10

HW Mark: 10 9 8 7 6 RE-Submit

Relations & Functions

164 160 78 145

interval notation = (smallest, biggest)

This booklet belongs to: Marissa Period 4

LESSON #	DATE	QUESTIONS FROM NOTES	Questions that I find difficult
1	NOV.12/14	Pg. 4-7	
2	Nov.13/14	Pg. 8-12	16, 17, 18
3	NOV.17/14	Pg. 13-20	58, 59, 66
4	NOV.24/14	Pg. 20-24	80, 78, 81, 90
5	NOV.26/14	Pg. 25-26	
6	NOV.27/14	Pg. 27-34	109, 137, 147
7	Dec.1/14	Pg. 34-41	153, 155, 151, 159,
		Pg.	167, 168, 175, 177, 178
		Pg.	
		Pg.	
		REVIEW	
		TEST	

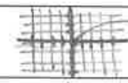
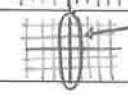
Your teacher has important instructions for you to write down below.

Relations & Functions

Relations SPECIFIC OUTCOMES		TOPICS	REVIEW EXAMPLE
C1. Interpret and explain the relationships among data, graphs and situations.	1.1	Graph, with or without technology, a set of data, and determine the restrictions on the domain and range.	
	1.2	Explain why data points should or should not be connected on the graph for a situation.	
	1.3	Describe a possible situation for a given graph.	
	1.4	Sketch a possible graph for a given situation.	
	1.5	Determine, and express in a variety of ways, the domain and range of a graph, a set of ordered pairs, or a table of values.	
C2. Demonstrate an understanding of relations and functions.	2.1	Explain, using examples, why some relations are not functions, but all functions are relations.	
	2.2	Determine if a set of ordered pairs represents a function.	
	2.3	Sort a set of graphs as functions or non-functions.	
	2.4	Generalize and explain rules for determining whether graphs and sets of ordered pairs represent functions.	
C4. Describe linear relations.	4.1	Identify independent and dependent variables in a given context.	
C5. Characteristics of graphs.	5.3	Determine the slope of the graph of a linear relation.	

[C] Communication [PS] Problem Solving, [CN] Connections [R] Reasoning, [ME] Mental Mathematics [T] Technology, and Estimation, [V] Visualization

Key Terms

Term	Definition	Example
Relation	The set of ordered pairs that connects two sets.	(0,4), (1,10), (2,16)
Function	A special class of relations where every x value has a unique y value.	(0,20), (1,25), (2,30)
Ordered pair	(x,y)	(2,12)
Coordinate Plane	Has 2 axes: x axis and y axis. The point where the axes meet is the origin at (0,0).	 origin
x-axis	The horizontal axis on the coordinate plane	 x axis
y-axis	The vertical axis on the coordinate plane	 y axis
Domain	The first set of elements in the ordered pair	(x, y)
Range	The second set of elements in the ordered pair.	(x, y)
Element	A number in the ordered pair/graph.	(2,12) = 2 is element of domain 12 is element of range.
Permissible values	possible values?	2 is a permissible value of x
Dependent Variable	y	y
Independent Variable	x	x

Introduction to Relations

Relationships exist everywhere we look…

- There is a relationship between the lengths of lineups at the fair and how exciting the rides are.
- There is a relationship between the height of a ball and how long ago it was kicked.
- There is a relationship between traffic and the time of day.
- There is a relationship between distance travelled and the speed of the car.

Some relationships don't even seem to have a mathematical relationship but are connected in some other way.
For example: The students in your class all have a birth month and height. We could write a list matching each student's birth month and height.

As ***ordered pairs***…(3, 155), (5,138), (11, 162), (12, 135), (7, 142), …

(March, 155 cm tall)

Some notes here…

Challenge Question:

1. Give examples of three other relationships you see on an everyday basis:
   1. the hours you work and the money you earn
   2. the hour of the day and the temperature
   3. your age and your height
2. Write a set of **3 ordered pairs** for one of your relationships above. Explain what the ordered pair means.

#1: a) (2, 20) → you work 2 hrs for \$20 (\$10/h)
b) (4, 40) → you work 4 hrs for \$40
c) (10, 100) → you work 10 hrs for \$100

Use the following information to answer questions below.

Consider the data given in following table:

Student	Height (cm)	Arm Span (cm)
Lulu	135	137
Bones	144	151
Phat Charlie	150	148
Lucky	150	156
Dizzy Dee	165	165
Crash	155	152
Anjohkinu	160	164
Sam	200	210
Talloola	125	127

Written as ***ordered pairs.***

✓(135, 137)
✓(144, 151)
✓(150, 148)
✓(150, 156)
✓(165, 165)
✓(155, 152)
✓(160, 164)
✓(200, 210)
✓(125,127)

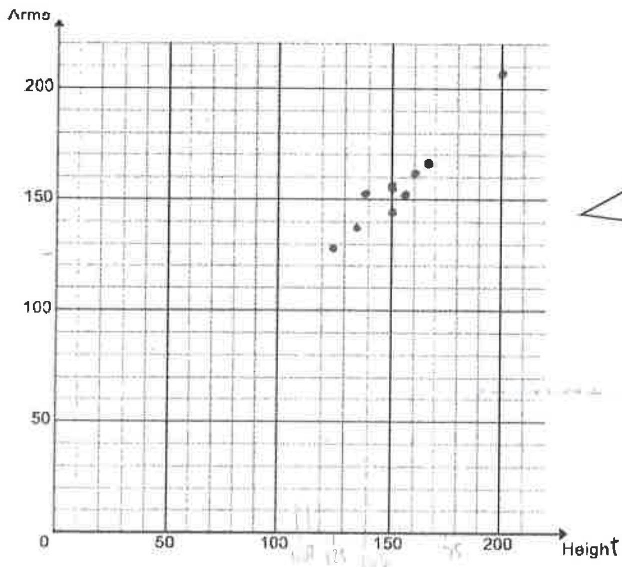
3. Why do you think the numbers in brackets are called "ordered pairs"?

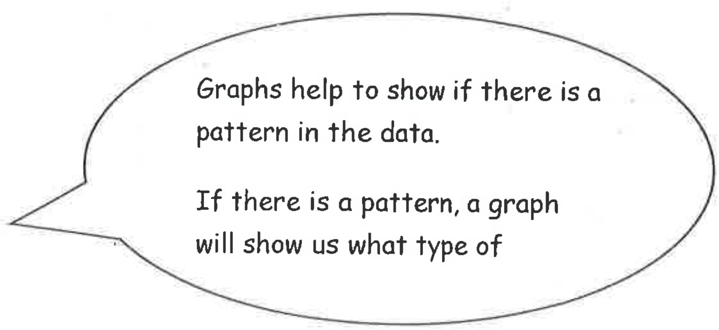
Because they are paired numbers written in the alphebetical order of the x an y axis.

4. The data above represents a relation between what two quantities?

A students' height and their arm span (cm).

5. Graph the data in the table above (Trouble graphing? See next page).





6. Describe the relationship you see on the graph above. (What does it look like? What shape is it?)

It is a cluster, but if you leave out some points you can make a line ← ANSWER KEY

★ IT IS GOING UP DIAGONALLY ★

What is a relation?

Definition 1:
If two groups of items are related, the set of all possible pairings is called a **relation.**

For example:

A person's height and their arm span.
Distance travelled and driving time.
Exam score and study time.

Definition 2:
A **relation** is the set of ordered pairs that connects two sets.

Definition 3:
7. Write your own...

The "relationship" between sets of information.

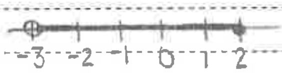
Domain:
The set of first items in a relation.

Range:
The set of second items in a relation.

Some notes here (possibly)...

x "belongs to the set of" Real Numbers
$= x \in \mathbb{R}$

SET NOTATION $\{x \mid -3 < x \le 2,\ x \in \mathbb{R}\}$ →



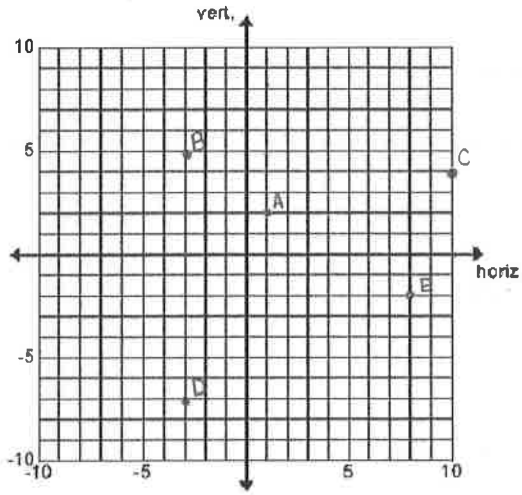
$\mid$ = "is such that"

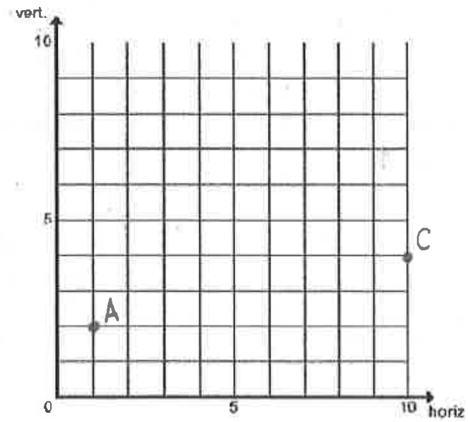
N = natural Q = rational
W = whole $\overline{Q}$ = irrational
Z = integer $\mathbb{R}$ = real

Graphing Relations on a Coordinate Plane

Below are two examples of the Coordinate Plane

8. The vertical line with numbers on it is called the y-axis
9. The horizontal line with numbers on it is called the x-axis





10. What is the difference between each of the graphs shown above?

The graph on the left is the whole graph (has negative coordinates) while the one on the right just has positive coordinates.

11. Describe a scenario where it is more appropriate to use the graph on the right.

If you were showing the hours you worked and the money you earned (income).

12. Describe a scenario where it is more appropriate to use the graph on the left.

when doing temperatures.

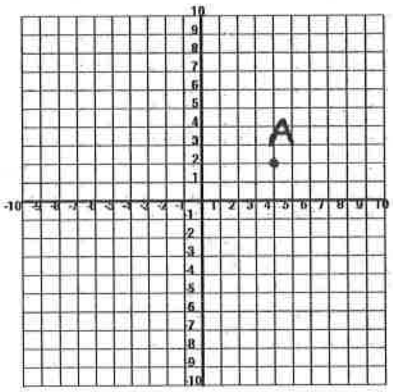
13. How could you describe to another student where to plot a point on the plane? For example: (2, 5)

The first number (2) find on the x-axis and then find the second number (5) on the y-axis and find the point where they interesect (plot there).

14. Plot and label the following ordered pairs on each of the grids above (whenever possible):
A(1,2), B(-3,5), C(10, 4), D(-3, -7), E(8, -2)

Coordinate Geometry (Cartesian Coordinate Geometry)

Based on the ***coordinate plane.***



The coordinate plane has two **axes.**

The vertical ***y-axis.***

The horizontal **x-*axis*.**

The point where the axes meet is called the ***origin*.**

Every point on the plane can be located using two numbers called ***coordinates*** or an ***ordered pair***.

The ordered pair is always given as (x, y). The coordinates of the origin are (0,0).

Example: Point A has coordinates (4,2).
4 is an ***element*** of the domain. 2 is an ***element*** of the range.

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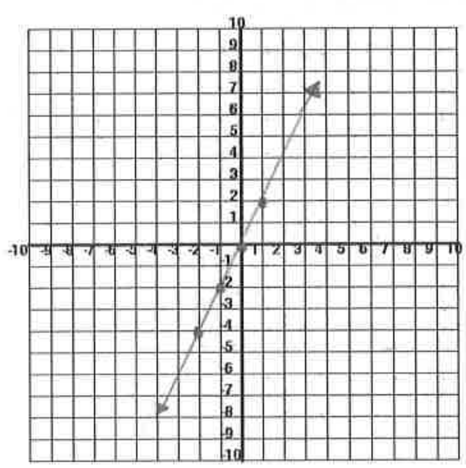
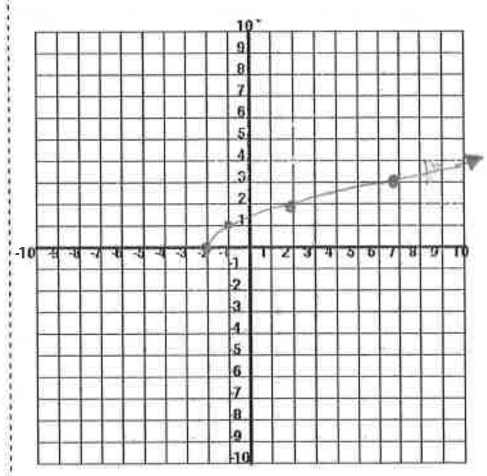
15. Challenge Question:
Using the graph below, plot the relation described by the equation $y = 2x$.



x	y
-2	-4
-1	-2
0	0
1	2

$y = 2x$
$y = 2(-2)$
$y = 2(-1)$
$y = 2(0)$
$y = 2(1)$

16. Challenge Question:
Using the graph below, plot the relation described by the equation $y = \sqrt{x+2}$.



x	y
-2	0
-1	1
2	2
7	3

x	y
~~-3~~	$\cancel{\sqrt{-1}}$
-2	0
-1	1
~~0~~	$\cancel{\sqrt{2}}$
2	2

$y = \sqrt{x+2}$
$y = \sqrt{-2+2} : y = \sqrt{0} = 0$
$y = \sqrt{2+2} : y = \sqrt{4} = 2$

$D: \{x \mid x \geq -2, x \in \mathbb{R}\}$

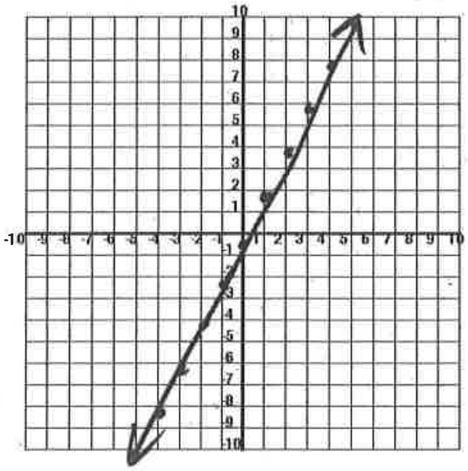
Graphing relations using a *Table of Values*.

Using the graph below, plot the relation described by the equation $y = 2x$.

$y = 2x$	
x	y
-2	-4
-1	-2
0	0
1	-2
2	-4

A Table of Values: Choose a few reasonable input values (x), then calculate output values (y).

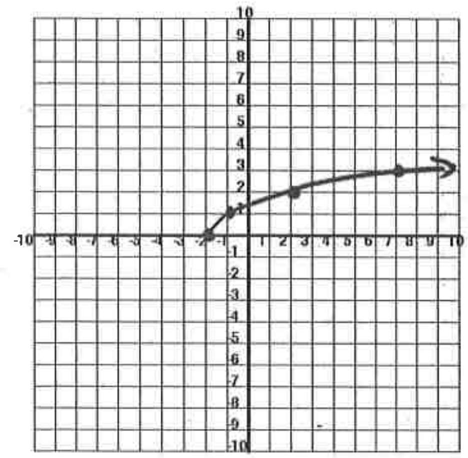
This produces some ordered pairs to plot our relation.



Using the graph below, plot the relation described by the equation $y = \sqrt{x+2}$.

$y = \sqrt{x+2}$	
x	y
-2	0
-1	1
2	2
7	3

Can you see why I chose these particular input (x) values?



17. Are there any input values that would not make sense? Are there any that are "not permitted"?

no?

★ANSWER KEY: NO★

18. Are there any input values that would not make sense? Are there any that are "not permitted"?

$x < -2$

19. Consider your answers to the previous two questions. What effect do "not permitted" input values have on the graph of the relation?

They are impossible to plot on the graph

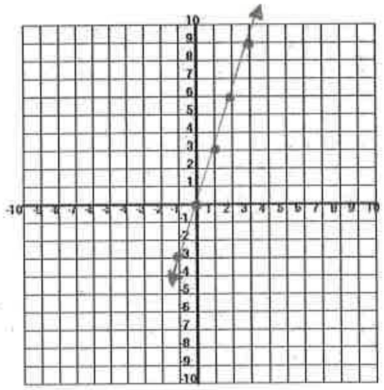
★ANSWER KEY: GRAPH WILL HAVE STOPS AND BREAKS WHERE VALUES ARE NOT PERMITTED★

When creating a table of values, you should consider:

- Positive Inputs (domain)
- Negative Inputs (domain)
- Zeros
- Non-permitted input values

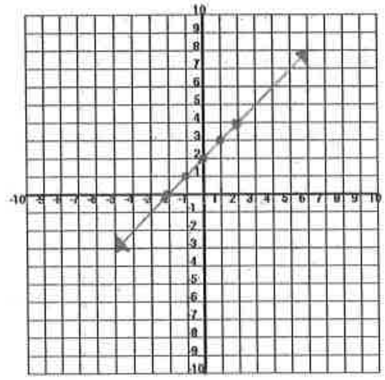
Graphing Relations continued...

20. Using the table and graph below, plot the relation described by the equation $y = 3x$.



$y = 3x$.	
x	y
-1	-3
0	0
1	3
2	6
3	9

21. Using the table and graph below, plot the relation described by the equation $y = x + 2$.



$y = x + 2$.	
x	y
-2	0
-1	1
0	2
1	3
2	4

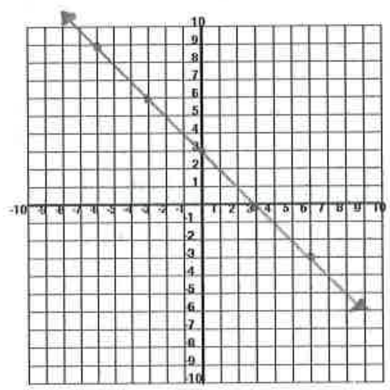
22. What are two possible items in the domain above?

n/a

23. What are two possible items in the range above?

n/a

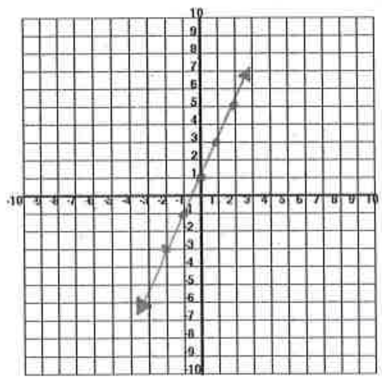
24. Using the table and graph below, plot the relation described by the equation $y = 3 - x$.



$y = 3 - x$	
x	y
-6	9
-3	6
0	3
3	0
6	-3

$x = -3 \quad y = 3-(-3)$
$= 3+3$
$= 6$

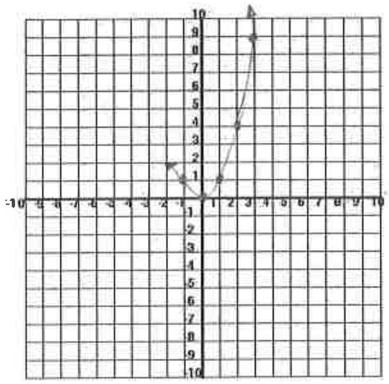
25. Using the table and graph below, plot the relation described by the equation $y = 2x + 1$.



$y = 2x + 1$	
x	y
-2	-3
-1	-1
0	1
1	3
2	5

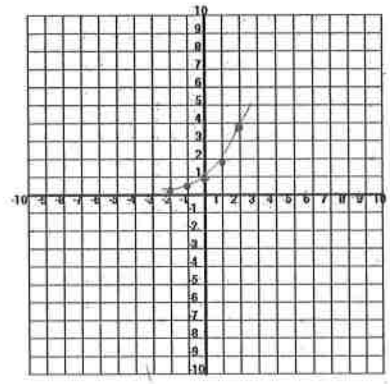
Graphing Relations continued...

26. Using the table and graph below, plot the relation described by the equation $y = x^2$.



$y = x^2$	
x	y
-1	1
0	0
1	1
2	4
3	9

27. **CHALLENGE**Using the table and graph below, plot the relation described by the equation $y = 2^x$.



$y = 2^x$	
x	y
-2	$\frac{1}{4}$
-1	$\frac{1}{2}$
0	1
1	2
2	4

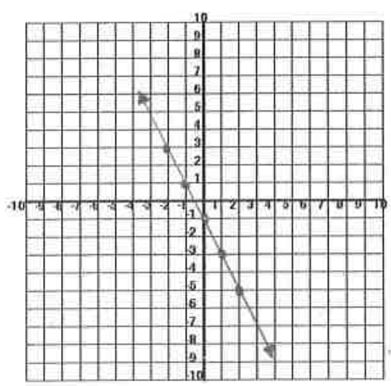
28. What is a value that **cannot** be part of the range above?

n/a

29. What is a value that **cannot** be part of the range above?

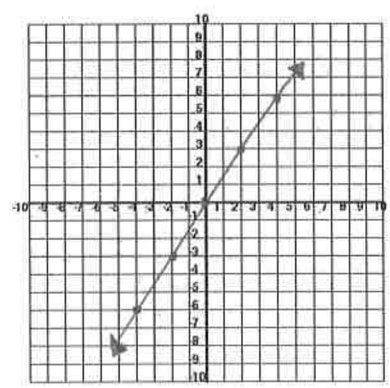
n/a

30. Using the table and graph below, plot the relation described by the equation $y = -2x - 1$.



$y = -2x - 1$	
x	y
-2	3
-1	1
0	-1
1	-3
2	-5

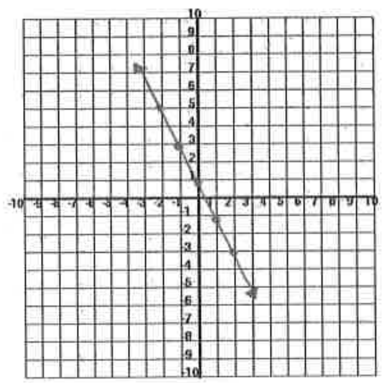
31. Using the table and graph below, plot the relation described by the equation $y = \frac{3}{2}x$.



$y = \frac{3}{2}x$	
x	y
-4	-6
-2	-3
0	0
2	3
4	6

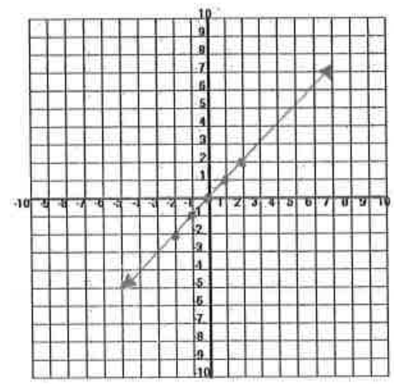
Graphing Relations continued…

32. Using the table and graph below, plot the relation described by the equation $y = 1 - 2x$.



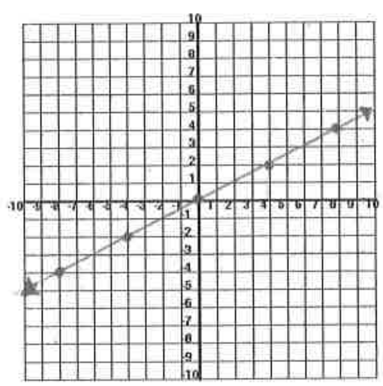
$y = 1 - 2x$	
x	y
-2	5
-1	3
0	1
1	-1
2	-3

33. Using the table and graph below, plot the relation described by the equation $y = x$.



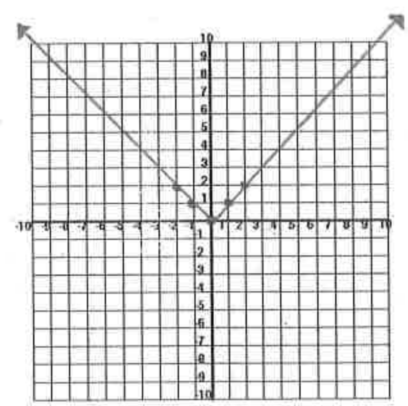
$y = x$	
x	y
-2	-2
-1	-1
0	0
1	1
2	2

34. Using the table and graph below, plot the relation described by the equation $x = 2y$.



$x = 2y$.	
x	y
-8	-4
-4	-2
0	0
4	2
8	4

35. **CHALLENGE**Using the table and graph below, plot the relation described by the equation $y = |x|$.



$y = \|x\|$.	
x	y
-2	2
-1	1
0	0
1	1
2	2

☆ ABSOLUTE VALUE GRAPHS ARE V-SHAPED ☆

$D: \{x \mid x \in \mathbb{R}\}$

$R: \{y \mid y \geq 0, y \in \mathbb{R}\}$

Domain & Range (continued)

Recall, (2,5) and (-3,7) are called ***ordered pairs*** because the order of the two ***elements*** is important.

- The first set of elements in the ordered pair is called the ***domain*** of the relation.
- The second set of elements in the ordered pair is called the ***range*** of the relation.

36. **Challenge Question:**
List the domain and range for the relation (1,1), (2, 4), (3,9), (4,16)

Domain: $\{1, 2, 3, 4\} \rightarrow \{x \mid x: 1, 2, 3, 4\}$
Range: $\{1, 4, 9, 16\} \rightarrow \{y \mid y: 1, 4, 9, 16\}$

Answer:	
Domain: {1,2,3,4}	Range: {1,4,9,16}

37. Which of the following is/are true?

a. The domain is the set of permissible values of x.
~~b.~~ The domain is the set of permissible values of y.
~~c.~~ The range is the set of permissible values of x.
d. The range is the set of permissible values of y.

Your notes here…

Domain & Range of Discrete Data (points): [Definition on page 25]

Remember, domain is all "first elements" and range is all "second elements".

Since we are often working with graphs that have an x-axis and a y-axis.

Domain is often described as all permissible values of x.

Range is often described as all permissible values of y.

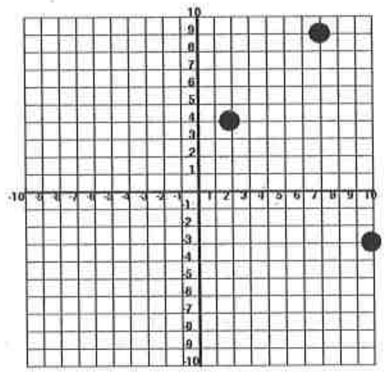
Find the domain and range:

Example:
Find the domain and range of the relation:
(2,3), (3,4), (4,5), (5, 6)

Solution:
Simply list the first elements, then second:

domain: {2,3,4,5} range:{3,4,5,6}

Example:
Find the domain and range of the following relation.



Solution:
First, find the coordinates of the BIG points:
(2,4), (7,9), (10,-3)

Domain: {2,7,10}

Range: {4,9,-3}

**It's OK that there is no apparent pattern...this is still a relation.

Find each of the following.

38. Find the domain for the following relation.

(−2,4), (3,5), (5,7), (8,11)

~~Domain: {-2,3,5,8}~~

$\{x \mid x = -2, 3, 5, 8\}$

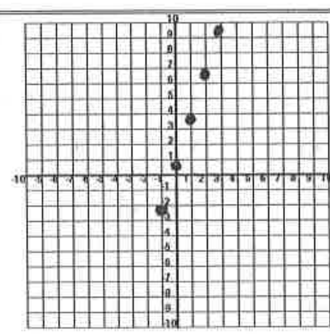
39. Find the range for the relation below.

(2,3), (4,3), (6,3), (8,3)

~~Range: {3,3,3,3}~~

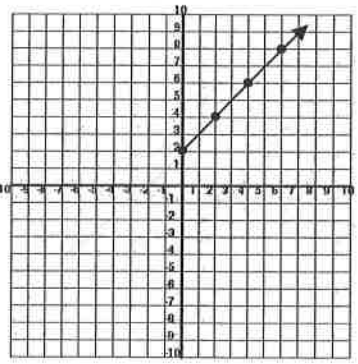
$\{y \mid y = 3\}$

40. Find the domain for the graphed relation.



$\{x \mid x = -1, 0, 1, 2, 3\}$

41. **Challenge Question:** Find the domain of the following graph.



~~$\{x \mid x = 0, 2, 4, 6\}$~~

$\{x \mid x \geq 0, x \in \mathbb{R}\}$

$[0, \infty)$

42. How many items are there in the domain of the relation above?

~~4~~ infinite ∞

43. What is the smallest item in the domain?

0

44. What is the biggest value in the domain?

~~6~~ infinite

45. How many items are there in the range?

infinite ∞

46. What is the smallest item in the range?

2

47. What is the biggest item in the range?

infinite ∞

Domain & Range of Continuous Data (Lines and Curves): [Definition on page 25]

When the graph of a relation is a line or curve, the domain and range cannot be expressed as a list of numbers as in the earlier questions. Why is this so?

Consider Example A and B.

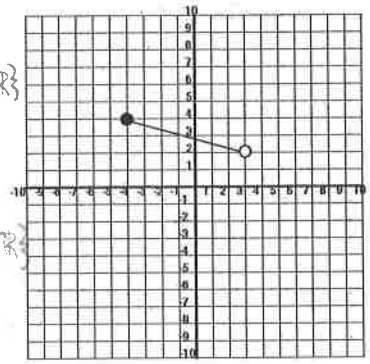
Example A

{x | -4 ≤ x < 3, x∈R}

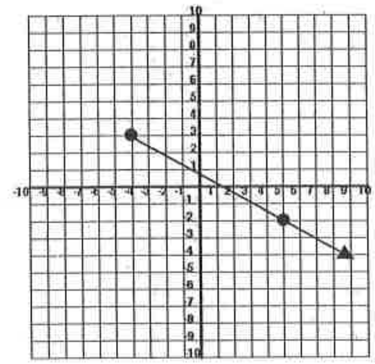
[-4, 3)

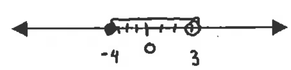
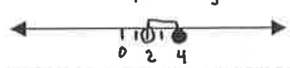
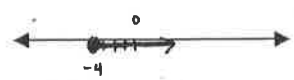
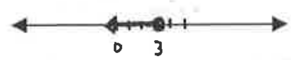
{y | 2 < y ≤ 4, y∈R}

(2, 4]

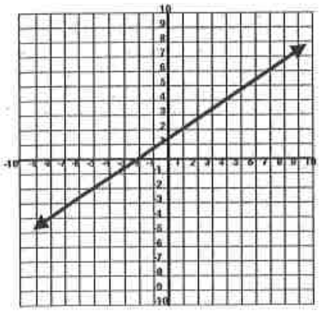


Example B



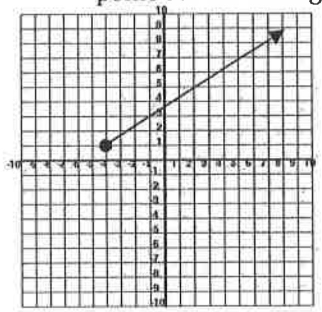
Use Inequalities	Use Interval Notation	Use a number line
Example A Domain: $-4 \le x < 3$ Range: $2 < y \le 4$	Example A Domain: $[-4,3)$ Range: $(2,4]$	Example A Domain:  Range: 
Example B Domain: $x \ge -4$ Range: $y \le 3$	Example B Domain: $[-4,\infty)$ Range: $(\infty,3]$	Example B Domain:  Range: 
The inequality symbols: $<, >, \le, \ge, \ne$ **Set Notation:** $x\epsilon R$: The domain is the set of real numbers. $\{y \mid y \le 0, y\epsilon R\}$: The range is the set of real numbers less than or equal to zero.	Brackets are used to show the interval. [if the number is included (if the number is not included ∞ is used if the set does not end. (∞,∞): No upper or lower limit, or, "all real numbers". $(3,\infty)$: All real numbers greater than 3.	<u>Solid</u> circles indicate the number is included. <u>Hollow</u> circles indicate the number is not included.

48. If a relation continues in both directions:



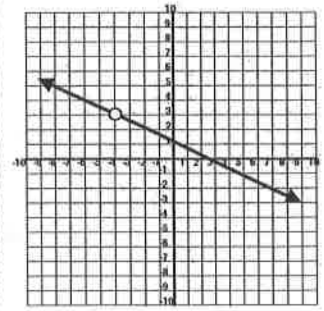
Use Interval Notation:
Domain: (∞, ∞)
Range: (∞, ∞)

49. The relation has a starting point but no ending point:

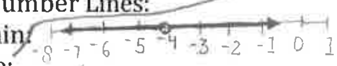


Use Inequalities:
Domain: $\{x \mid x \geq -4, x \in \mathbb{R}\}$
Range: $\{y \mid y \geq 1, y \in \mathbb{R}\}$

50. The relation has a **non-permissible** value:

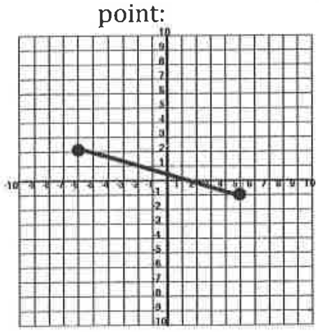


Use Number Lines:
Domain:



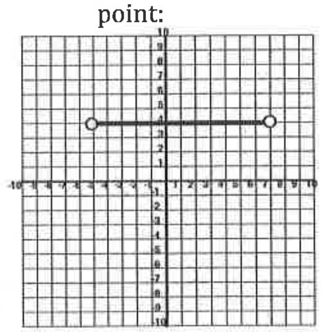
Range:

51. The relation has a starting point and a finishing point:



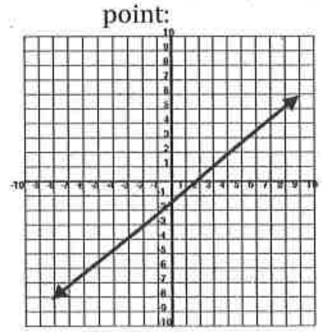
Use words:
Domain: x is such that it is greater or equal to -6 and less than or equal to 5, x belongs to the set of real numbers.
Range: y is such that y is greater than or equal to -1 and less than or equal to 2, y belongs to the set of real numbers.

52. The relation has a starting point and a finishing point:



Use Inequalities:
Domain: $\{x \mid -5 < x < 7, x \in \mathbb{R}\}$
Range: $\{y \mid y = 4, y \in W\}$

53. The relation has no starting point or finishing point:



Use Interval Notation:
Domain: (∞, ∞)
Range: (∞, ∞)

Write a set of instructions for finding the domain of a function in:

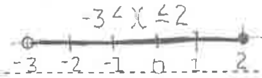
54. Interval Notation:

lower limit, upper limit

exclude : ( )
include : [ ]

$[0, \infty)$ used for infinity : ∞)

$-3 < x \leq 2$



$= (-3, 2]$

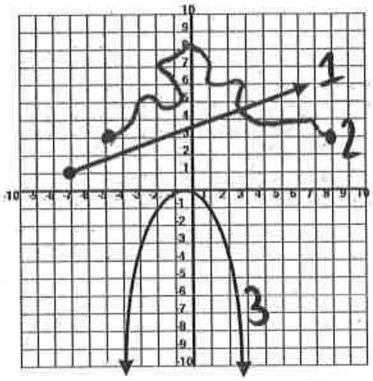
55. Using a Number Line:

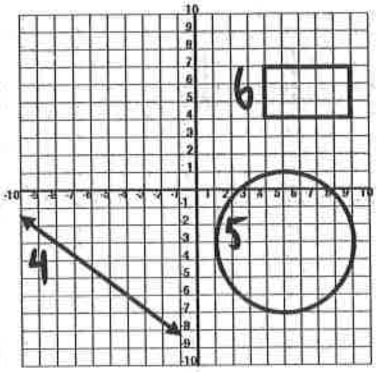
D:  $\{x \mid x > -4, x \in \mathbb{R}\}$ $(-4, \infty)$

R:  $\{y \mid y < 9, y \in \mathbb{R}\}$ $(\infty, 9)$

56. Using Inequalities:

57. Try to match each of the following graphs with domain and range below. (There are three on each graph)





4 **A.** $x \epsilon R, \quad y \epsilon R$

5 **B.** $[1,9]$ *and* $[-7,1]$

3 **C.** $\{x|x\epsilon R\}, \ \{y|y \leq 0, y\epsilon R\}$

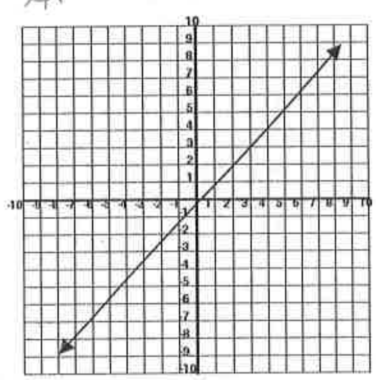
6 **D.** *domain*$[4,9]$, *range*$[4,7]$

1 **E.** $\{x|x \geq -7, x\epsilon R\}, \quad \{y|y \geq 1, y\epsilon R\}$

2 **F.** Domain is all real numbers from -5 to 8. Range is all real numbers from 3 to 8.

Find the domain and range for each of the following graphs.

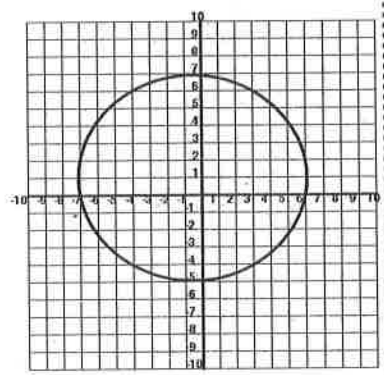
58. Use set notation:



domain $\{x \in R\}$

range $\{y \in R\}$

59. Use interval notation:

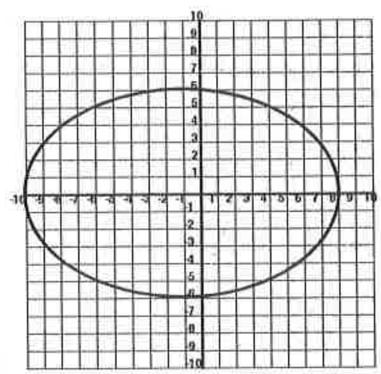


domain $[-7, 6]$

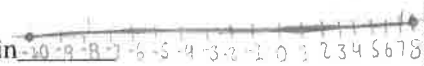
range $[-5, 7]$

☆ Write smallest first ☆

60. Use number lines:



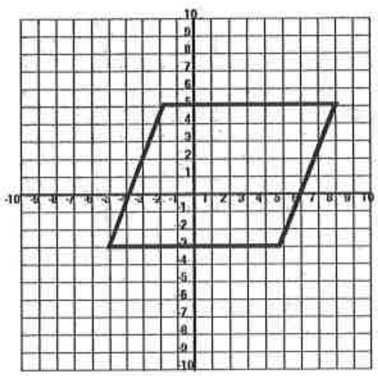
domain



range

Find the domain and range for each of the following graphs.

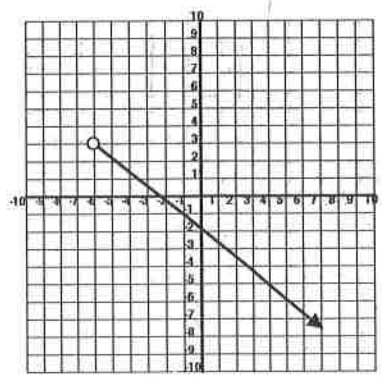
61. Use set notation:



domain $\{x \mid -5 \le x \le 8, x \in \mathbb{R}\}$

range $\{y \mid -3 \le y \le 5, y \in \mathbb{R}\}$

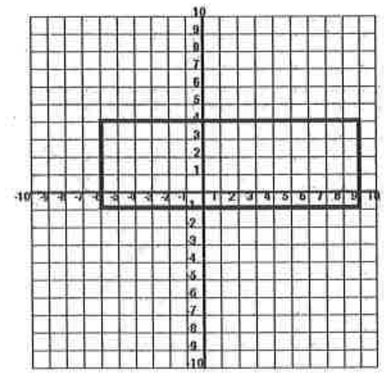
62. Use interval notation:

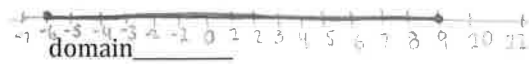


domain $(-6, \infty)$

range $(\infty, 3)$

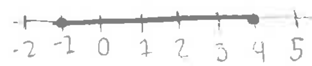
63. Use number lines:



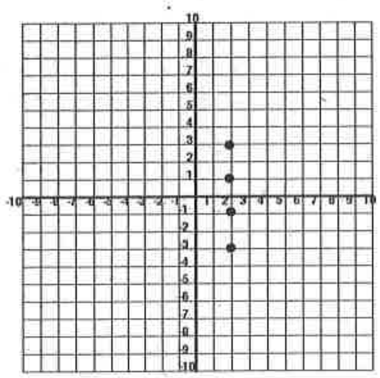


domain__________

range__________



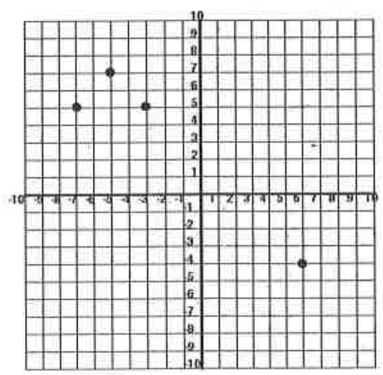
64. Use a list (discrete):



domain $\{x \mid x = 2, x \in \mathbb{Z}\}$

range $\{y \mid y = -3, -1, 1, 3, y \in \mathbb{Z}\}$

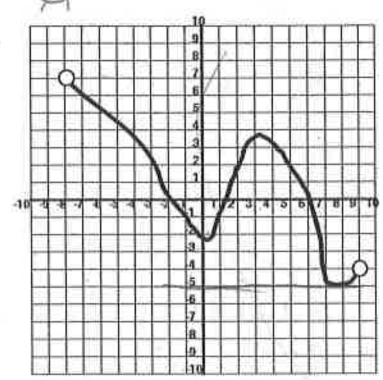
65. Use a list (discrete):



domain $\{x \mid x = -7, -5, -3, 6, x \in \mathbb{Z}\}$

range $\{y \mid y = -4, 5, 7, y \in \mathbb{Z}\}$

66. Use interval notation:

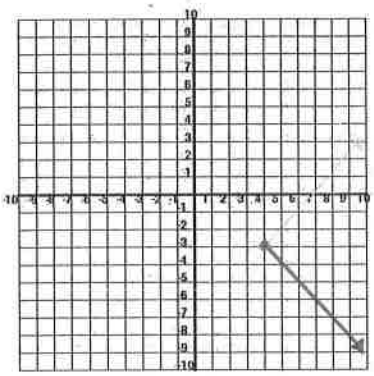


domain $(-8, 9)$

range $[-5, 7)$

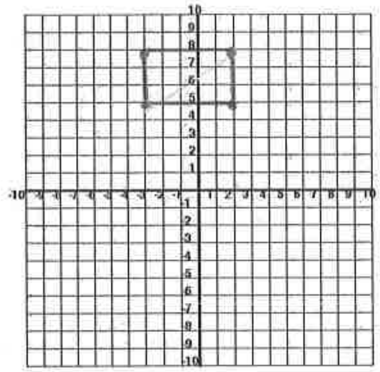
-3 5
-2 6
-1 7
0 8

67. Draw a graph using the following domain and range.



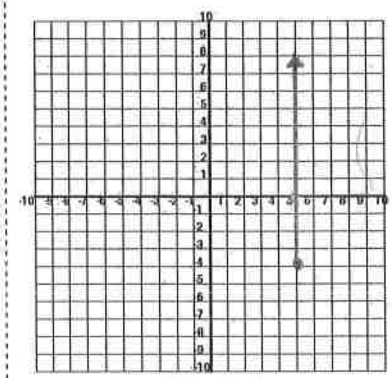
$\{x|x \geq 4, x \epsilon R\}$,
$\{y|y \leq -3, y \epsilon R\}$

68. Draw a graph using the following domain and range.



$\{x|-3 \leq x \leq 2, x \epsilon R\}$
$\{y|5 \leq y \leq 8, y \epsilon R\}$

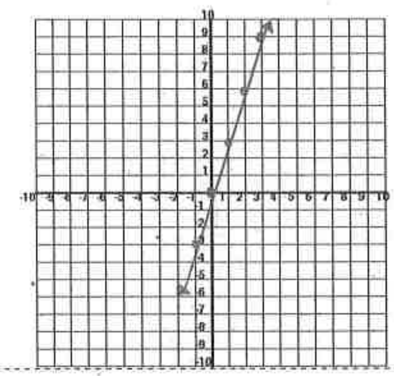
69. Draw a graph using the following domain and range.



$\{x|x = 5\}$,
$\{y|y \geq -4, y \epsilon R\}$

70. **Challenge Question:**
Graph the relation represented by the equation $y = 3x$.

6	2
9	3
-3	-1
0	0
3	1
-6	-2

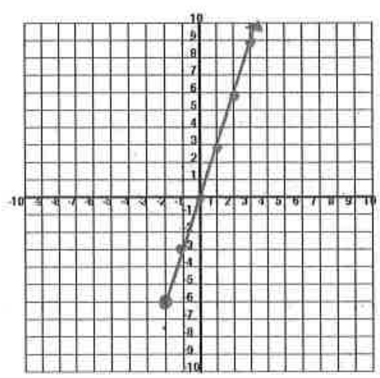


71. What is the domain of $y = 3x$?

★ $\{x|x \in \mathbb{R}\}$ ★ — book answer
(∞, ∞)

72. **Challenge Question:**

Graph the line represented by the equation $y = 3x$ if the domain is $x \geq -2$.



★NOT CLOSED CIRCLE★

✓

Graphing Relations and Domain:

When graphing equations earlier, we always assumed the domain was **all real numbers**. ..Not anymore.

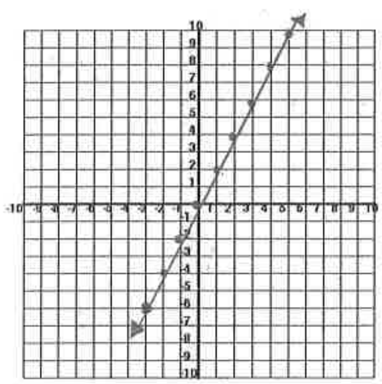
When graphing a relation, the domain may be:

- Given as $\boldsymbol{R}$ (all real numbers)
- Given as a list such as {-2,-1,0,1,2}
- Given as an inequality such as $x \geq 0$.

We will consider the impact each of these have when graphing the relation $y = 2x$

73. Graph the relation $y = 2x$.

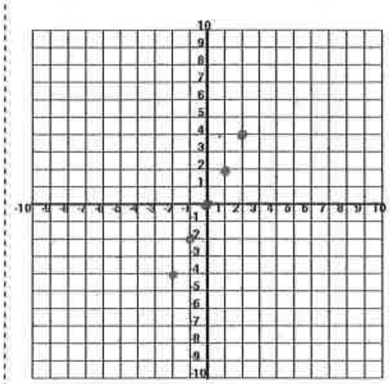
Domain: $\{x|x\epsilon R\}$



See next page for solution

74. Graph the relation $y = 2x$.

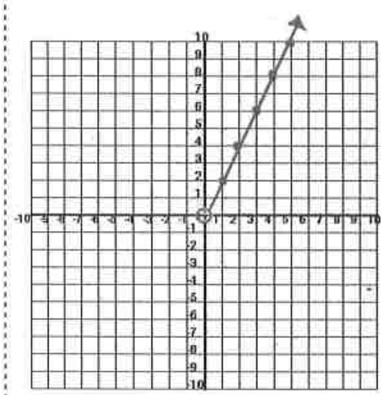
The domain is **{-2,-1,0,1,2}**



See next page for solution

75. Graph the relation $y = 2x$.

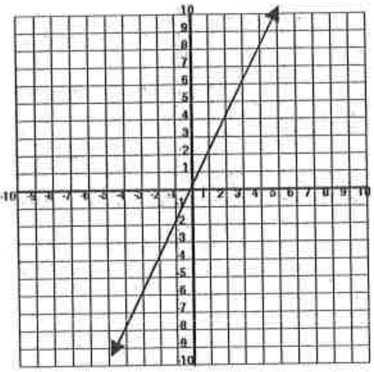
Domain: $(0, \infty)$



See next page for solution

Some notes here...

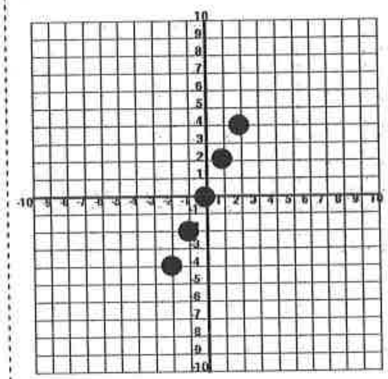
Graph the relation $y = 2x$.
The domain is $\{x|x\epsilon R\}$.



Any values of x would be permissible. This results in a continuous line in both directions.

Two arrow heads!

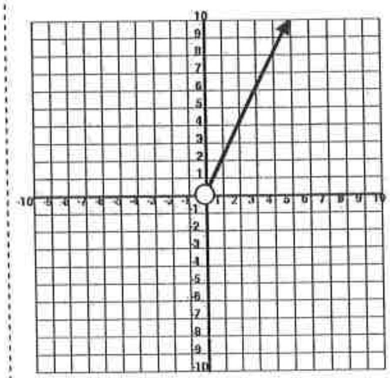
Graph the relation $y = 2x$.
The domain is **{-2,-1,0,1,2}**



Only the specified values of x would be permissible {-2,-1,0,1,2}. This results in five discrete points on the graph.

Find the y values that go with these five x values.

Graph the relation $y = 2x$.
Domain: $(0, \infty)$.

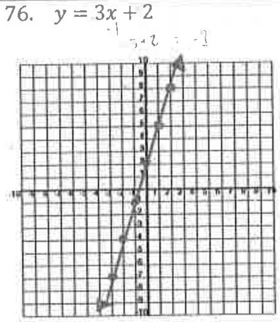


Any values of x greater than 0 would be permissible. This results in a continuous line starting at x=0 and moving in the positive direction.

One arrow head!

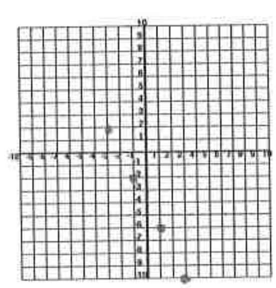
Graph each of the following for the given domain.

76. $y = 3x + 2$



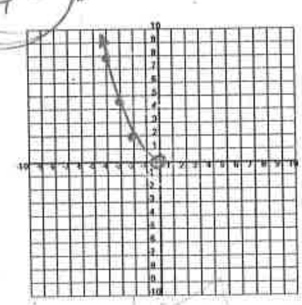
Domain: $\{x|x\epsilon R\}$

77. $y = -2x - 4$



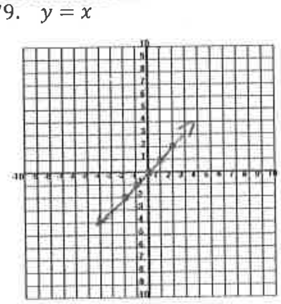
Domain is $\{-3, -1, 1, 3\}$.

78. $y = \frac{1}{2}x^2$



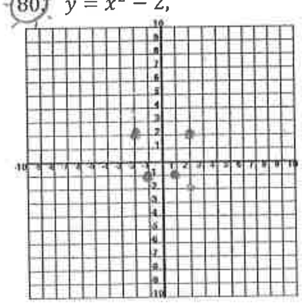
Domain is $(\infty, 0)$ — mean 0 = biggest?

79. $y = x$



Domain is $\{x| -2 \le x \le 2,\ x\epsilon R\}$

80. $y = x^2 - 2$,

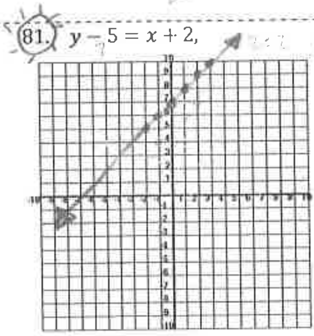


Domain is $\{-2, -1, 2, 1\}$.

x	y
-2	2
-1	-1
1	-1
2	2

X

81. $y - 5 = x + 2$,



Domain is $\{x|x\epsilon R\}$

$y - 5 = x + 2$
$+5 \quad +5$
$y = x + 7$

X	Y
0	7
1	8
2	9
3	10
4	11

82. In your own words, describe the different ways a relation may look due to restrictions on the domain.

- It will not have aline
- just points
- no arrows

Finding the domain and range of an equation.

Becoming more familiar with the equation of particular relations (assuming there is one) allows you to quickly determine the domain or range.

Possible Strategies:

- Visualize the graph from memory (or actually plot it).
- Consider possible restrictions based on the equation. For example, $y = \sqrt{x}$ has a domain $x \geq 0$ because all negative values of x produce a "not real" output.

83. Find the domain of the relation:
$y = 3x$

(∞, ∞)

OR

$\{x \mid x \in \mathbb{R}\}$

84. Find the domain of the relation:
$y = \sqrt{x - 2}$

$\{x \mid x \geq 2, x \in \mathbb{R}\}$

OR

$[2, \infty)$

85. Find the domain of the relation:
$y = x^2$

$\{x \mid x \in \mathbb{R}\}$

OR

(∞, ∞)

86. Find the range of the relation:
$y = 3x$

$\{y \mid y \in \mathbb{R}\}$

OR

(∞, ∞)

87. Find the range of the relation:
$y = \sqrt{x - 2}$

$\{y \mid y \geq 0, y \in \mathbb{R}\}$

OR

$[0, \infty)$

88. Find the range of the relation:
$y = x^2$

$\cancel{\{y \mid y \in \mathbb{R}\}}$

~~OR~~

$\cancel{(\infty, \infty)}$

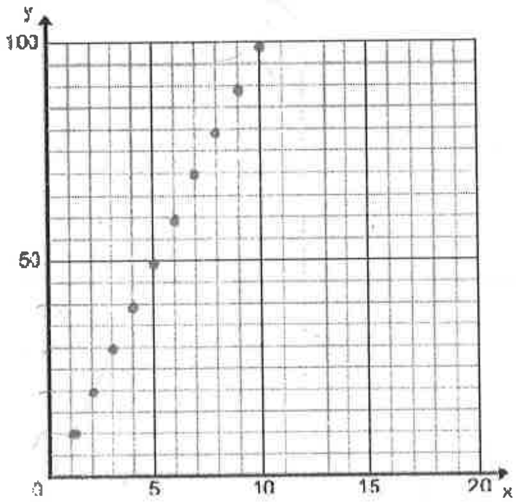
$\{y \mid y \geq 0, y \in \mathbb{R}\}$

$[0, \infty)$

89. **Challenge Question:**

Consider the various ways graphs look because of the restrictions on their domain before you answer the following question.

Use the equation $C = 10n$ to graph the cost, C, of a family with 'n' people to go to the movies.



★NO LINE- CAN'T HAVE 1/2 PPL! ★

90. **Challenge Question:**

Find a reasonable domain for the function above.

$\{x \mid x = 1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$

$x \mid x \in W$ $\{x \mid x \geq 0, x \in W\}$? why not?

$[0, \infty)$

Find a reasonable range for the function above.

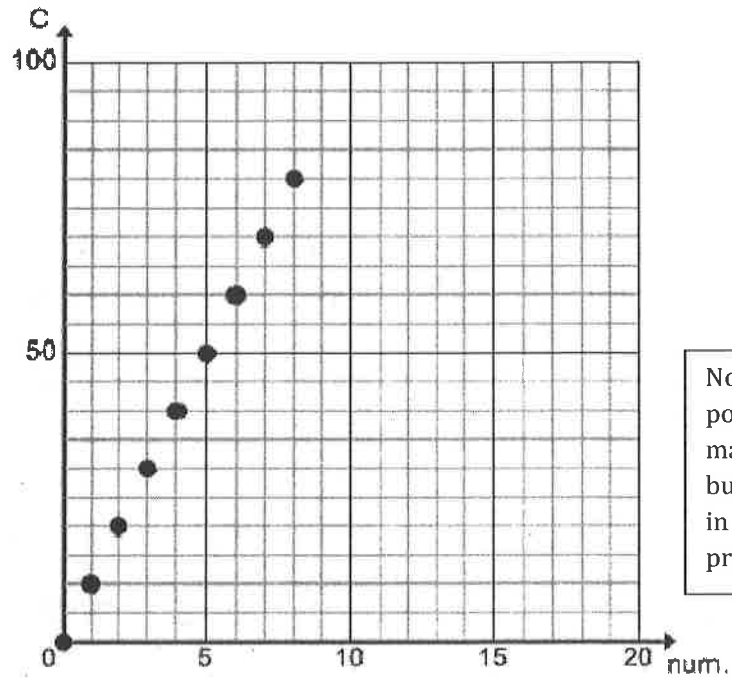
$\{y: 10, 20, 30, 40, 50, 60, 70, 80, 90, 100\}$

Some notes here...

Discrete and Continuous Data

From previous page...

Use the equation $C = 10n$ to graph the cost, C, of a family with 'n' people to go to the movies.



Note: The first two points are mathematically correct but do not make sense in terms of the problem.

91. Why is the graph above a series of dots, not a continuous graph?

Because there can't be decimals of people, just whole numbers (ex. 2, 3, 4...)

Continuous Data:	**Discrete Data**:
• Graph will appear as a line. • Occurs when quantities don't "skip" values...continuous things like time and temperature.	• Graphs will appear as a series of dots. • Quantities such as whole items (people, cars, hamburgers, etc.) • When part numbers don't "make sense."

Answer the following questions.

92. A cup of coffee sits on the counter for several hours. Describe what would happen to the temperature of the coffee in the cup.

The temperate would go down (become colder), gradually.

93. A cup of coffee sits on the counter for several hours. Temperature is a function of the time the coffee is on the counter.

a) What would be a reasonable domain for this function?

D {x | x ≥ 1, x ∈ ℝ}

b) Discrete or Continuous (Circle one)

94. A high school student is surveying the volume of traffic in the school parking lot. The number of cars in the parking lot is a function of the time of day.

a) What would be a reasonable range for this function?

R{0 ≤ y ≤ 100, y ∈ W} ~~R {y | 7 ≤ x ≤ 4~~

b) Discrete or Continuous (Circle one)

95. Dalleep is plotting his height as a function of time.

a) What quantity would represent the domain? Height or Time (Circle one)

b) Height: Discrete or Continuous (Circle one)

c) Time: Discrete or Continuous (Circle one)

d) What would be a reasonable range?

R {1ft ≤ y ≤ 7ft, y ∈ ℝ}

* remember: put upper limit!!

96. The Cost of Energy Bars:

Cost is a function of the number of energy bars.

Discrete or Continuous (Circle one)

97. Total Earnings (with hourly wage):

Earnings are a function of hours worked

Discrete or ~~Continuous~~ (Circle one)

★ ALL FUNCTIONS ARE RELATIONS, BUT ONLY SOME

Functions: RELATIONS ARE FUNCTIONS ★

A special class of relation in which **there is only one *output* (*y*) for every valid *input* (*x*).**

I find it easiest to determine if a relation is a function from a graph!

Consider the following tables:

Table 1

x	*y*
-1	3
0	0
2	-6
4	-12

Domain (*x*): {-1,0,2,4}

Range(*y*): {3,0,-6,-12}

$y = x^2$

Table 2

x	*y*
-2	4
-1	1
0	0
1	1

Domain (*x*): {-2,-1,0,1}

Range(*y*): {4,1,0,1}

★ NOT A FUNCTION ★

Table 3

x	*y*
4	2
3	7
2	12
4	17

Domain (*x*): {4,3,2,4}

Range(*y*): {2,7,12,17}

Notice in Table 3, when the input (*x*) is 4, there are two possible outputs, 2 <u>or</u> 17. This is **NOT** a function.

Tables 1 and 2 are both functions. Each element in the domain produces only **one** element in the range.

Some notes here possibly...

∴ = therefore

∵ = because

Which of the following relations are functions? Indicate why or why not.

98. (2,1), (3, 5), (7, 8), (9, 12)

YES or NO

Why? unique output points for each input

99. (1, 3), (5,3), (7,3), (12,3)

YES or NO

Why? unique output points for each input

100. (3,1), (3,5), (2,6), (1,4)

YES or NO

Why? ∵ for a valid input of 3 for x, there is not a unique output

101. $y = \pm\sqrt{x}$

$y = -\sqrt{5}$ $y = -\sqrt{x}$

$y = -\sqrt{5}$ $y = +\sqrt{x}$

YES or NO

$x = 4$

Why? ∵ there are 2 outputs for a given input.

102.

x	y
-3	7
2	3
5	5
1	7

YES or NO

Why? ∵ every input has a unique output

103.

x	y
-3	11
2	5
-3	-6
8	7

YES or NO

Why? ∵ -3 does not have a unique output.

x	y
4	-2
4	+2

$y = -\sqrt{4} = -2$

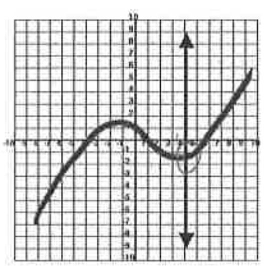
$= +\sqrt{4} = +2$

The Vertical Line Test:

You can test whether a relation is a function by using the vertical line test.

If you move a vertical line through the relation from left to right, the vertical line will only ever contact a function once. If the vertical line contacts the graph more than once at a given time, it is not a function.

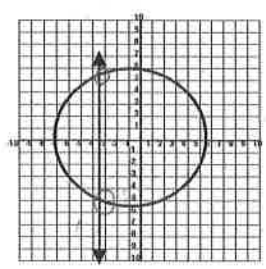
Eg.1.



Yes, it is a function.

A vertical line will contact the function only once.

Eg.2.

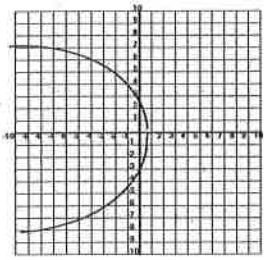


No, not a function.

A vertical line may contact the function twice.

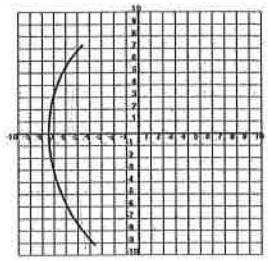
Determine if each of the following relations is a function or not.

104.



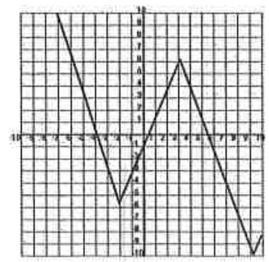
NO

105.



NO

106.



YES

107. Is the relation $y = 2x - 3$ a function?

YES or NO

How do you know?

∵ Every input will have a unique output.

108. Is the relation $y = x^2$ a function?

YES or ~~NO~~

How do you know?

~~∵ each input will have two outputs. when squaring a number, the answer will be positive.~~

b/c every input will have a unique output.

109. Is the relation $\sqrt{x} = \sqrt{y^2}$ a function?

~~Y: √x~~

YES or NO

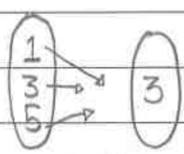
How do you know?

∵ each input will have a positive AND negative output

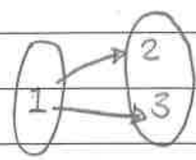
Ex. $4 = (2)^2$ or $4 = (-2)^2$

Some notes here possibly…

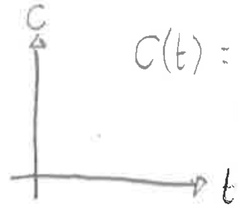
If you have a domain of numbers that equal the same numbers that's okay



OKAY!



NOT A FUNCTION!



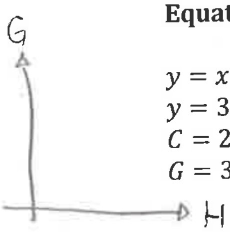
$C(t) = 20t + 1200$

Function Notation:

☆ time (always on horizontal - x axis) ☆

There is a special way to write functions. This is called function notation.

Consider the following comparisons:



Equation		**Function Notation**
$y = x + 2$	⟶	$f(x) = x + 2$
$y = 3x - 5$	⟶	$f(x) = 3x - 5$
$C = 20t + 1200$	⟶	$C(t) = 20t + 1200$
$G = 3h^2 - 2$	⟶	$g(h) = 3h^2 - 2$

> Notice the $f(x)$ part simply replaces the y. There is no new operation; it is only a new way of writing the equation. It immediately tells you "this is a function."
> Notice the letter in brackets is always the same as the one on the right.

- Function notation allows us to use letters appropriate to our function and differentiate between several functions (give them unique names).
- Also the notation tells us which variable is **dependent** on the other.

Eg. $g(h) = 3h^2 - 2$ tells us that function g is written in terms of h. That is, *g depends on h.*

Function notation can also be used to tell us to perform an operation.

Evaluate $f(2), f(-3), f(x + 2)$ for the function $f(x) = 3x + 7$

$f(2) = 3(2) + 7$

$f(2) = 13$

$f(-3) = 3(-3) + 7$

$f(-3) = -2$

$f(x + 2) = 3(x + 2) + 7$

$f(x + 2) = 3x + 6 + 7$

$f(x + 2) = 3x + 13$

If $f(x) = 5x - 6$, find

110. $f(4)$

$f(x) = 5x - 6$
$f(4) = 5(4) - 6$
$\boxed{f(4) = 14}$

111. $f(-1)$

$f(x) = 5x - 6$
$f(-1) = 5(-1) - 6$
$f(-1) = -5 - 6$
$\boxed{f(-1) = -11}$

112. $f(-3 + x)$

$f(x) = 5x - 6$
$f(-3+x) = 5(-3+x) - 6$
$f(-3+x) = -15 + 5x - 6$
$\boxed{f(-3+x) = 5x - 21}$

If $g(x) = 2x - 4$, find

113. $g(4)$

$g(x) = 2x - 4$
$g(4) = 2(4) - 4$
$g(4) = 8 - 4$
$\boxed{g(4) = 4}$

114. $g(-1)$

$g(x) = 2x - 4$
$g(-1) = 2(-1) - 4$
$g(-1) = -2 - 4$
$\boxed{g(-1) = -6}$

115. $g(x - 1)$

$g(x) = 2x - 4$
$g(x-1) = 2(x-1) - 4$
$g(x-1) = 2x - 2 - 4$
$\boxed{g(x-1) = 2x - 6}$ ✓

y, when x = x - 1

✓