} \qquad = \frac{1}{81}$$

25. $(-3)^{-4}$

$\frac{1}{(-3)^4} = \frac{1}{-3\times-3\times-3\times-3}$

$= \boxed{\frac{1}{81}}$

26. -3^{-4}

$\frac{1}{-3^4} = \frac{1}{-1\times3^4} = \boxed{\frac{1}{-81}}$

☆BOOKS SAYS $-\frac{1}{81}$☆

27. 4^2 $4\times4=16$

$\boxed{16}$

28. $(-4)^2$

$-4 \times -4 = \boxed{16}$

29. $-(4)^2$

$-1\times4\times4 = -1\times16$

$= \boxed{-16}$

30. 5^0 $= \boxed{1}$

31. -5^0 $= -1\times1 = \boxed{-1}$

32. $\left(\frac{34a^2}{2x}\right)^0$ $= \frac{1}{1} = \boxed{1}$

✓

The Exponent Laws:

Challenge #3

33. Multiply.

$a^3 \times a^6$

$3+6=9$

$\boxed{a^9}$

Explain your steps.

when bases are the same and powers are being multiplied, add exponents

Challenge #4

34. Divide.

$g^7 \div g^3$

$7-3=4$

$g^7 \div g^3$

$= \boxed{g^4}$

Explain your steps.

when bases are the same and powers are being divided, subtract exponents.

Challenge #5

35. Multiply.

$5m^4 \times 3m^2$

$5m^4 \times 3m^2$

$(5 \times 3) \times (m^4 \times m^2)$

$\boxed{15m^6}$

Explain your steps.

when powers are multiplied, and bases are the same, multiply the coefficients and add the exponents.

CORRECTION: $\frac{2^3}{3^3} \times \frac{(-6)^2}{4^2} = \frac{8}{27} \times \frac{36}{16} = \frac{4}{6} = \boxed{\frac{2}{3}}$

Simplify the following, write your answers using exponents.

36. $a^3 \times a^6$
$= a^{3+6}$
$= a^9$

37. $a^2 \times a^{-4}$
$2 + -4$
$\boxed{a^{-2}}$

38. $f^2 \times f^x$
$\cancel{f^{2x}}$ → CORRECTION: $\boxed{f^{2+x}}$

39. $x^{\frac{1}{4}} \times x^{\frac{6}{8}}$ ★USING EXPONENTS★
$\frac{1}{4} = \frac{2}{8} + \frac{6}{8} = \frac{8}{8}$
$x^{\frac{8}{8}} = x^1 = \cancel{x} = \boxed{x^1}$

40. $2^3 \times 2^{-5}$
$3 + -5$
$2^3 \times 2^{-5} = \boxed{2^{-2}}$

41. $g^7 \div g^3$
$= g^{7-3}$
$= g^4$

42. $m^4 \div m^0$
$m^4 \div m^0 = \boxed{m^4}$

43. $t^0 \div t^{-5}$
$0 - (-5) = 0 + 5 = 5$
$t^0 \div t^{-5} = \boxed{t^5}$

44. $\frac{x^{13}}{x^3}$
$x^{13} \div x^3 = \boxed{x^{10}}$

45. $5m^4 \times 3m^2$
$= 5 \times 3 \times m^{4+2}$
$= 15m^6$

46. $-10x^4 \div -2x^{-2}$
$= (-10 \div -2) \times (x^4 \div x^{-2})$
$= 5 \times x^6$
$= \boxed{5x^6}$

47. $\frac{4a^{4}{}^{2}}{-8a^{2}}$ -2
$\boxed{-\frac{1a^2}{2}} = \boxed{-\frac{a^2}{2}}$

48. $\frac{2}{3}x^3 \times \frac{6}{5}x^4$
$\left(\frac{2}{3} \times \frac{6}{5}\right) \times (x^3 \times x^4)$
$\frac{4}{5} \times x^7 = \boxed{\frac{4x^7}{5} \text{ or } \frac{4}{5}x^7}$
★ ★ ★

49. $\frac{2}{a^3} \div \frac{6}{a^6}$
$\frac{2}{a^3} \div \frac{6}{a^6}$
$\frac{2}{a^3} \times \frac{a^6}{6} = \boxed{\frac{a^3}{3}}$

50. Evaluate.
$\left(\frac{2}{3}\right)^3 \left(\frac{-6}{4}\right)^2$?
$\frac{8}{27} \times \frac{-36}{16} = \boxed{\frac{-4}{6}} = \frac{-2}{3}$ no negative?
$\boxed{\frac{2}{3}}$

Multiplying Powers with the same Base:	Dividing Powers with the same Base:
Add the exponents. Eg. $x^5 \times x^2 = x^{5+2} = x^7$ $a^{\frac{2}{3}} \times a^{\frac{1}{3}} = a^{\frac{3}{3}} = a^1 = a$ $3x^2 \times 2x^5 = 3 \times 2 \times x^2 \times x^5 = 6x^7$	Subtract the exponents. Eg. $d^4 \div d^3 = d^{4-3} = d^1 = d$ $\frac{y^6}{y^{-2}} = y^{6-(-2)} = y^8$

✓

Challenge #6

51. Evaluate. = get # answer ★

$(5^2)^3$

$2 \times 3 = 6$

$\boxed{5^6} \rightarrow 5^6 = \boxed{15625}$

[Power of a Power]

Explain your steps.

When a power is raised to an exponent, multiply the exponents

15625

Challenge #7

52. Simplify.

$(m^3)^2$

$3 \times 2 = 6$

$\boxed{m^6}$

[Power of a Power]

Explain your steps.

When a power is raised to an exponent, multiply the exponents

Challenge #8

53. Simplify.

$(2m^4)^3$

$2^3 \times m^{4 \times 3}$

$8 \times m^{12}$

$\boxed{8m^{12}}$

[Power of a ~~Product~~]
Power?

Explain your steps.

When a power is raised to an exponent, put the exponent on the coefficient and evaluate and multiply the exponents.

√

★ A number multiplied by 0 is 0 ★

★ not negative? ★

$-3^{-4} \times x^{-2\times-4} \times y^{3\times-4}$

$\frac{1}{-3^4} \times x^8 \times y^{-12}$

$\frac{1}{-81} \times \frac{x^8}{1} \times \frac{1}{y^{12}} = \boxed{\frac{x^8}{-81y^{12}}}$

Simplify the following.

54. $(m^3)^2$

$= m^3 \times m^3$
$= m^6$

$= m^{3\times2}$
$= m^6$

55. $(t^4)^0$

$t^{4\times0}$

$t^0 = \boxed{1}$

56. $(x^2y^3)^{-3}$

$x^{2\times-3} \times y^{3\times-3}$

$x^{-6} \times y^{-9}$

★ $\boxed{x^{-6}y^{-9}}$ ★ $\to \boxed{\frac{1}{x^6y^9}}$

NOT DONE!!

57. $(2m^4)^3$

$2m^4 \times 2m^4 \times 2m^4$
$= 2 \times 2 \times 2 \times m^4 \times m^4 \times m^4$
$= 8m^{12}$

OR

$= 2^3m^{4\times3}$
$= 8m^{12}$

58. $(2c^4d^3)^{-3}$

$2^{-3} \times c^{4\times-3} \times d^{3\times-3}$

$\frac{1}{8} \times c^{-12} \times d^{-9}$

$\frac{1}{8}c^{-12}d^{-9} = \frac{1}{8} \times \frac{1}{c^{12}d^9}$ $\boxed{\frac{1}{8c^{12}d^9}}$

★ do these go together?

59. $(-3x^{-2}y^3)^{-4}$

$-3^{-4} \times x^{-2\times-4} \times y^{3\times-4}$

$\frac{1}{-3^4} \times x^8 \times y^{-12}$

$\frac{1}{-81} \times x^8 \frac{1}{y^{12}}$

$\frac{1}{-81} \times \frac{1}{y^{12}} \times x^8$

$\boxed{\frac{1}{-81y^{12}}x^8}$ X

$(-3x^{-2}y^3)^{-4}$

$(-3^{-4}x^{-2\times-4}y^{3\times-4}$

$\frac{1}{(-3)^4} \times x^8 y^{-12}$

$\frac{1}{(-3)^4} \times \frac{x^8}{1} \times \frac{1}{y^{12}}$

$\boxed{\frac{x^8}{81y^{12}}}$

60. $(3x^{-2}y^{-3})^{-3}$

$3^{-3} \times x^{-2\times-3} \times y^{-3\times-3}$

$\frac{1}{27} \times x^6 \times y^9$

$\boxed{\frac{x^6y^9}{27}}$

61. $(-2xy^3)(-3x^2y^3)^2$

$= -2xy^3 \times (-3)^2 x^4y^6$

$= -2xy^3 \times 9x^4y^6$

$= -18x^{1+4}y^{3+6}$

$= \boxed{-18x^5y^9}$

62. $(2a^2)^3(4a^3b)^2$

$2^3a^6 \times 4^2a^6b^2$

$8a^6 \times 16a^6b^2$

$128a^{6+6}b^2$

$\boxed{128a^{12}b^2}$

Power of a Power:
Multiply the exponents.

Eg$(5^2)^3 = (5 \times 5)^3$.
$= (5 \times 5)(5 \times 5)(5 \times 5)$
$= 5 \times 5 \times 5 \times 5 \times 5 \times 5$
$= 5^6$

THE RULE:

$$(a^m)^n = a^{m\times n}$$

If you have a power of a power … multiply exponents.

Eg. $(x^2)^5 = x^{2\times5} = x^{10}$

Power of a Product:
Apply the exponent to all factors.

Eg. $(5 \times 2)^3$
$= (5 \times 2) \times (5 \times 2) \times (5 \times 2)$
$= 5 \times 5 \times 5 \times 2 \times 2 \times 2$
$= 5^3 \times 2^3$

THE RULE:

$$(ab)^m = a^mb^m$$

If you have a power of a product … apply the exponent to EVERY factor in the product.

Eg. $(a^2b^3)^{-3} = a^{2\times-3}b^{3\times-3} = a^{-6}b^{-9}$

$(2c^4d^3)^{-3} = 2^{-3}c^{4\times-3}d^{3\times-3}$

$= \frac{1}{8}c^{-12}d^{-9} = \frac{1}{8} \times \frac{1}{c^{12}} \times \frac{1}{d^9} = \boxed{\frac{1}{8c^{12}d^9}}$

✓

Challenge #9

63. Evaluate.

$$\left(\frac{2}{5}\right)^3$$

$\frac{2^3}{5^3} = \boxed{\frac{8}{125}}$

$\frac{8}{125}$

Explain your steps.

Apply exponent to numerator and denominator

Challenge #10

64. Evaluate.

$$\left(\frac{2}{5}\right)^{-3} = \frac{5^3}{2^3} = \frac{125}{8}$$

$\frac{2^{-3}}{5^{-3}} = \frac{1}{2^3} \div \frac{1}{5^3}$

$= \frac{1}{8} \div \frac{1}{125} = \frac{1}{8} \times \frac{125}{1}$

$= \boxed{\frac{125}{8}}$

$\frac{125}{8}$

Explain your steps.

1) Apply exponent to numerator and denominator

2) flip reprical

3) Simplify / multiply

Challenge #11

65. Simplify.

$$\left(\frac{x}{2}\right)^3$$

$\frac{x^3}{2^3} = \boxed{\frac{x^3}{8}}$

Explain your steps.

Apply exponent to numerator (variable) and denominator

Challenge #12

Can divide 8 and 6 first!

66. Simplify.

$$\left(\frac{6x^5y^3}{8y^4}\right)^{-2}$$

$\frac{8^2 y^{8^2}}{6^2 x^{10} y^6}$

$\frac{\cancel{64}^{16} y^2}{\cancel{36}^{9} x^{10}}$

$\boxed{\frac{16y^2}{9x^{10}}}$

$\frac{6^{-2} x^{-10} y^{-6}}{8^{-2} y^{-8}}$

$= \left(\frac{1}{36} \times \frac{1}{x^{10}} \times \frac{1}{y^6}\right) \div \left(\frac{1}{64} \times \frac{1}{y^8}\right)$

$\frac{1}{64y^8}$

$\frac{1}{\cancel{36}^{9} x^{10} \cancel{y^6}} \times \frac{\cancel{64}^{16} \cancel{y^8}^{2}}{1}$

$\boxed{\frac{16y^2}{9x^{10}}}$

Explain your steps.

Apply exponent to the numerator and denominator.

Power of a Quotient:

Apply the exponent to numerator AND denominator.

Eg. $\left(\frac{2}{5}\right)^3 = \left(\frac{2}{5}\right) \times \left(\frac{2}{5}\right) \times \left(\frac{2}{5}\right)$

$= \frac{2\times2\times2}{5\times5\times5}$

$= \frac{2^3}{5^3}$ If asked to write using exponents

$= \frac{8}{125}$ If asked to simplify.

$\left(\frac{2}{5}\right)^{-3}$ The negative exponent means "flip the base".

$= \frac{5\times5\times5}{2\times2\times2}$

$= \frac{5^3}{2^3}$

$= \frac{125}{8}$

THE RULE:

$$\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}$$

$$\left(\frac{a}{b}\right)^{-m} = \frac{b^m}{a^m}$$

Simplify the following.

67. $\left(\frac{x}{2}\right)^3$

$= \frac{x^3}{2^3}$

$= \frac{x^3}{8}$

68. $\left(\frac{a}{b}\right)^4$

$\boxed{\frac{a^4}{b^4}}$

69. $\left(\frac{x^2}{y^3}\right)^5$

$\boxed{\frac{x^{10}}{y^{15}}}$

70. $\left(\frac{-2a^2}{3y^3}\right)^3$

$\frac{(-2)^3a^6}{3^3y^9}$

$\boxed{\frac{-8a^6}{27y^9}}$

71. $\left(\frac{a^{-3}}{b^{-2}}\right)^{-2}$

$\boxed{\frac{a^6}{b^4}}$

72. $\left(\frac{4x}{3y}\right)^2$

$\frac{4^2x^2}{3^2y^2} = \boxed{\frac{16x^2}{9y^2}}$

73. $\left(\frac{6x^5y^3}{8y^4}\right)^{-2}$

$= \frac{(8)^2(y^4)^2}{(6)^2(x^5)^2(y^3)^2}$

$= \frac{64y^8}{36x^{10}y^6}$

$= \frac{16y^2}{9x^{10}}$

74. $\left(\frac{5ab^2c^3}{2a^{-2}c^{-3}}\right)^2$

$\left(\frac{5ab^2c^3}{2a^{-2}c^{-3}}\right)^2 = \left(\frac{5a^3b^2c^6}{2}\right)^2$

$= \boxed{\frac{25a^6b^4c^{12}}{4}}$

75. $\left[\left(\frac{2m^2n^2}{mn^3}\right)^{-1}\right]^3$

$\left(\frac{mn^3}{2m^2n^2}\right)^3$

$\frac{m^3n^9}{8m^6n^6}$ $\boxed{\frac{n^3}{8m^3}}$

$\boxed{\text{REMEMBER: } a = a^1}$

Simplify the following.

76. $\left(\frac{6ab^3}{2ab}\right)^3$

$\frac{6^3a^3b^9}{2^3a^3b^3}$

$= \frac{\overset{27}{\cancel{216}}\cancel{a^3}b^{\cancel{9}\,6}}{{}_1\cancel{8}\cancel{a^3}\cancel{b^3}} \rightarrow \boxed{27b^6}$

77. $\left(\frac{4x^{-3}y^4}{8x^2y^{-2}}\right)^{-2}$

$\left(\frac{8x^2y^{-2}}{4x^{-3}y^4}\right)^2$

$\frac{{}^4\cancel{64}x^4x^{+4}}{{}_1\cancel{16}x^{-6}y^{8\,12}}$

$= \boxed{\frac{4x^{10}}{y^{12}}}$

78. Show why $\frac{2a^2}{b^3}$ is the same as $2a^2 \times b^{-3}$.

$\frac{2a^2}{b^3} = \frac{2a^2}{1} \times \frac{1}{b^3}$

$= \frac{2a^2}{b^3} = \frac{2a^2}{b^3}$

79. Show why $\frac{12x^3}{y}$ is the same as $12x^3 \times y^{-1}$.

$\frac{12x^3}{y} = \frac{12x^3}{1} \times \frac{1}{y^1}$

$\frac{12x^3}{y} = \frac{12x^3}{y}$

Challenge #13

80. Write the following without using <u>any</u> negative exponents.

$3a^2b^{-5}$

$\frac{3a^2}{1} \times \frac{1}{b^5}$

$= \boxed{\frac{3a^2}{b^5}}$

81. Write the following without using <u>any</u> negative exponents.

$\frac{3}{a^{-2}b^5}$

$\boxed{\frac{3a^2}{b^5}}$

Challenge #14

82. Simplify using positive exponents.

$\left(\frac{2x^{-2}y^4}{x^{-3}y^3}\right)^{-3}$

$\left(\frac{x^{-3}y^3}{2x^{-2}y^4}\right)^3$

$\frac{\cancel{x^{-9}}\cancel{y^9}}{8x^{\cancel{-6}\,3}y^{\cancel{12}\,3}}$

$= \boxed{\frac{1}{8x^3y^3}}$

Explain your steps

1. Flip/recipricol to make exponent (-3) positive.
2. Apply exponent to numerator and denominator
3. simplify

$3^2 = \frac{1}{3^{-2}}$ / $3^{-2} = \frac{1}{3^2}$ $\frac{3x}{yz^{-2}} = \frac{3xz^2}{y}$

Writing Expressions with Positive Exponents. (Why? Because it is standard practice.)

An expression with powers is simplified if there are no brackets and no negative exponents.

Sometimes you will use the laws above and end up with an answer with negative exponents. The quick way to convert a negative exponent into a positive exponent is to ***move it across the division line***. The solution in question 83 shows why this works.

Simplify the following. (No brackets, no negative exponents)

83. $3a^2b^{-5}$

$$= 3a^2 \times \frac{1}{b^5}$$

$$= \frac{3a^2}{b^5}$$

84. a^2b^{-3}

$\frac{a^2}{1} \times \frac{1}{b^3} = \boxed{\frac{a^2}{b^3}}$

85. $\frac{2xy^5}{x^{-4}}$

$\frac{2xy^5}{1} \div \frac{x^{-4}}{1}$

$\frac{2xy^5}{1} \times \frac{1}{xy^3} = \cancel{\boxed{\frac{2y^5}{x^3}}}$

$= \frac{2xy^5}{x^{-4}} = \frac{2xy^5x^4}{} \boxed{2x^5y^5}$

86. $3a^2b^{-3}c^{-5}$

$\boxed{\frac{3a^2}{b^3c^5}}$

87. $(x^4y^{-3}z^{-1})^{-2}$

$\left(\frac{1}{x^4y^{-3}z^{-1}}\right)^2$

$\frac{1}{x^8y^{-6}z^{-2}} = \boxed{\frac{y^6z^2}{x^8}}$

88. $\frac{(3x^{-3}y^{-5})^2}{2xy}$

$\frac{3^2x^{-6}y^{-10}}{2xy}$ $\frac{9}{2 \times x^1 \times y^1 \times x^6 \times y^{10}}$

$= \boxed{\frac{9}{2x^7y^{11}}}$

89. $\left(\frac{2x^{-2}y^4}{x^{-3}y^3}\right)^{-3}$

$$= \left(\frac{x^{-3}y^3}{2x^{-2}y^4}\right)^3$$

$$= \frac{x^{-9}y^9}{8x^{-6}y^{12}}$$

$$= \frac{x^{-3}y^{-3}}{8}$$

$$= \frac{1}{8x^3y^3}$$

90. $\left(\frac{2a^3b^2}{4a^{-2}b^{-1}}\right)^{-3}$

$\frac{\left(\frac{4a^{-2}b^{-1}}{8a^3b^2}\right)^3}{2}$

$\frac{(2a^{-5}b^{-3})^3}{2}$

$\frac{2^3a^{-15}b^{-9}}{2}$

$\frac{8a^{-15}b^{-9}}{2}$

$= \frac{4}{1} \times \frac{1}{a^{15}} \times \frac{1}{b^9} = \boxed{\frac{4}{a^{15}b^9}}$

91. $\frac{(4m^2n^2)(7m^{-3}n^2)}{14mn^5}$

$\frac{4 \times m^2 \times n^2 \times 7 \times m^{-3} \times n^2}{14 \times m \times n^5}$

$\frac{28m^{-1}n^4}{14mn^5}$

$\frac{28n^4}{14mn^5m^1}$

$\frac{28n^4}{14m^2n^5}$

$\boxed{\frac{2}{m^2n}}$

92. Why does moving a power across the division line in a fraction change the sign on the exponent?

B/c the base is reciprocated

Simplify the following. (No brackets, no negative exponents)

93. $\left(\frac{12x^3y^{-1}}{-8x^{-1}y^5}\right)^{-2}$

$\left(\frac{-8x^{-1}y^5}{12x^3y^{-1}}\right)^2$

$\left(\frac{(-2)x^{-4}y^6}{3}\right)^2$

$\frac{4x^{-8}y^{12}}{3^2}$

$\boxed{\frac{4y^{12}}{9x^8}}$

94. $\left(\frac{4a^3b^{-2}}{6a^2b^{-1}}\right)^{-3}$

$\left(\frac{6a^2b^{-1}}{4a^3b^{-2}}\right)^3$

$\frac{216a^6b^{-3}}{64a^9b^{-6}}$

$\frac{27}{8}\frac{\cancel{216}b^3}{\cancel{64}a^3}$

$= \boxed{\frac{27b^3}{8a^3}}$

95. $\left(\frac{8x^2y^{-3}}{4x^{-1}y^{-5}}\right)^{-3}$

$\left(\frac{4x^{-1}y^{-5}}{8x^2y^{-3}}\right)^3$

$\left(\frac{1}{2x^3y^2}\right)^3$

$= \boxed{\frac{1}{8x^9y^6}}$

96. $\left(\frac{12x^{-3}y^5}{16x^3y^{-2}}\right)^{-1}$

$\left(\frac{16x^3y^{-2}}{12x^{-3}y^5}\right)^1$

$\left(\frac{4x^6}{3y^7}\right)^1$

$\boxed{\frac{4x^6}{3y^7}}$

97. **Challenge #15**

If $\sqrt{9} \times \sqrt{9} = 9$,

and $9^a \times 9^a = 9$

Then what is the value of 'a'?

$\frac{1}{2}$

Explain:

$9^{\frac{1}{2}} = \sqrt[2]{9^1} = \sqrt[2]{9} \times \sqrt[2]{9} = 3 \times 3 = 9$

98. **Challenge #16**

If $\sqrt[3]{2} \times \sqrt[3]{2} \times \sqrt[3]{2} = 2$,

and $2^a \times 2^a \times 2^a = 2$

Then what is the value of 'a'?

$\frac{1}{3}$

Explain:

$2^{\frac{1}{3}} = \sqrt[3]{2^1} = \sqrt[3]{2} \times \sqrt[3]{2} \times \sqrt[3]{2} = 2$

99. Write a "rule" that relates a **rational** (fraction) **exponent** to an equivalent **radical expression**.

A rational (fraction) exponent can be written as an equivalent radical expression by making the denominator the index and the numerator the exponent of the radicand.

Ex. $x^{\frac{1}{2}} = \sqrt[2]{x^1} \rightarrow \sqrt[2]{x}$

$\sqrt{}$ square root "2" is implied

index $\sqrt[3]{}$ The denominator in a rational exponent is the index.

☆ DENOMINATOR IS THE INDEX! ☆

Rational Exponents in the form: $x^{\frac{1}{n}}$

radical form

$\sqrt[5]{x} = x^{\frac{1}{5}}$ → exponential form.

Remember, *rational* often refers to fractions.

What does a rational exponent mean?

Recall: $\sqrt{9} \times \sqrt{9} = 9$.

But $9^{\frac{1}{2}} \times 9^{\frac{1}{2}} = 9$

And $3 \times 3 = 9$

So, $\sqrt{9} = 9^{\frac{1}{2}} = 3$

If $\sqrt[3]{2} \times \sqrt[3]{2} \times \sqrt[3]{2} = 2$

But $2^{\frac{1}{3}} \times 2^{\frac{1}{3}} \times 2^{\frac{1}{3}} = 2$

So, $\sqrt[3]{2} = 2^{\frac{1}{3}}$

100. Write another statement like the one to the left.

Recall: $\sqrt{16} \times \sqrt{16} = 16$

But: $16^{\frac{1}{2}} \times 16^{\frac{1}{2}} = 16$

And: $4 \times 4 = 16$

So: $\sqrt{16} = 16^{\frac{1}{2}} = 4$

If: $\sqrt[3]{8} \times \sqrt[3]{8} \times \sqrt[3]{8} = 8$

But: $8^{\frac{1}{3}} \times 8^{\frac{1}{3}} \times 8^{\frac{1}{3}} = 8$

SO: $\sqrt[3]{8} = 8^{\frac{1}{3}}$

The Rule...

$$a^{\frac{1}{n}} = \sqrt[n]{a} \quad \text{and} \quad a^{-\frac{1}{n}} = \frac{1}{\sqrt[n]{a}}$$

Evaluate or simplify the following.

101. $49^{\frac{1}{2}}$

$\sqrt[2]{49} = \boxed{7}$

102. $-(16^{\frac{1}{2}})$

$-1 \times \sqrt[2]{16} = \boxed{-4}$

103. $(-16)^{\frac{1}{2}}$

$\sqrt[2]{-16} = \boxed{4i}$

(no real solution)

104. $64^{\frac{1}{3}}$

$\sqrt[3]{64} = \boxed{4}$

105. $27^{-\frac{1}{3}}$

$\frac{1}{\sqrt[3]{27}} = \boxed{\frac{1}{3}}$

106. $32^{-\frac{1}{5}}$

$\frac{1}{\sqrt[5]{32}} = \boxed{\frac{1}{2}}$

107. $10000^{\frac{1}{4}}$

$\sqrt[4]{10\,000} = \boxed{10}$

108. $(4x^2)^{\frac{1}{2}}$

$\sqrt[2]{4x^2} = \boxed{2x}$

109. $(27x^6)^{-\frac{1}{3}}$

$\frac{1}{\sqrt[3]{27x^6}} = \boxed{\frac{1}{3x^2}}$

✓

Write in radical form.

110. $7^{\frac{1}{2}}$ $\boxed{\sqrt[2]{7}}$ $\sqrt{7}$

111. $(3x)^{\frac{1}{3}}$ $\boxed{\sqrt[3]{(3x)}}$ $\sqrt[3]{3x}$

112. $4^{\frac{1}{5}}$ $\boxed{\sqrt[5]{4}}$

113. $4^{-\frac{1}{5}}$ $\boxed{\frac{1}{\sqrt[5]{4}}}$

114. $-64^{\frac{1}{3}}$ $-1 \times \sqrt[3]{64}$ $= \boxed{-\sqrt[3]{64}}$

115. $64^{-\frac{1}{3}}$ $\boxed{\frac{1}{\sqrt[3]{64}}}$

Write in exponential form.

116. $\sqrt[2]{13}$ $\boxed{13^{\frac{1}{2}}}$

117. $-3\sqrt{x}$ $\boxed{-3x^{\frac{1}{2}}}$

118. $\sqrt[2]{2y}$ $\boxed{(2y)^{\frac{1}{2}}}$

119. $\sqrt[4]{4}$ $\boxed{4^{\frac{1}{4}}}$

120. $\sqrt[7]{4}$ $\boxed{4^{\frac{1}{7}}}$

121. $\frac{1}{\sqrt[5]{3x}}$ $\boxed{(3x)^{-\frac{1}{5}}}$

Consider the following...

Step 1: $32^{\frac{3}{5}} = \left(32^{\frac{1}{5}}\right)^3$

Step 2: $32^{\frac{3}{5}} = \left(\sqrt[5]{32}\right)^3$

Step 3: $32^{\frac{3}{5}} = (2)^3$

Step 4: $32^{\frac{3}{5}} = 8$

122. Challenge #17. Complete the following as shown above.

Step 1: $27^{\frac{2}{3}} = \left(27^{\frac{1}{3}}\right)^2$

Step 2: $27^{\frac{2}{3}} = \left(\sqrt[3]{27}\right)^2$

Step 3: $27^{\frac{2}{3}} = (3)^2$

Step 4: $27^{\frac{2}{3}} = 9$

Explain: - Make $\frac{2}{3} \rightarrow \left(\frac{1}{3}\right)^2 = \frac{1}{3} \times \frac{2}{1} = \frac{2}{3}$
- Turn into equivalent radical expression
- solve radical
- solve exponent.

Rational Exponents in the form: $x^{\frac{m}{n}}$ where m is not 1.

Consider the power $27^{\frac{2}{3}}$. To understand the meaning of the rational exponent we can use the exponent law:
$(a^m)^n = a^{m\times n}$.

If we take $27^{\frac{2}{3}}$ and split the exponent into two parts we get the following...

$$27^{\frac{2}{3}} = \left(27^{\frac{1}{3}}\right)^2$$

This can then be written as...

$$\left(\sqrt[3]{27}\right)^2$$

The power can be evaluated from this point...

$$\left(\sqrt[3]{27}\right)^2 = (3)^2 = 9$$

The Rule...

$$a^{\frac{m}{n}} = \sqrt[n]{a^m} = \left(\sqrt[n]{a}\right)^m \qquad and \qquad a^{-\frac{m}{n}} = \frac{1}{\sqrt[n]{a^m}} = \frac{1}{\left(\sqrt[n]{a}\right)^m}$$

Two more examples:

Eg.1 Evaluate $8^{\frac{2}{3}}$ without using a calculator.

$$8^{\frac{2}{3}} = \left(8^{\frac{1}{3}}\right)^2 = \left(\sqrt[3]{8}\right)^2 = (2)^2 = 4$$

Means square of the cube root of 8.

Eg.2 Evaluate $9^{-\frac{3}{2}}$ without using a calculator.

$$9^{-\frac{3}{2}} = \left(\frac{1}{9}\right)^{\frac{3}{2}} = \frac{\left(1^{\frac{1}{2}}\right)^3}{\left(9^{\frac{1}{2}}\right)^3} = \frac{1}{(\sqrt{9})^3} = \frac{1}{(3)^3} = \frac{1}{27}$$

Means "the reciprocal" of the cube of the square root of 9.

Write each of the following using radicals. (Do not evaluate)

123. $4^{\frac{2}{5}}$

$\boxed{\sqrt[5]{4^2}}$

124. $4^{\frac{3}{5}}$

$\boxed{\sqrt[5]{4^3}}$

125. $4^{\frac{4}{5}}$

$\boxed{\sqrt[5]{4^4}}$

126. $4^{-\frac{2}{5}}$

$\boxed{\frac{1}{\sqrt[5]{4^2}}}$

127. $4^{-\frac{3}{5}}$

$\boxed{\frac{1}{\sqrt[5]{4^3}}}$

128. $4^{-\frac{4}{5}}$

$\boxed{\frac{1}{\sqrt[5]{4^4}}}$

Evaluate each of the following.

129. $4^{\frac{1}{2}}$

$4^{\frac{1}{2}} = \sqrt{4} = \boxed{2}$

130. $125^{\frac{1}{3}}$

$125^{\frac{1}{3}} = \sqrt[3]{125} = \boxed{5}$

131. $8^{\frac{2}{3}}$

$(8^{\frac{1}{3}})^2$

$(\sqrt[3]{8})^2 = (2)^2 = \boxed{4}$

132. $81^{\frac{3}{4}}$

$(81^{\frac{1}{4}})^3$

$(\sqrt[4]{81})^3$

$(3)^3 = \boxed{27}$

133. $4^{\frac{3}{2}}$

$(4^{\frac{1}{2}})^3$

$(\sqrt[2]{4})^3 = (2)^3 = \boxed{8}$

134. $16^{-\frac{3}{4}}$

$\frac{1}{(16^{\frac{1}{4}})^3} = \frac{1}{(\sqrt[4]{16})^3} = \frac{1}{(2)^3}$

$= \boxed{\frac{1}{8}}$

135. $(-27)^{-\frac{2}{3}}$

$\frac{1}{((-27)^{\frac{1}{3}})^2}$

$\frac{1}{(\sqrt[3]{-27})^2} = \frac{1}{(-3)^2} = \boxed{\frac{1}{9}}$

136. $(-8)^{-\frac{5}{3}}$

$\frac{1}{((-8)^{\frac{1}{3}})^5} = \frac{1}{(\sqrt[3]{-8})^5} = \frac{1}{(-2)^5}$

$= \boxed{\frac{1}{-32}}$

137. $9^{2.5}$

$9^{\frac{5}{2}} = (9^{\frac{1}{2}})^5 = (\sqrt[2]{9})^5$

$= (3)^5 = \boxed{243}$

$\left(\sqrt[3]{\frac{8}{27}}\right)^2 = \left(\frac{2}{3}\right)^2$

$= \boxed{\frac{4}{9}}$

138. $(-1)^{-\frac{8}{5}}$

$= \frac{1}{((-1)^{\frac{1}{5}})^8}$

$= \frac{1}{(\sqrt[5]{-1})^8} = \frac{1}{(-1)^8}$

$= \frac{1}{1} = \boxed{1}$

139. $\left(\frac{100}{9}\right)^{\frac{3}{2}}$

$(100^{\frac{1}{2}})^3$ $\quad \sqrt[2]{100^3}$

$(\sqrt[2]{100})^3$

$(10)^3 = 1000$

$(9^{\frac{1}{2}})^3$

$(\sqrt[2]{9})^3$

$(3)^3$

$= 27 \rightarrow \boxed{\frac{1000}{27}}$

140. $\left(\frac{27}{8}\right)^{-\frac{2}{3}}$ → JUST FLIP ☆

$\cancel{\frac{1}{((\frac{27}{8})^{\frac{1}{3}})^2} = \frac{1}{(\sqrt[3]{\frac{27}{8}})^2}}$

$\cancel{\frac{1}{((\frac{3}{2}))^2} = \frac{1}{(\frac{9}{4})} = \boxed{\frac{1}{\frac{9}{4}}}}$

$\frac{1}{(\sqrt[3]{27})^2} = \frac{1}{3^2} = \frac{1}{9}$

$\frac{1}{(\sqrt[3]{8})^2} = \frac{1}{2^2} = \frac{1}{4}$

$= \frac{1}{9} \div \frac{1}{4} = \frac{1}{9} \times \frac{4}{1} = \boxed{\frac{4}{9}}$ ✓

Write each of the following using exponents. (Do not evaluate)

Eg. $\sqrt{12} = 12^{\frac{1}{2}}$ Eg. $\left(\sqrt[3]{7}\right)^4 = 7^{\frac{4}{3}}$ Eg. $\frac{1}{\left(\sqrt[3]{7}\right)^2} = 7^{-\frac{2}{3}}$

141. $\sqrt{7}$

$\boxed{7^{\frac{1}{2}}}$

142. $\sqrt[3]{34}$

$\boxed{34^{\frac{1}{3}}}$

143. $\sqrt[3]{-11}$

$\boxed{(-11)^{\frac{1}{3}}}$

144. $\sqrt[5]{a^2}$

$\cancel{(a^2)}^{\frac{1}{5}} = \boxed{a^{\frac{2}{5}}}$

145. $\sqrt[3]{6^4}$

$\boxed{6^{\frac{4}{3}}}$

146. $\left(\sqrt[3]{x}\right)^2$

$\left(x^{\frac{1}{3}}\right)^2 = \boxed{x^{\frac{2}{3}}}$

147. $\left(\sqrt[5]{6}\right)^3$

$\left(6^{\frac{1}{5}}\right)^3 = \boxed{6^{\frac{3}{5}}}$

148. $\left(\sqrt[4]{2x}\right)^5$

$\left((2x)^{\frac{1}{4}}\right)^5$

$= \boxed{(2x)^{\frac{5}{4}}}$

149. $\frac{1}{\sqrt[3]{a}}$

$\boxed{a^{-\frac{1}{3}}}$

$(2b^3)^{\frac{1}{3}} = 2^{\frac{1}{3}} \cdot b$

$2^{\frac{1}{3}} \times b^{\frac{3}{1} \times \frac{1}{3}} = 2^{\frac{1}{3}} b^{\frac{3}{3}}$

$= 2^{\frac{1}{3}} b^{\frac{1}{1}}$

$\boxed{2^{\frac{1}{3}} b}$

150. $\frac{1}{\left(\sqrt[5]{x}\right)^4}$

$\left(x^{-\frac{1}{5}}\right)^4$

$= \boxed{x^{-\frac{4}{5}}}$

151. $\frac{1}{\sqrt[4]{x^3}}$

$\boxed{x^{-\frac{3}{4}}}$

152. $\sqrt[3]{2b^3}$

$2b^{\frac{3}{3}}$

$\boxed{2b^{\frac{1}{1}}}$?

Evaluate if possible.

153. $(-9)^{\frac{1}{2}}$

$\sqrt[2]{-9} = 3i$

$\boxed{\text{no solution}}$

154. $100000^{\frac{3}{5}}$

$\left(\sqrt[5]{100000}\right)^3$

$= (10)^3 = \boxed{1000}$

155. $\left(\frac{27}{8}\right)^{\frac{2}{3}}$

$27^{\frac{2}{3}} = \left(\sqrt[3]{27}\right)^2 = (3)^2 = 9$

$8^{\frac{2}{3}} = \left(\sqrt[3]{8}\right)^2 = (2)^2 = 4$

$= \boxed{\frac{9}{4}}$

156. $3^{\frac{1}{2}} \times 3^{\frac{1}{2}}$

$\sqrt[2]{3} \times \sqrt[2]{3} = \sqrt{3} \times \sqrt{3}$

$= \sqrt{9} = \boxed{3}$

157. $-9^{\frac{1}{2}}$

$-1 \times \sqrt[2]{9} = -1 \times 3$

$= \boxed{-3}$

158. $(2^5)^{0.4}$

$(2^5)^{\frac{4}{10}} = (2^5)^{\frac{2}{5}}$

$\left(\sqrt[5]{32}\right)^2 = (2)^2 = \boxed{4}$

$\sqrt[2]{7} \times \sqrt[2]{7}$

Evaluate if possible.

159.

a. $-8^{\frac{4}{3}}$

$-1 \times \left(\sqrt[3]{8}\right)^4$

$= -1 \times (2)^4 = -1 \times 16$

$= \boxed{-16}$

b. $(-8)^{\frac{4}{3}}$

$\left(\sqrt[3]{(-8)}\right)^4 = (-2)^4 = \boxed{16}$

What important rule is explored above? The exponent only effects the thing closest to it.

160. $4^{\frac{3}{2}} \div 16^{\frac{1}{4}}$

$\left(\sqrt[2]{4}\right)^3 \div \sqrt[4]{16}$

$(2)^3 \div 2$

$8 \div 2 = \boxed{4}$

161. $(-1)^{-\frac{3}{2}}$

$\frac{1}{\left(\sqrt[3]{-1}\right)^3} = \frac{1}{i^3}$

no real solution

162. $(\sqrt[3]{5^2})(\sqrt[3]{5})$

$\cancel{\sqrt[3]{5^2}}$? $\sqrt{5}$?

$\left(\sqrt[3]{5}\right)^2\left(\sqrt[3]{5}\right)$?

$= \sqrt[3]{5^2} = \sqrt[3]{25} \times \sqrt[3]{5} = \sqrt[3]{5} \times \sqrt[3]{5} \times \sqrt[3]{5} = \boxed{5}$ ✓

163. $(\sqrt[4]{16})(\sqrt[5]{32})$

$(2)(2) = \boxed{4}$

164. $\sqrt{\sqrt[3]{729}}$

$\sqrt{9} = \boxed{3}$

165. Evaluate to two decimal places using a calculator

$\frac{1}{\sqrt[5]{300}}$

$\boxed{\frac{1}{3.13}} = \boxed{0.32}$

166. Evaluate to two decimal places using a calculator

$\frac{5}{\sqrt[6]{256}}$

$\boxed{1.98}$

167. Evaluate to two decimal places using a calculator

$\frac{1}{\sqrt[13]{2500}}$

$\boxed{0.55}$

168. Challenge

Write the following radicals as a single power.

$(\sqrt[2]{x^3})(\sqrt[3]{x})$

$(x^{\frac{3}{2}})(x^{\frac{1}{3}})$

$(x^{\frac{9}{6}})(x^{\frac{2}{6}})$

$\boxed{x^{\frac{11}{6}}}$

$\cancel{(x^{\frac{3}{2}})(x^{\frac{1}{3}})}$

$\cancel{x^{\frac{3}{3}} = \boxed{x^1}}$

$\cancel{(x^{\frac{3}{2}})(x^{\frac{1}{3}})}$

$\cancel{x^{\frac{3}{6}} = x^{\frac{1}{2}}}$

Write each of the following radicals as a single power.

169. $(\sqrt{x^3})(\sqrt[3]{x})$

$\left(x^{\frac{3}{2}}\right)\left(x^{\frac{1}{3}}\right)$ Write as powers (both base-x).

$\left(x^{\frac{9}{6}}\right)\left(x^{\frac{2}{6}}\right)$ Create common denominators.

$\left(x^{\frac{9+2}{6}}\right)$ Add numerators.

$\left(x^{\frac{11}{6}}\right)$

170. $(\sqrt[3]{x^2})(\sqrt[4]{x^3})$

$x^{\frac{2}{3}} \times x^{\frac{3}{4}}$

$= x^{\frac{8}{12}} \times x^{\frac{9}{12}}$

$= \boxed{x^{\frac{17}{12}}}$

171. $(\sqrt[5]{x^3})(\sqrt[3]{x^2})$

$= x^{\frac{3}{5}} \times x^{\frac{2}{3}}$

$= x^{\frac{9}{15}} \times x^{\frac{10}{15}}$

$= \boxed{x^{\frac{19}{15}}}$

More rational exponents...

172. The height and the base of a triangle each measure $2^{\frac{3}{2}}$ cm. Without using a calculator, what is the area of the triangle?

$2^{\frac{3}{2}}$  $2^{\frac{3}{2}}$

$\frac{2^{\frac{3}{2}} \times 2^{\frac{3}{2}}}{2} = \frac{2^{\frac{6}{2}}}{2} = \frac{2^{3}}{2} = \frac{8}{2} = \boxed{4cm^2}$

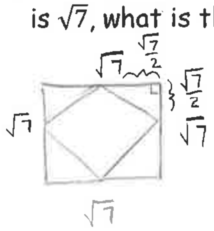
173. Find the area of a rectangle if the length is $5^{\frac{2}{3}}$ and the width is $5^{\frac{2}{5}}$. Write your answer in exponential form, then approximate to two decimal places.

$5^{\frac{2}{5}}$  $5^{\frac{2}{3}}$

$= 5^{\frac{2}{3}} \times 5^{\frac{2}{5}} = 5^{\frac{6}{15}} \times 5^{\frac{10}{15}}$

$= \boxed{5^{\frac{16}{15}} cm^2}$ $= \boxed{5.57cm^2}$

174. Inscribe a square inside another square such that the corners of the internal square contact the midpoint of sides of the larger square. If the side length of the larger square is $\sqrt{7}$, what is the area of the inscribed square? Answer in exact form.



$= \left(\frac{\sqrt{7}}{2}\right)^2 + \left(\frac{\sqrt{7}}{2}\right)^2 = 1.75 + 1.75 = \sqrt{3.5} = 1.870828693^2 = \boxed{3.5 \text{ units}^2}$

175. Simplify(write as a single power.)

$\left[(\sqrt[3]{x^4})(\sqrt[5]{x})\right]^{-2}$

$\frac{23}{15}$?

$\frac{1}{\left[(\sqrt[3]{x^4})(\sqrt[5]{x})\right]^2}$

$\frac{1}{\left[(x^{\frac{4}{3}})(x^{\frac{1}{5}})\right]^2} = \frac{1}{\left[(x^{\frac{20}{15}})(x^{\frac{3}{15}})\right]^2}$

$= \frac{1}{(x^{\frac{23}{15}})^2} = \boxed{\frac{1}{x^{\frac{46}{15}}}}$

176. Simplify (write as a single power.)

$\left[(\sqrt[4]{x^9})(\sqrt[3]{x^6})\right]^{\frac{2}{3}}$

$\left[(x^{\frac{9}{4}})(x^{\frac{6}{3}})\right]^{\frac{2}{3}}$

$\left[(x^{\frac{9}{4}})(x^{\frac{2}{1}})\right]^{\frac{2}{3}} = \left[(x^{\frac{9}{4}})(x^{\frac{8}{4}})\right]^{\frac{2}{3}}$

$= \left[(x^{\frac{17}{4}})\right]^{\frac{2}{3}} = x^{\frac{34}{12}} = \boxed{x^{\frac{17}{6}}}$

177. Ei-Q evaluated $64^{\frac{3}{2}}$ using the following steps. In which step did she make her first error?

Step 1: $64^{\frac{3}{2}} = (\sqrt{64})^3$

Step 2: $64^{\frac{3}{2}} = (8)^3$

Step 3: $64^{\frac{3}{2}} = 24$ $\boxed{512}$

a) In step 1.
b) In step 2.
c) In step 3.
d) She made no error.

178. Flinflan started to evaluate $81^{-\frac{3}{4}}$ in two different ways shown below. Which of the following statements is correct?

Method 1: $81^{-\frac{3}{4}} = (\sqrt[4]{81})^{-3}$

Method 2: $81^{-\frac{3}{4}} = \frac{1}{\sqrt[4]{81^3}}$

a) Method 1 will produce the correct answer but method 2 will not.
b) Method 2 will produce the correct answer but method 1 will not.
c) Both methods will produce the correct answer.
d) Neither method will produce the correct answer.

179. Simplify: $[(\sqrt[3]{x^4})(\sqrt[5]{x^2})]^{-1}$

$\frac{1}{(\sqrt[3]{x^4})(\sqrt[5]{x^2})} = \frac{1}{(x^{\frac{4}{3}})(x^{\frac{2}{5}})} = \frac{1}{(x^{\frac{20}{15}})(x^{\frac{6}{15}})}$

$\boxed{\frac{1}{x^{\frac{26}{15}}}}$

180. Simplify: $[(\sqrt[3]{a^5})(\sqrt[4]{a^3})]^{-2}$

$\frac{1}{[(a^{\frac{5}{3}})(a^{\frac{3}{4}})]^2} = \frac{1}{[(a^{\frac{20}{12}})(a^{\frac{9}{12}})]^2} = \frac{1}{(a^{\frac{29}{12}})^2} =$

$\frac{1}{a^{\frac{58}{12}}} = \boxed{\frac{1}{a^{\frac{29}{6}}}}$

181. Simplify:

$$\sqrt[3]{\left(a^{\frac{2}{3}}\right)^{\frac{1}{4}}}$$

$\sqrt[3]{a^{\frac{2}{12}}} = \sqrt[3]{a^{\frac{1}{6}}}$ $6 \times 3 = 18$

$\boxed{a^{\frac{1}{18}}}$?

182. Simplify:

$$\sqrt[4]{\left(x^{\frac{1}{3}}\right)^{\frac{1}{5}}}$$

$\sqrt[4]{x^{\frac{1}{15}}}$ $4 \times 15 = 60$

$\boxed{x^{\frac{1}{60}}}$?

Match each item in column 1 with an equivalent item in column 2

Column 1

183. $\left(\frac{t}{j}\right)^{\frac{2}{3}}$ = F

184. $\left(\frac{j}{t}\right)^{\frac{3}{2}}$ = C

185. $\left(\frac{t}{j}\right)^{-\frac{2}{3}}$ $\sqrt[3]{\frac{j^2}{t^2}}$ = A

186. $\left(\frac{j}{t}\right)^{-\frac{3}{2}}$ = $\sqrt{\frac{t^3}{j^3}}$ = E

187. $\left(\frac{t}{j}\right)^{-\frac{3}{2}}$ = $\sqrt{\frac{j^3}{t^3}}$ = C

Column 2

A. $\sqrt[3]{\frac{j^2}{t^2}}$

B. $-\left(\frac{j}{t}\right)^{\frac{3}{2}}$

C. $\sqrt{\frac{j^3}{t^3}}$

D. $-\left(\frac{t}{j}\right)^{\frac{2}{3}}$

E. $\sqrt{\frac{t^3}{j^3}}$

F. $\sqrt[3]{\frac{t^2}{j^2}}$

G. $-\left(\frac{t}{j}\right)^{\frac{3}{2}}$

188. Which of the following is equivalent to $3a^{\frac{1}{2}} \times (5a)^{\frac{1}{2}}$

$15a^{\frac{2}{2}}$ = $15a$

a. ~~$15a$~~
b. $a\sqrt{15}$
c. $3\sqrt{5a}$
d. $3a\sqrt{5}$

~~$3\sqrt{a} \times 1\sqrt{5a}$~~

~~$= 3$~~

$3 \times a^{\frac{1}{2}} \times 5^{\frac{1}{2}} \times a^{\frac{1}{2}}$

$= 3 \times a^{\frac{1}{2}} \times a^{\frac{1}{2}} \times 5^{\frac{1}{2}}$

$= 3 \times a^{\frac{2}{2}} \times 5^{\frac{1}{2}}$

$= 3 \times a \times 5^{\frac{1}{2}}$

$= 3a\sqrt{5}$

189. Which of the following is equivalent to $2x^{\frac{1}{2}} \times (3x)^{\frac{1}{2}}$

a. $6x$
b. $x\sqrt{6}$
c. $2\sqrt{3x}$
d. $2x\sqrt{3}$

$2 \times x^{\frac{1}{2}} \times 3^{\frac{1}{2}} \times x^{\frac{1}{2}}$

$= 2 \times x^{\frac{1}{2}} \times x^{\frac{1}{2}} \times 3^{\frac{1}{2}}$

$= 2 \times x^{\frac{2}{2}} \times 3^{\frac{1}{2}}$

$= 2 \times x \times 3^{\frac{1}{2}}$

$= 2x\sqrt{3}$

190. Which of the following is not equivalent to $x^{\frac{2}{3}}$?

a. $\sqrt[3]{x^2}$ ✓ $= \boxed{x^{\frac{2}{3}}}$ ✓

b. $\left(\sqrt[6]{x}\right)^4$ ✓ $= x^{\frac{4}{6}} = \boxed{x^{\frac{2}{3}}}$ ✓

c. $(x^2)(\sqrt[3]{x})$ why? $= x^{\frac{2}{1}} \times x^{\frac{1}{3}} = x^{\frac{6}{3}} \times x^{\frac{1}{3}} = \boxed{x^{\frac{7}{3}}}$ ×

d. $\sqrt{x^3}$ $= \boxed{x^{\frac{3}{2}}}$ ×

191. Which of the following is not equivalent to $a^{\frac{3}{2}}$?

a. $\sqrt[4]{a^6}$ ✓ $= a^{\frac{6}{4}} = \boxed{a^{\frac{3}{2}}}$ ✓

b. $\sqrt[3]{a^2}$ × $= a^{\frac{2}{3}}$ ×

c. $a\sqrt{a}$ ✓ $= a^{\frac{1}{1}} \times a^{\frac{1}{2}} = a^{\frac{2}{2}} \times a^{\frac{1}{2}} = \boxed{a^{\frac{3}{2}}}$ ✓

d. $\sqrt{\sqrt{a^6}}$ ✓ $= \sqrt[2]{a^{\frac{6}{2}}} = \sqrt[2]{a^3} = \boxed{a^{\frac{3}{2}}}$ ✓

192. Evaluate. Answer in simplest fraction form.

$$\frac{3^0 + 2^{-1}}{3^2 + 2^2}$$

$$\frac{2}{3^2+2^3} = \frac{2}{9+8} = \frac{2}{17}$$

$$\frac{3^0 + 2^{-1}}{3^2 + 2^2} = \frac{1}{3^2+2^2+2^1} = \frac{1}{9+4+2} = \frac{1}{13 \times 2} = \boxed{\frac{1}{26}}\ ?$$

$$\frac{3^0 + 2^{-1}}{3^2 + 2^2} = \frac{1+\frac{1}{2}}{9+4} = \frac{1\frac{1}{2}}{13} = \frac{\sqrt[2]{1}}{13}$$

$$\frac{3^0 + 2^{-1}}{3^2 + 2^2} = \frac{1+\frac{1}{2}}{9+4} = \frac{1\frac{1}{2}}{13} = \frac{\frac{3}{2}}{13} = \frac{3}{2} \div \frac{13}{1} = \frac{3}{2} \times \frac{1}{13} = \boxed{\frac{3}{26}}$$ ✓

193. Evaluate. Answer in simplest fraction form.

$$\frac{3^{-2} + 3^2}{3^{-2} + 2^0}$$

$$\frac{3^2 \times 3^2}{2^0 \times 3^2} = \frac{3^4}{1 \times 9} = \frac{81}{9} = \frac{9}{1}$$

$$\frac{3^{-2} + 3^2}{3^{-2} + 2^0} = \frac{\frac{1}{3^2} + 9}{\frac{1}{3^2} + 1} = \frac{\frac{1}{9} + 9}{\frac{1}{9} + 1} = \frac{9\frac{1}{9}}{1\frac{1}{9}} = \frac{\frac{82}{9}}{\frac{10}{9}}$$

$$= \frac{82}{9} \div \frac{10}{9} = \frac{82}{9} \times \frac{9}{10} = \frac{82}{10} = \boxed{\frac{41}{5}}$$

Answers:

1. 81
2. 2
3. x^8
4. $2x$
5. $9 \times 9 = 81$ or $3 \times 3 \times 3 \times 3 = 81$ or $3^4 = 81$
6. Answers vary. Similar to above.
7. $16, 8, 4, 2, 1, \frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{1}{16}$
8. Divide by 2 as you go down the list
9. Fits the pattern above.
10. Yes follows the division pattern.
11. Decreasing exponent value is like dividing by two in this case.
12. 4
13. 2^5
14. 2
15. -4^2
16. -9^2
17. $\frac{2x^3}{2x^3}, (5x)^0$
18. $(-3)^2$
19. -64
20. -27
21. -16
22. $\frac{1}{16}$
23. $-\frac{1}{16}$
24. $\frac{1}{81}$
25. $\frac{1}{81}$
26. $-\frac{1}{81}$
27. 16
28. 16
29. -16
30. 1
31. -1
32. 1
33. a^9
34. g^4
35. $15m^6$
36. a^9
37. a^{-2}
38. f^{2+x}
39. x^1
40. 2^{-2}
41. g^4
42. m^4
43. t^5
44. x^{10}
45. $15m^6$
46. $5x^6$
47. $-\frac{1}{2}a^2 = -\frac{a^2}{2}$
48. $\frac{4x^7}{5}$
49. $\frac{a^3}{3}$
50. $\frac{2}{3}$
51. 15625
52. m^6
53. $8m^{12}$
54. m^6
55. 1
56. $x^{-6}y^{-9} = \frac{1}{x^6y^9}$
57. $8m^{12}$
58. $2^{-3}c^{-12}d^{-9} = \frac{1}{8c^{12}d^9}$
59. $(-3)^{-4}x^8y^{-12} = \frac{x^8}{81y^{12}}$
60. $3^{-3}x^6y^9 = \frac{1}{27}x^6y^9$ or $\frac{x^6y^9}{27}$
61. $-18x^5y^9$
62. $128a^{12}b^2$
63. $\frac{8}{125}$
64. $\frac{125}{8}$
65. $\frac{x^3}{8}$
66. $\frac{16y^2}{9x^{10}}$
67. $\frac{x^3}{8}$
68. $\frac{a^4}{b^4}$
69. $\frac{x^{10}}{y^{15}}$
70. $\frac{-8a^6}{27y^9}$
71. $\frac{a^6}{b^4}$
72. $\frac{16x^2}{9y^2}$
73. $\frac{16y^2}{9x^{10}}$
74. $\frac{25a^6b^4c^{12}}{4}$
75. $\frac{n^3}{8m^3}$
76. $27b^6$
77. $\frac{4x^{10}}{y^{12}}$
78. $\frac{2a^2}{b^3} = \frac{2a^2}{1} \times \frac{1}{b^3}$ and $\frac{1}{b^3} = b^{-3}$
79. $\frac{12x^3}{y} = \frac{12x^3}{1} \times \frac{1}{y}$ and $\frac{1}{y} = y^{-1}$
80. $\frac{3a^2}{b^5}$
81. $\frac{3a^2}{b^5}$
82. $\frac{1}{8x^3y^3}$
83. $\frac{3a^2}{b^5}$
84. $\frac{a^2}{b^3}$
85. $2x^5y^5$
86. $\frac{3a^2}{b^3c^5}$
87. $\frac{y^6z^2}{x^8}$
88. $\frac{9}{2x^7y^{11}}$
89. $\frac{1}{8x^3y^3}$
90. $\frac{4}{a^{15}b^9}$
91. $\frac{2}{m^2n}$
92. Remember that a negative exponent can be evaluated by reciprocating the base, therefore expressions like a^{-3} become $\frac{1}{a^3}$. Notice the exponent became positive.
93. $\frac{4y^{12}}{9x^8}$
94. $\frac{27b^9}{8a^9}$
95. $\frac{1}{8x^9y^6}$
96. $\frac{4x^6}{3y^7}$
97. $\frac{1}{2}$
98. $\frac{1}{3}$
99. $x^{\frac{1}{n}} = \sqrt[n]{x}$
100. Possible answer:
$$\sqrt[4]{3} \times \sqrt[4]{3} \times \sqrt[4]{3} \times \sqrt[4]{3} = 3$$
$$3^{\frac{1}{4}} \times 3^{\frac{1}{4}} \times 3^{\frac{1}{4}} \times 3^{\frac{1}{4}} = 3$$
$$\therefore \sqrt[4]{3} = 3^{\frac{1}{4}}$$
101. 7
102. -4
103. *no real number*
104. 4
105. $\frac{1}{3}$
106. $\frac{1}{2}$
107. 10
108. $2x$
109. $\frac{1}{3x^2}$
110. $\sqrt{7}$
111. $\sqrt[3]{3x}$
112. $\sqrt[5]{4}$
113. $\frac{1}{\sqrt[5]{4}}$
114. $-\sqrt[3]{64}$
115. $\frac{1}{\sqrt[3]{64}}$
116. $13^{\frac{1}{2}}$
117. $-3x^{\frac{1}{2}}$
118. $(2y)^{\frac{1}{2}}$
119. $4^{\frac{1}{4}}$
120. $4^{\frac{1}{7}}$
121. $(3x)^{-\frac{1}{5}}$
122. $27^{\frac{2}{3}} = \left(27^{\frac{1}{3}}\right)^2$
$$27^{\frac{2}{3}} = \left(\sqrt[3]{27}\right)^2$$
$$27^{\frac{2}{3}} = (3)^2$$
$$27^{\frac{2}{3}} = 9$$
123. $\sqrt[5]{4^2}$ or $\left(\sqrt[5]{4}\right)^2$
124. $\sqrt[5]{4^3}$ or $\left(\sqrt[5]{4}\right)^3$
125. $\sqrt[5]{4^4}$ or $\left(\sqrt[5]{4}\right)^4$
126. $\frac{1}{\sqrt[5]{4^2}}$ or $\frac{1}{\left(\sqrt[5]{4}\right)^2}$
127. $\frac{1}{\sqrt[5]{4^3}}$ or $\frac{1}{\left(\sqrt[5]{4}\right)^3}$
128. $\frac{1}{\sqrt[5]{4^4}}$ or $\frac{1}{\left(\sqrt[5]{4}\right)^4}$
129. $\sqrt{4} = 2$
130. $\sqrt[3]{125} = 5$
131. $\left(\sqrt[3]{8}\right)^2 = 4$
132. $\left(\sqrt[4]{81}\right)^3 = 27$
133. $\left(\sqrt{4}\right)^3 = 8$
134. $\frac{1}{\left(\sqrt[4]{16}\right)^3} = \frac{1}{8}$
135. $\frac{1}{\left(\sqrt[3]{-27}\right)^2} = \frac{1}{9}$
136. $\frac{1}{\left(\sqrt[3]{-8}\right)^5} = -\frac{1}{32}$
137. $9^{\frac{5}{2}} = \left(\sqrt{9}\right)^5 = 243$

138. 1
139. $\frac{1000}{27}$
140. $\frac{4}{9}$
141. $7^{\frac{1}{2}}$
142. $34^{\frac{1}{3}}$
143. $(-11)^{\frac{1}{3}}$
144. $a^{\frac{2}{5}}$
145. $6^{\frac{4}{3}}$
146. $x^{\frac{2}{3}}$
147. $6^{\frac{3}{5}}$
148. $(2x)^{\frac{5}{4}}$
149. $a^{-\frac{1}{3}}$
150. $x^{-\frac{4}{5}}$
151. $x^{-\frac{3}{4}}$
152. $2^{\frac{1}{3}}b$
153. *no real solution*
154. 1000
155. $\frac{9}{4}$
156. 3
157. -3
158. 4
159. a)-16 b) 16
160. 4
161. *no real solution*
162. 5
163. 4
164. 3
165. 0.32
166. 1.98
167. 0.55
168. $x^{\frac{11}{6}}$
169. *Answered on page.*
170. $x^{\frac{17}{12}}$
171. $x^{\frac{19}{15}}$
172. $4\ cm^2$
173. $5^{\frac{16}{15}}$ cm² $\cong$ 5.57 cm²
174. $\frac{7}{2}$ *or* $3.5\ cm^2$
175. $x^{-\frac{46}{15}}$ or $\frac{1}{x^{\frac{46}{15}}}$
176. $x^{\frac{17}{6}}$
177. c
178. c
179. $x^{-\frac{26}{15}} = \frac{1}{x^{\frac{26}{15}}}$
180. $a^{-\frac{29}{6}} = \frac{1}{a^{\frac{29}{6}}}$
181. $a^{\frac{1}{18}}$
182. $x^{\frac{1}{60}}$
183. F
184. C
185. A
186. E
187. C
188. D
189. D
190. C,D
191. B
192. $\frac{3}{26}$
193. $\frac{41}{5}$