O-B. → student info → student links- Hmw by 7 → MS-Fisher → Blk 4 → Holiday Hmw → signup

10

HW Mark:	10	9	8	7	6	RE-Submit

Characteristics of Linear Relations

+MIDPOINT PG.30

SLOPE & INTERCEPTS

45 31 103 69

This booklet belongs to: Marissa Period 4

LESSON #	DATE	QUESTIONS FROM NOTES	Questions that I find difficult
1	Dec.3/14	Pg. 4-11	
2	Dec.10/14	Pg. 12-18	42, 47, 54, 56
3	Dec.11/14	Pg. 19-25	76, 97, 98, 101, 103
		Pg.	
		Pg.	
		Pg.	
		Pg.	
		Pg.	
		Pg.	
		Pg.	
		REVIEW	
		TEST	

60 58 59 69 70

Your teacher has important instructions for you to write down below.

~~58, 59, 60~~, 69, 70, 76, 97, 98, 101, 103

Characteristics of Linear Relations

Relations SPECIFIC OUTCOMES		TOPICS	REVIEW Note or Example
Demonstrate an understanding of slope with respect to: • rise and run • line segments and lines • rate of change • parallel lines • perpendicular lines.	3.1	Determine the slope of a line segment by measuring or calculating the rise and run.	
	3.2	Classify lines in a given set as having positive or negative slopes.	
	3.3	Explain the meaning of the slope of a horizontal or vertical line.	
	3.4	Explain why the slope of a line can be determined by using any two points on that line.	
	3.5	Explain, using examples, slope as a rate of change.	
	3.6	Draw a line, given its slope and a point on the line.	
	3.7	Determine another point on a line, given the slope and a point on the line.	
	3.8	Generalize and apply a rule for determining whether two lines are parallel or perpendicular.	
	3.9	Solve a contextual problem involving slope.	
Determine the characteristics of the graphs of linear relations, including the: • intercepts • slope • domain • range.	5.1	Determine the intercepts of the graph of a linear relation, and state the intercepts as values or ordered pairs.	
	5.2	Determine the slope of the graph of a linear relation.	

[C] Communication [PS] Problem Solving, [CN] Connections [R] Reasoning, [ME] Mental Mathematics [T] Technology, and Estimation, [V] Visualization

Characteristics of Linear Relations

Key Terms

Term	Definition	Example
Line		
Line segment		
Linear relation		
Slope		
Positive slope		
Negative slope		
Zero slope		
Undefined slope		
Intercepts		
Parallel lines		
Parallel slopes		
Perpendicular lines		
Perpendicular slopes		
Midpoint formula		
Distance formula		
Parallelogram		

Linear Relations:

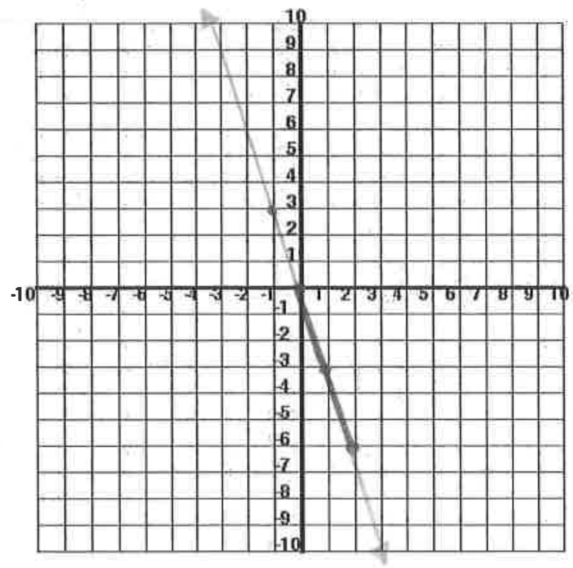
- A relationship between two quantities that when graphed will produce a **straight line**.
- One quantity **increases or decreases at a constant rate** with respect to another.

Eg. $y = -3x$ $C = 50n + 1000$ $p = 40q$

LINE SEGMENT: A part of a line that has two endpoints and includes all the points between the endpoints.

1. Using a dashed or coloured line, graph the relation represented by the equation $y = -3x$.

2. Using a solid or different coloured line graph the same relation if the domain is $0 \le x \le 2$.



The solid section you just plotted is a line segment, a section of the dashed line.

3. What are the endpoints of the line segment? (0,0) and (2,-6)

4. What are the endpoints of the dashed line? (∞, ∞)

5. What are 5 properties you could use to describe the line segment above?

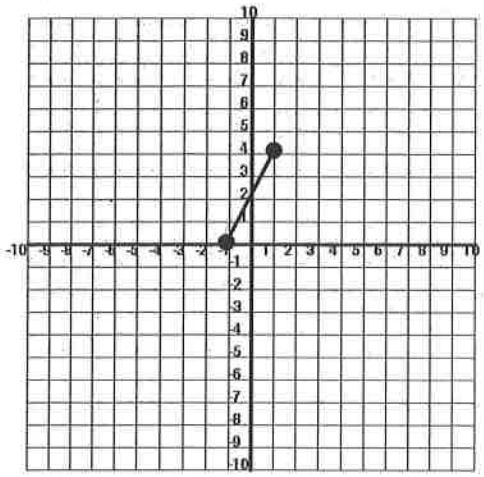
1. negative slope
2. definite endpoint.
3. no gaps
4. straight line
5. slope $= \frac{3}{1}$

6. Which of these properties are also true for the dashed line above?

- negative slope
- no gaps
- straight line
- slope $= \frac{3}{1}$

Slope of a Line (or Line Segment): (Rate of Change)

Consider the line segment below.



7. What is the vertical change (rise) between the endpoints?

4

8. What is the horizontal change between the two endpoints?

2

9. What is the ratio of rise to run as a fraction?

$\frac{4}{2} = \frac{2}{1} = 2$

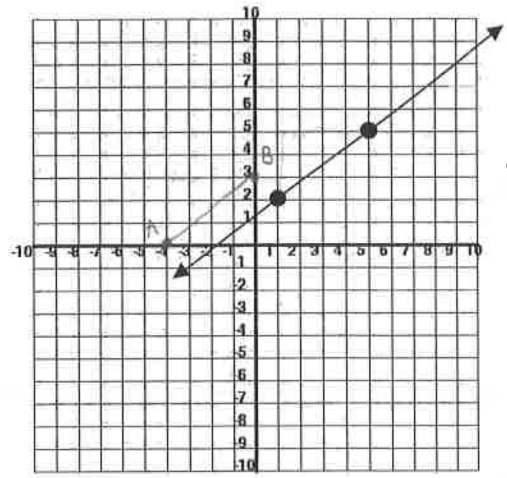
10. How fast does the relationship change in the vertical direction when compared to the horizontal direction?

☆ twice as fast ☆

Your notes here...

11. **Challenge Question:**

Find the slope (rate of change) of the line below.



$m = \boxed{\frac{3}{4}}$

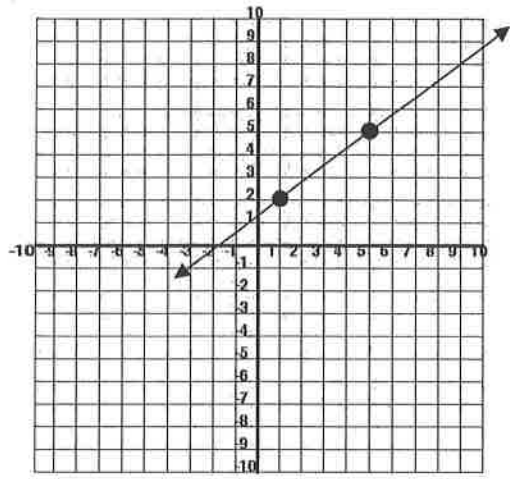
12. **Challenge Question:**

Find the slope (rate of change) of the line segment with end points at A(-4,0) and B(0,3).

$m = \boxed{\frac{3}{4}}$

$\frac{3-0}{0-(-4)} = \boxed{\frac{3}{4}}$

Find the slope (rate of change) of the line below.



Recall:

Slope is the ratio of $\frac{Rise}{Run}$. We can count gridlines from point-to-point to get $\frac{rise:\ +3}{run:\ +4} = \frac{3}{4}$.

NOTE:

If you started at the right point…

$\frac{rise: -3}{run:\ -4} = \frac{3}{4}$ we would be moving in the "negative" direction but the slope calculated would be the same.

Find the slope (rate of change) of the line segment with end points at A(-4,0) and B(0,3).

Strategy 1: Plot the points on a grid and follow the same solution strategy to the left.

Strategy 2:

We can see the rise is actually a change in the y-direction…a difference in the y-values.
For the points: A(-4,0) and B(0,3)

$$rise: y - y = 3 - 0 = 3$$
$$run: x - x = 0 - (-4) = 4$$

Therefore $slope = \frac{Rise}{Run} = \frac{3}{4}$.

IMPORTANT
TO USE THIS STRATEGY…you must be consistent with your "starting" x and y values in calculating rise and run.

Note the formula on the next page to help you do this.

Slope of a Line (or Line Segment)

Slope is the measure of the "steepness" of a line. It is represented with the symbol (m).
Slope also describes the direction of the line.

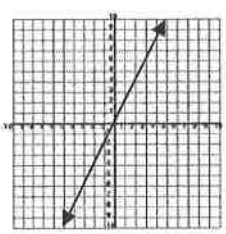
The slope is found by dividing the vertical change (the rise or fall) by the horizontal change (the run).

$$m = \frac{rise}{run} \quad or \quad m = \frac{y_2 - y_1}{x_2 - x_1}$$

Be Careful with Negatives!

Positive Slope

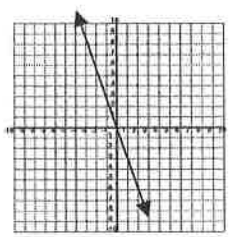
Eg. $m = 2$



Rises from left to right.

Negative Slope

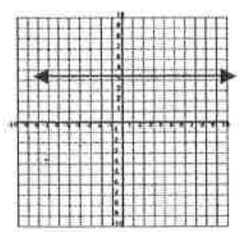
Eg. $m = -3$



Falls from left to right.

Zero Slope

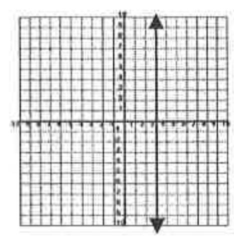
Eg. $m = 0$



Rise is 0.
0 divided by any "run" will still = 0.

Undefined Slope

Eg. $m = \infty$



Think... the run is 0.
Division by 0 is undefined.

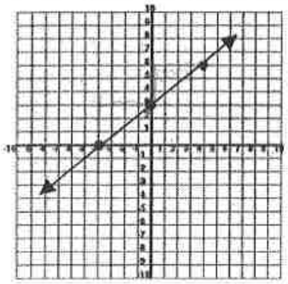
13. Describe, in your own words, how you find the slope of a line segment.

find the difference of the rise and the difference of the run and divide.

14. How does a line segment differ from a line?

A line segment has definite end points whereas a line has indefinite end points.

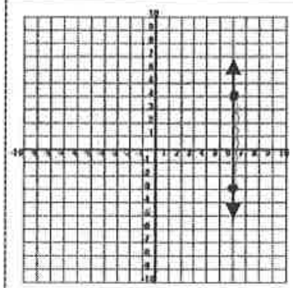
Find the rise, the run and the slope for the following lines by counting units.
In most cases, you will need to pick two points on the line to use.



15. rise= 3

run= 4

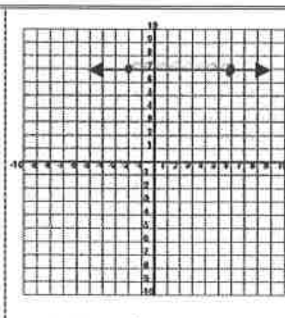
slope= $\frac{3}{4}$



16. rise= 7

run= 0

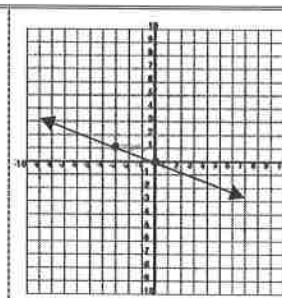
slope= ~~indefinite~~
★ undefined ★



17. rise= 0

run= 8

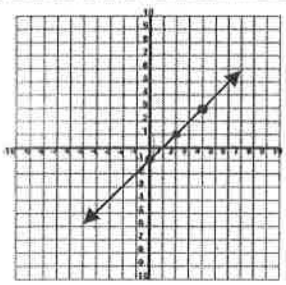
slope= 0



18. rise= 1

run= -3

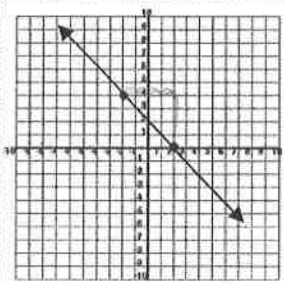
slope= $-\frac{1}{3}$



19. rise= 2

run= 2

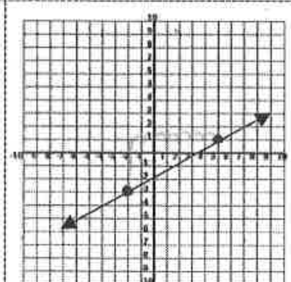
slope= $\frac{2}{2} = \frac{1}{1} = \boxed{1}$



20. rise= 4

run= -4

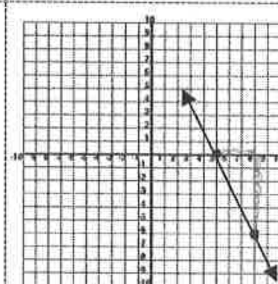
slope= $-\frac{4}{4} = \boxed{-1}$



21. rise= 4

run= 7

slope= $\boxed{\frac{4}{7}}$



22. rise= 6

run= -3

slope= $-\frac{6}{3} = \boxed{-2}$

Use the formula $m = \frac{y_2 - y_1}{x_2 - x_1}$ to find the slopes of line segments with the following endpoints.

23. (0,0) and (2,3)

$$\frac{y_2 - y_1}{x_2 - x_1}$$

$$\frac{3-0}{2-0} = \boxed{\frac{3}{2}}$$

24. (1,3) and (2,7)

$$\frac{y_2 - y_1}{x_2 - x_1}$$

$$\frac{7-3}{2-1} = \frac{4}{1} = \boxed{4}$$

25. (−5,7) and (−4,−2)

$$\frac{y_2 - y_1}{x_2 - x_1}$$

$$\frac{-2-7}{-4-(-5)} = \frac{-9}{1} = \boxed{-9}$$

$\frac{-6-4}{\frac{2}{1}-\frac{1}{2}} = \frac{-6-4}{\frac{4}{2}-\frac{1}{2}} = \frac{-10}{\frac{3}{2}} \rightarrow \frac{-10}{1} \times \frac{2}{3} = \boxed{\frac{-20}{3}}$

Use the formula $m = \frac{y_2 - y_1}{x_2 - x_1}$ to find the slopes of line segments with the following endpoints.

26. (5,7) and (5,3)

$\frac{y_2 - y_1}{x_2 - x_1}$

$\frac{3-7}{5-5} = \frac{-4}{0} = \boxed{\text{undefined}}$

27. Find the coordinates of any another point on this line.

(5, -1)

28. (−4,5) and (6,5)

$\frac{y_2 - y_1}{x_2 - x_1}$

$\frac{5-5}{6-(-4)} = \frac{0}{10} = \boxed{0}$

29. Find the coordinates of any another point on this line.

(16, 5)

30. $\left(\frac{1}{2}, 4\right)$ and (2, −6)

$\frac{y_2 - y_1}{x_2 - x_1}$

$\frac{-6-4}{2-0.5} = \frac{-10}{1.5} \rightarrow \frac{-10}{1} \times \frac{1.5}{1}$

$\frac{-15}{1} \rightarrow \cancel{\boxed{-15}}$

31. Find the coordinates of any another point on this line.

(5, -26)

32. The slope of a line is -2. If the line passes through (t, -1) and (-4,9), find the value of t.

$\frac{y_2 - y_1}{x_2 - x_1} = \frac{9-(-1)}{-4-t} = \frac{-2}{1} \quad \frac{10}{5} \quad \boxed{t = 1}$

33. The slope of a line is $-\frac{3}{2}$. If the line passes through (5,2) and (b,-4), find the value of b.

$\frac{y_1 - y_2}{x_1 - x_2} = \frac{-4-2}{b-5} = \frac{-3}{2} \rightarrow \boxed{b = 9}$

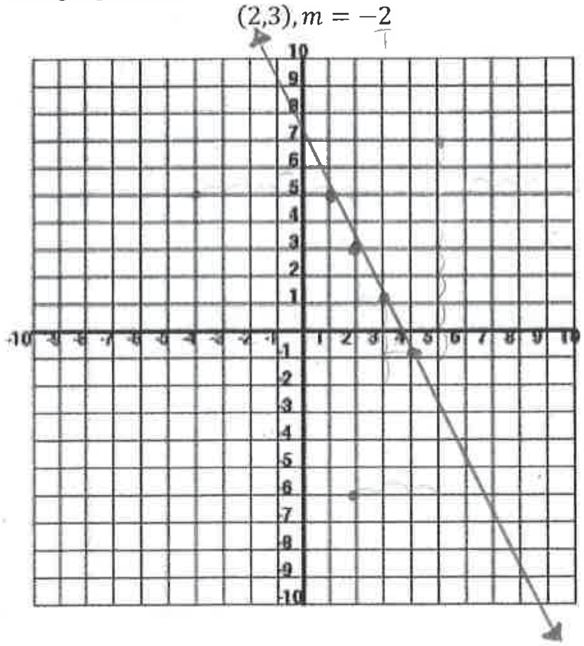
9

-6

34. Challenge

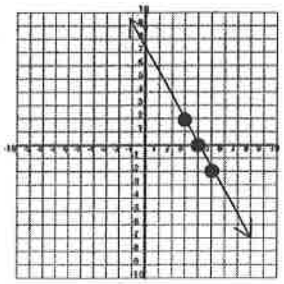
Given a point on the line and the slope, sketch the graph of the line.

(2,3), $m = -2$



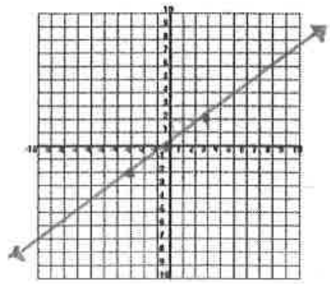
Given a point on the line and the slope, sketch the graph of the line.

35. $(2,3), m = -2$

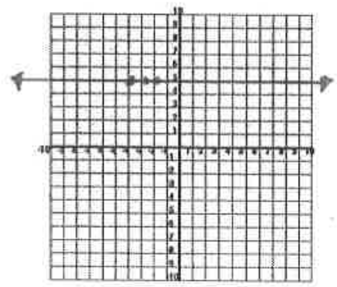


1. Plot the point: (2,3)
2. Use $\frac{rise}{run} = \frac{-2}{1}$ to get a second point…and a third.
3. Connect with a line.

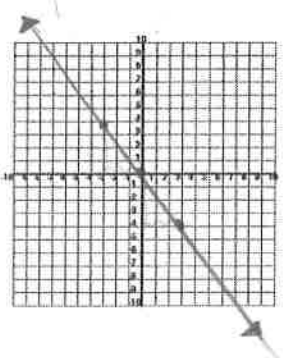
36. $(-3,-2), m = \frac{2}{3}$



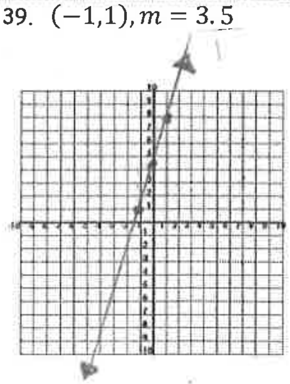
37. $(-4,5), m = 0$



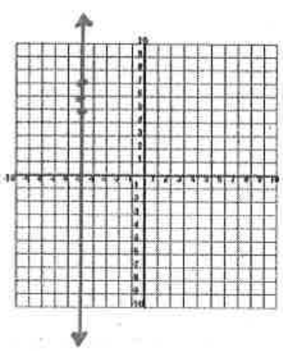
38. $(-3,4), m = -\frac{4}{3}$



39. $(-1,1), m = 3.5$



40. $(-5,7), m\ is\ undefined$

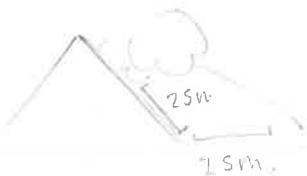


Slope is a measure of **Rate of Change** for a relation. That is, how fast one quantity increases or decreases in respect to another.

Answer the following questions regarding slope and rate of change.

41. A fallen tree leans against a vertical cliff. The tree was 15 m from the cliff and now rests against the cliff 25 m from the ground.

Find the positive slope of the fallen tree.



$\frac{25}{15} = \boxed{\frac{5}{3}}$

42. A section of roller coaster falls 52 m in a horizontal distance of 4 m.

Find the slope of this section of track?

$-\frac{52}{4} = -\frac{13}{1} = \boxed{-13}$ X

? ☆ 13 ☆ ?

43. The cost for 8 students to go to the movies is $80.

What is the cost per student, or rate?

$\frac{\$80}{8} = \frac{8n}{8}$

$10 = n

$\boxed{\$10 \text{ per student}}$

44. Write two ordered pairs for this relation.

(2, 20), (3, 30)

45. To fill my gas tank that holds 70 litres, I paid $68.53.

What is the rate for gasoline per litre (in cents to the nearest tenth)?

$\frac{68.53}{70} = \frac{70n}{70}$

$0.979 = n$

X $\boxed{\$0.9 \ /L}$

$\boxed{97.9¢/L}$

46. TSpray drove 735 kilometres in 7 hours.

Find his rate of travel per hour.

$\frac{735}{7} = \frac{7h}{7}$

$105 = h$

$\boxed{105 \text{ km/h}}$

47. What name is given to this quantity?

☆ speed? ☆

☆ $\frac{\text{distance}}{\text{time}}$ = speed ☆

$\frac{68.53\$}{70L}$

720 = F + 22h
210 = F + 5h
510 = 17h
17 17
30 = h

Answer the following questions regarding slope and rate of change.

48. A round of golf for a group of hackers consists of the "green fee" and the club rental fee. Clubs are rented on a fee per club basis. Jack pays $72.25 for his green fee and 3 clubs, and Jill pays $95 for her green fee and 10 clubs. What is the rate to rent one club?

Jill 95 = G + 10c
Jack 72.25 = G + 3C

$\frac{22.75}{7} = \frac{7C}{7}$

$\boxed{\$3.25 = C}$

95 = G + 10(3.25)
95 = G + 32.5
-32.5 -32.5
62.5 = G

What is the green fee?

$\boxed{\$62.50}$

49. Pro-lectric charges their customers a fixed cost plus an hourly rate. To work in my basement, they charged me $210 for 5 hours work. To complete my upstairs renovations they charged me $720 for 22 hours work. What is the hourly rate?

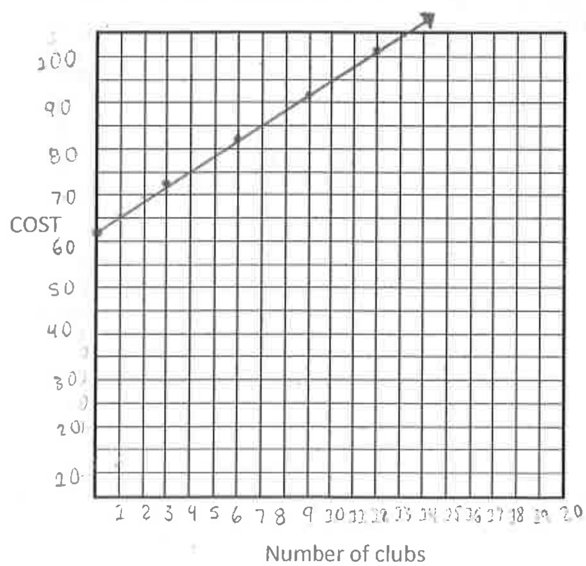
720 = F + 22h
210 = F + 5h
720 = 22h
210 = 5h
$\frac{510}{17} = \frac{17h}{17}$ → \$30 = h → $\boxed{\$30/h}$

What is the fixed cost?

$\boxed{\$60}$

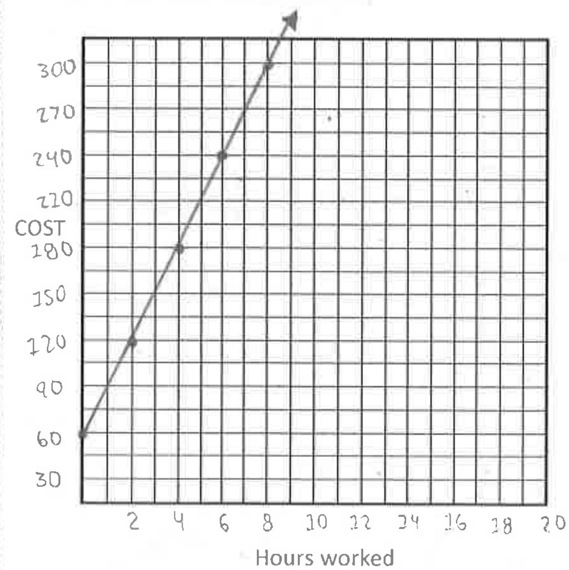
720 = F + 22(30)
720 = F + 660
-660 -660
60 = F

50. Plot the relation above.



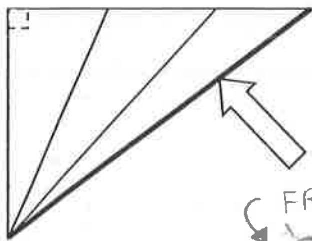
y = 62.50 + 3.25C

51. Plot the relation above.



C(h) = 60 + 30h

52. Below is a scale drawing of a bridge support. Perform the necessary measurements to determine the slope of the indicated beam.



FROM ANSWER KEY

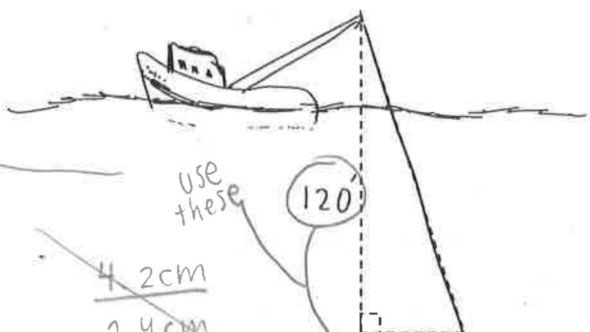
$\boxed{\frac{3\text{cm}}{4\text{cm}}}$ → $\frac{3}{4}$ or 0.75

53. Below is a scale diagram of a section of road between Sidney and Victoria. Measure and calculate the slope of the road.

FROM ANSWER KEY

$\cancel{\frac{0.70\text{m}}{7\text{cm}}}$ → $\frac{0.8\text{cm}}{6.9\text{cm}} = 0.116$

54. A fishing boat moving at 12 knots is shown below. Calculate the slope of the line in the water behind the boat.



use these

120

30

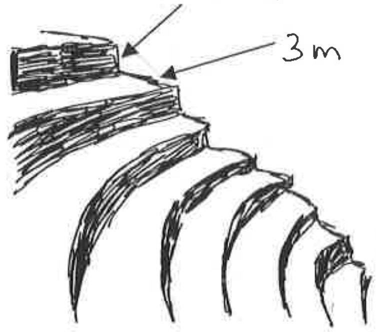
$\cancel{\frac{4.2\text{cm}}{1.4\text{cm}}}$

$= \boxed{3}$ ~~AWAY~~ 4?

$\frac{120}{30} = \frac{4}{1} = \boxed{4}$

55. Terraced landscapes are used by farmers to create useable space from seemingly unusable geography. Calculate the slope of the hill that has been terraced to support crops.

or $\frac{1}{2}$



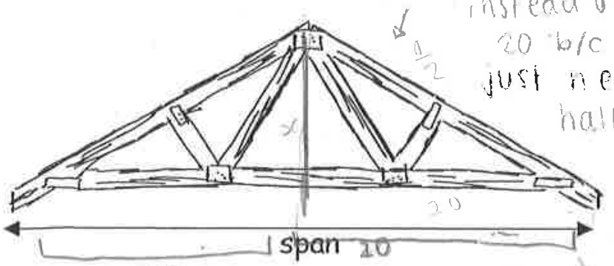
1.5 m

3m

$\frac{1.5\text{m}}{3\text{m}} = \frac{1.5}{3} = \boxed{0.5}$

56. The pitch of a roof is a measure of its "steepness". Calculate the height of the roof truss below if its total span is 20 feet and the pitch (slope)is 6/12.

use 10 instead of 20 b/c just need half.



span 10

$\frac{x}{20} = \frac{\cancel{6}^{3}}{12}$ $\frac{1}{2}$

$\frac{x}{20} = \frac{1}{2}$

$\frac{12x}{12} = \frac{30}{12}$

$x = \boxed{2.5\text{ft}}$ X ?

$\frac{\cancel{6}^{3}}{\cancel{12}} = \frac{x}{\cancel{10}\,5}$

$12x = 15$

$x = 1.25$

$\frac{6}{12} = \frac{x}{10}$

$\frac{60}{12} = \frac{12x}{12}$

$5 = x$

$\boxed{\text{height} = 5\text{ft}}$

57. Mr. J is building a hide-away cabin with a roof that has a pitch of 9/12. T-spray is also building a hut but his roof is one-third as steep. If both roofs have the same total height, how many times wider is T-spray's roof?

$\frac{9}{12} \times \frac{1}{3} = \frac{9}{36}$ ✓

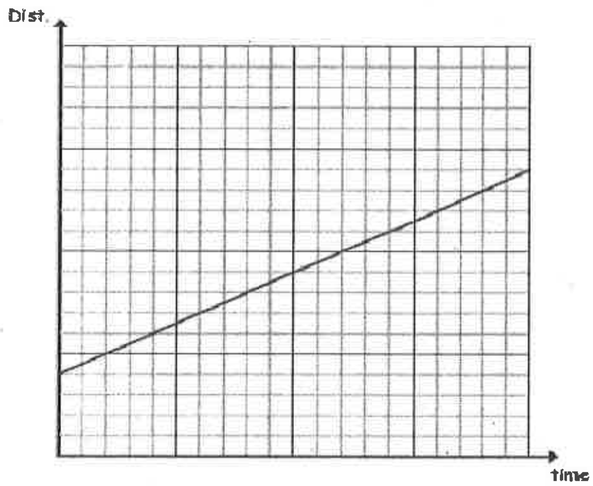
$\boxed{3\times \text{ wider}}$

Since slope compares two quantities, it is a *measure of rate of change*.

For each of the following scenarios, what rate does the slope represent?

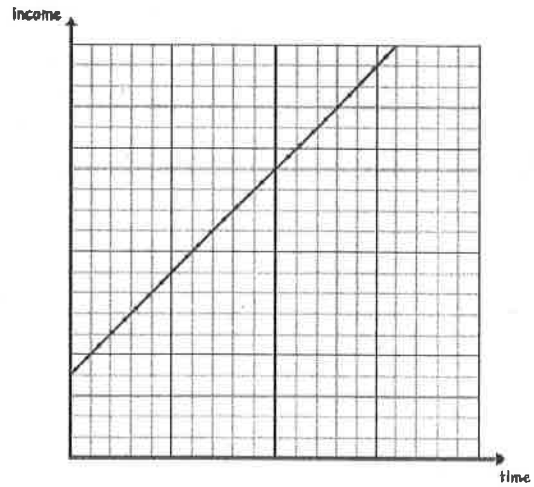
58 Rate: Speed?

What are the units of the slope? Km/h

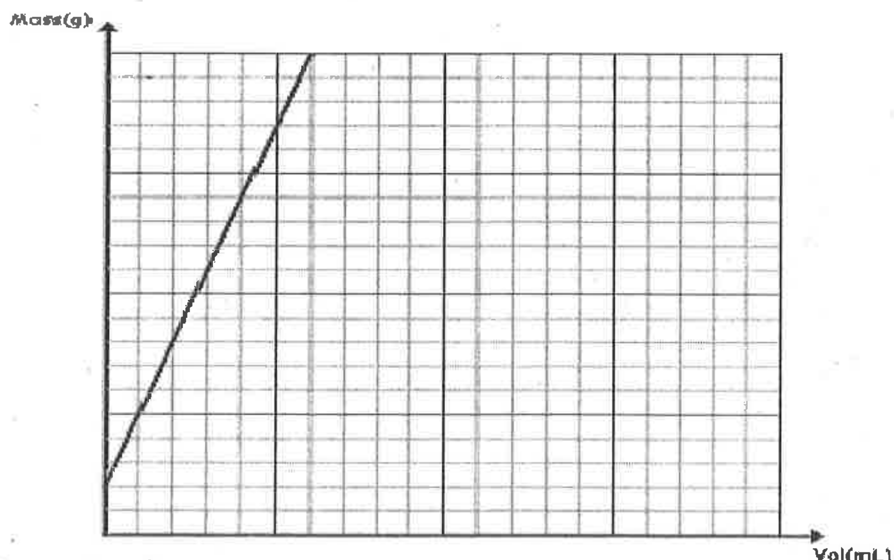


59 Rate: salary?

What are the units of the slope? \$/h



60

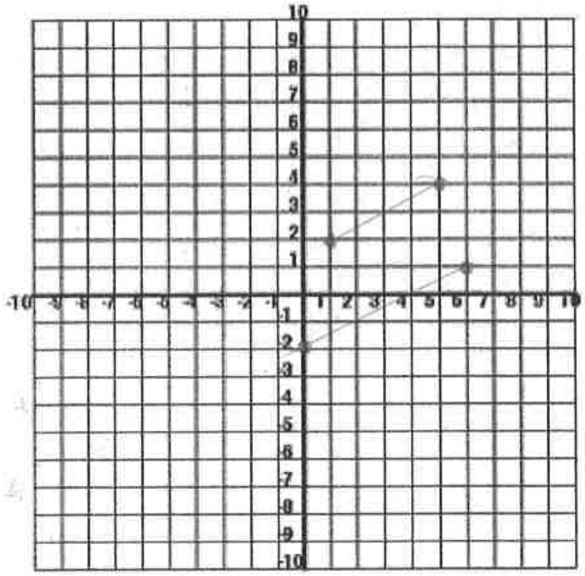


Rate: density?

What are the units of the slope? g/mL

61. Challenge # 5

Determine if AB is parallel to CD given the following points: A(1,2), B(5,4), C(0,-2), D(6,1).



AB : $\frac{2}{4}$: $\frac{1}{2}$

CD : $\frac{3}{6}$: $\frac{1}{2}$

yes, they are parallel

62. What can you say about the slopes of parallel line segments?

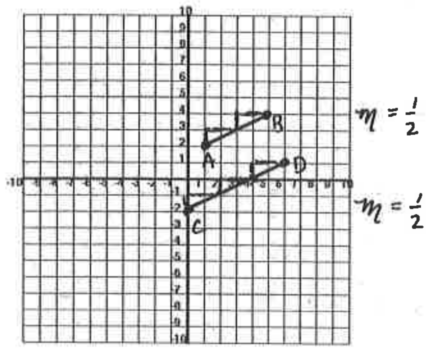
AB : $\frac{2}{4}$: $\frac{1}{2}$: 0.5

CD : $\frac{3}{6}$: $\frac{1}{2}$: 0.5

The slopes of parallel line segments are equal / the same.

Slopes of Parallel Lines (or segments)

Recall two lines are parallel if they do not ever intersect.



Parallel lines have *equal slopes.*

Any two horizontal lines are parallel.
Any two vertical lines are parallel.

To determine if line segments are parallel, calculate their slopes.

Eg.1. Determine if AB is parallel to CD. A(1,2), B(5,4), C(0,-2), D(6,1).

Slope of AB:
$m_{AB} = \frac{4-2}{5-1} = \frac{2}{4} = \frac{1}{2}$

Slope of CD:
$m_{CD} = \frac{1-(-2)}{6-0} = \frac{3}{6} = \frac{1}{2}$ SAME SLOPES ∴ PARALLEL

Eg.2. The following are slopes of two lines. Find the value of k so that the two lines are parallel.

$m_1 = 2$ and $m_2 = -\frac{6}{k}$ Since the lines are parallel, slopes must be equal. $2 = \frac{-6}{k}$

Cross Multiply: $\frac{2}{1} = \frac{-6}{k}$ $2k = -6$ $k = -3$

rise = y
run = x

Determine if the following pairs of line segments are parallel.

63. A(-2,-1), B(1,5) and C(2, -1), D(4,3)

$AB = \frac{5-(-1)}{1-(-2)} = \frac{6}{3} = \boxed{2}$

$CD = \frac{3-(-1)}{4-2} = \frac{4}{2} = \boxed{2}$

$2 = 2$ = $\boxed{YES}$ slope = 2

64. E(-3, 0), F(1, 5) and G(0, -6), H(2, -1)

$EF = \frac{5-0}{1-(-3)} = \boxed{\frac{5}{4}} = \boxed{1.25}$

$GH = \frac{-1-(-6)}{2-0} = \boxed{\frac{5}{2}} = \boxed{2.5}$

$\boxed{NO}$ slope of EF: $\frac{5}{4}$ or 1.25
slope of GH: $\frac{5}{2}$ or 2.5

65. I(-4,0), J(8, 2) and K(2, 8), L(-2, 4)

$IJ: \frac{2-0}{8-(-4)} = \frac{2}{12} = \boxed{\frac{1}{6}}$

$KL = \frac{4-8}{-2-2} = \frac{-4}{-4} = \boxed{1}$

$\boxed{NO}$ slope of IJ = $\frac{1}{6}$
slope of KL = 1

The following are slopes of two lines. Find the value of k so that the two lines are parallel.

66. $m_1 = -\frac{2}{3}$ and $m_2 = -\frac{k}{9}$

$-\frac{2}{3} = \frac{K}{-9}$

$\frac{3K}{3} = \frac{18}{3}$

$\boxed{K = 6}$ $-\frac{2}{3} = \frac{-6}{9}$ ✓

67. $m_1 = -3$ and $m_2 = \frac{k}{4}$

$\frac{-3}{1} = \frac{K}{4}$

$\boxed{K = -12}$

68. $m_1 = \frac{k}{3}$ and $m_2 = \frac{1}{2}$

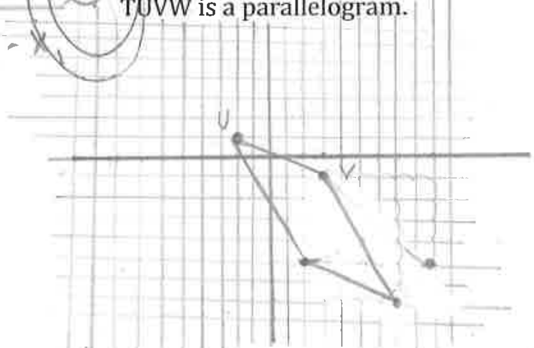
$\frac{1}{2} = \frac{K}{3}$

$\frac{2K}{2} = \frac{3}{2}$

$\boxed{K = \frac{3}{2}}$ ✓

$m = \frac{Y_2 - Y_1}{X_2 - X_1}$

69. The points T(2, -5), U(-2, 1), and V(3, -1) are given. Determine the coordinates of point W so that TUVW is a parallelogram.



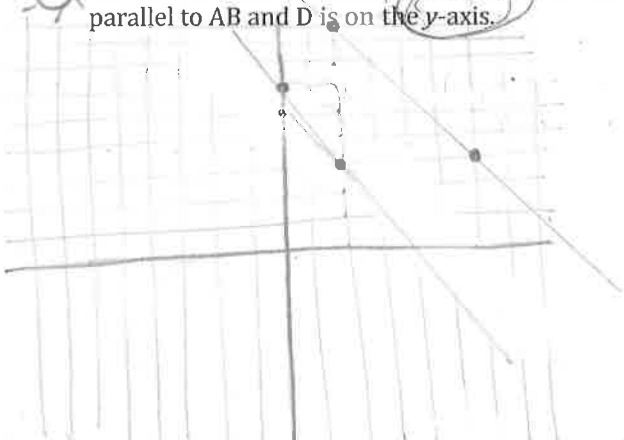
slope of TU = $\frac{1-(-5)}{-2-2} = \frac{6}{-4} = -\frac{3}{2}$

$\boxed{(7, -7)}$

VW = $\frac{Y - -1}{X - 3}$

$\frac{-4}{5}$

70. The points A(6,3), B(2,9), and C(2,3) are given. Determine the coordinates of point D so that CD is parallel to AB and D is on the y-axis.



$\frac{6}{4} = \frac{3}{2}$

$\frac{9-3}{2-6} = \frac{6}{-4} = -\frac{3}{2}$

$\boxed{(0, 6)}$

$\frac{9-3}{2-6} = \frac{6}{-4} = -\frac{3}{2}$ ✓

$\frac{K-3}{0-3}$

$\frac{K-3}{0-2} = -\frac{3}{2} = \frac{K-3}{-2} = \frac{-3}{2} = 2K - 6 = 6 \rightarrow \frac{2K}{2} = \frac{12}{2}$

$\boxed{K = 6} \rightarrow \frac{6}{0}$ (0, 6)

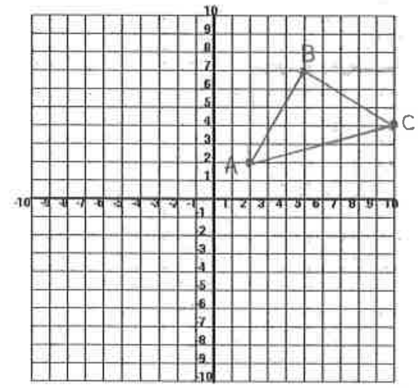
$\frac{2}{3} = \frac{K}{0}$ $\frac{K-2}{0-3} = \frac{-3}{2} \rightarrow \frac{K-2}{-3} = \frac{-3}{2}$ ~~$\frac{-3}{2} = \frac{K}{0} = 2K = 0$~~

$2K - 4 = 9 \rightarrow 2K = 13 = K = 9$

Slopes of Perpendicular Line Segments.

- The slopes of perpendicular lines are negative reciprocals.
- The product of perpendicular slopes is -1.

71. Plot the right triangle with vertices:
A(2,2), B(5,7), and C(10,4).



72. Find the slope of AB. $m = \frac{5}{3}$

73. Find the slope of BC. $m = \frac{-3}{5}$

These segments form the right angle in the triangle.

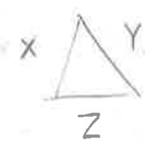
74. What do you notice about the slopes of the two segments.
They are the reciprocal with the opposite sign

75. Multiply the two slopes. What is the result?

$\frac{5}{3} \times \frac{-3}{5} = \frac{-1}{1} = \boxed{-1}$ The result is -1.

76. Is the triangle with vertices X(-9,-1), Y(-7,7), Z(3,-4) a right triangle?

$XY = \frac{7-(-1)}{-7-(-9)} = \frac{8}{2} = \boxed{\frac{4}{1}}$



$YZ = \frac{-4-7}{3-(-7)} = \frac{-11}{10}$ ~~NO!~~

$XZ = \frac{-4-(-1)}{3-(-9)} = \frac{-3}{12} = \boxed{\frac{-1}{4}}$

$\frac{4}{1} \rightarrow \frac{-1}{4}$ = YES

Perpendicular Lines will have slopes that are NEGATIVE RECIPROCALS.

Examples of perpendicular slopes are: $m_1 = 5$, $m_2 = -\frac{1}{5}$.

Examples of perpendicular slopes are: $m_1 = -\frac{5}{3}$, $m_2 = \frac{3}{5}$.

Perpendicular slopes will have a product of -1.

Look at the example above… $-\frac{5}{3} \times \frac{3}{5} = -\frac{15}{15} = -1$

★ FLIP m_2 (reciprocal) and change sign ★
or unknown

Determine the slope of a line segment perpendicular to a segment with each given slope.

77. $m = -3$

$\frac{1}{3}$

78. $m = -\frac{2}{3}$

$\frac{3}{2}$

79. $m = \frac{4}{5}$

$\frac{-5}{4}$

The following are slopes of two lines. Find the value of k so that the two lines are perpendicular.

80. $m_1 = -\frac{2}{3}$ and $m_2 = -\frac{k}{9}$

$\frac{2}{-3} = \frac{9}{K}$

$\frac{2K}{2} = \frac{-27}{2}$

$\boxed{K = \frac{-27}{2}}$

~~$-\frac{2}{3} = \frac{K}{-9}$~~

~~$\frac{3K}{3} = \frac{18}{3}$~~

~~$K = 6$~~

~~$-\frac{2}{3} = -\frac{6}{9} \to -\frac{2}{3} = -\frac{2}{3} \to m_2 = \boxed{\frac{3}{2}}$~~

81. $m_1 = -3$ and $m_2 = \frac{k}{4}$

$\frac{-3}{-1} = \frac{-4}{K}$

$\frac{3K}{3} = \frac{4}{3} \to \boxed{K = \frac{4}{3}}$

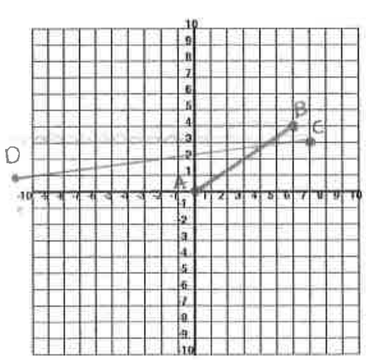
82. $m_1 = \frac{k}{3}$ and $m_2 = \frac{1}{2}$

$\frac{1}{2} = \frac{-3}{K}$

$\boxed{K = -6}$

Graph each pair of line segments. Determine if they are perpendicular or not.

83. A(0,0), B(6,4) and C(7,3), D(-11,1)

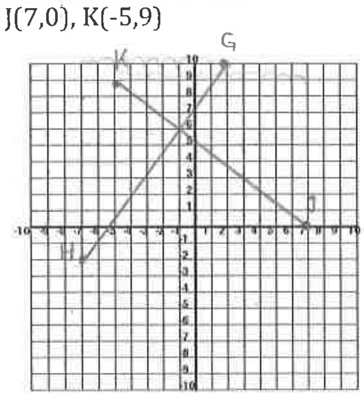


NO

$AB = \frac{4}{6} = \frac{2}{3} \neq$

$CD = \frac{2}{18} = \frac{1}{9}$

84. G(2,10), H(-7,-2) and J(7,0), K(-5,9)



YES

$GH = \frac{12}{9} = \frac{4}{3}$

$JK = \frac{9}{-12} = -\frac{3}{4}$

$\frac{2}{3} \times \frac{1}{9} = \frac{2}{27}$

$\frac{4}{3} \times \frac{-3}{4} = \frac{-1}{1} = -1$

✓

Intercepts

Non-vertical and non-horizontal lines are called **oblique** lines.

Oblique lines will cross both the x-axis and the y–axis.

These points are called the x-intercept and the y-intercept.

Line crosses x-axis here.

Line crosses y-axis here.

85. Challenge Question:

Find the intercepts for the line $y = 2x + 4$.

$x =$ $0 = 2x + 4$
$-4 \quad -4$
$\frac{-4}{2} = \frac{2x}{2}$
$-2 = x$
$\boxed{x \text{ intercept} = (-2, 0)}$

$y =$ $y = 2(0) + 4$
$y = 0 + 4$
$y = 4$
$\boxed{y \text{ intercept} = (0, 4)}$

86. Challenge Question:

Find the intercepts for the line $3x + 4y - 12 = 0$.

$x =$ $3x + 4(0) - 12 = 0$
$3x + 0 - 12 = 0$
$3x - 12 = 0$
$+12 \quad +12$
$\frac{3x}{3} = \frac{12}{3}$
$x = 4$
$\boxed{x \text{ intercept} : (4, 0)}$

$y =$ $3(0) + 4y - 12 = 0$
$0 + 4y - 12 = 0$
$-12 + 4y = 0$
$+12 \quad +12$

$\frac{4y}{4} = \frac{12}{4}$
$y = 3 \rightarrow$ $\boxed{y \text{ intercept} = (0, 3)}$

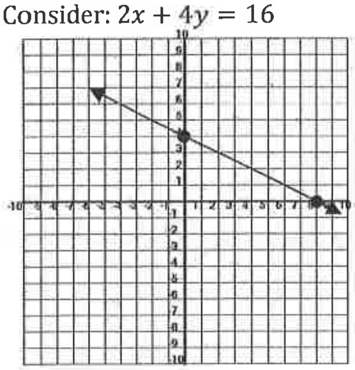
✓

Finding the Intercepts from a graph.

The location where a line passes through the x-axis is called the **x-intercept**. This point will have the coordinates $(x, 0)$

The location where a line passes through the y-axis is called the **y-intercept**. This point will have the coordinates $(0, y)$

Consider: $2x + 4y = 16$

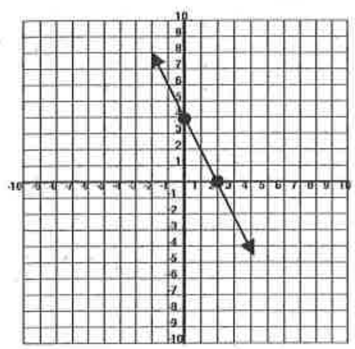


This line has an x-intercept at (8, 0).
And a y-intercept at (0, 4).

You may see this written as:
x-intercept is 8.
y-intercept is 4.

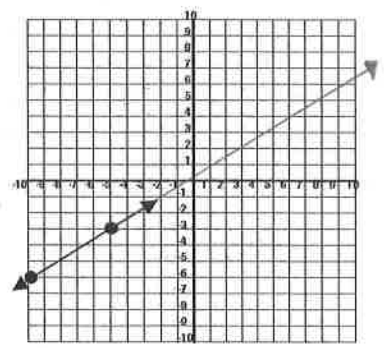
Find the x- and y-intercepts from the graph below.

87.



x-intercept: (2,0) y-intercept: (0,4)

88.



x-intercept: (0,0) y-intercept: (0,0)

Finding the Intercepts from an equation.

The x-intercept will have coordinates $(x, 0)$. This means we can substitute 0 in for y and solve to find the x-intercept. The y-intercept will have coordinates $(0, y)$.

Eg. Find the x-intercept for
$2x + 4y = 16$
$2x + 4(0) = 16$
$2x = 16$
$x = 8$

Find the y-intercept:
$2y + 4y = 16$
$2(0) + 4y = 16$
$4y = 16$
$y = 4$

Calculate the x- and y-intercepts.

89. $2x + 3y = 12$

x: $2x+3(0)=12$
$2x+0=12$
$\frac{2x}{2}=\frac{12}{2}$
$x=6$
$\boxed{x \text{ intercept} = (6,0)}$

y: $2(0)+3y=12$
$0+3y=12$
$\frac{3y}{3}=\frac{12}{3}$
$y=4$
$\boxed{y \text{ intercept} = (0,4)}$

90. $3x + 5y = 30$

x: $3x+5(0)=30$
$3x+0=30$
$\frac{3x}{3}=\frac{30}{3}$
$x=10$
$\boxed{x \text{ intercept} = (10,0)}$

y: $3(0)+5y=30$
$0+5y=30$
$\frac{5y}{5}=\frac{30}{5}$
$y=6$
$\boxed{y \text{ intercept} = (0,6)}$

91. $3x - 4y + 24 = 0$

x: $3x-4(0)+24=0$
$3x-0+24=0$
$3x+24=0$
$\frac{3x}{3}=\frac{-24}{3}$
$x=-8$
$\boxed{x \text{ intercept} = (-8,0)}$

y: $3(0)-4y+24=0$
y: $0-4y+24=0$
y: $-4y+24=0$
y: $\frac{-4y}{-4}=\frac{-24}{-4} \rightarrow y=6$
$\boxed{y \text{ intercept} = (0,6)}$

92. $4x + 5y = 10$

x: $4x+5(0)=10$
$4x+0=10$
$\frac{4x}{4}=\frac{10}{4}$
$x=\frac{10}{4} \rightarrow \frac{5}{2}$
$\boxed{x \text{ intercept} = (\frac{5}{2},0) \text{ or } (2.5,0)}$

y: $4(0)+5y=10$
$0+5y=10$
$\frac{5y}{5}=\frac{10}{5}$
$y=2$
$\boxed{y \text{ intercept} = (0,2)}$

93. $5y = 10x$

x: $5(0)=10x$
$\frac{0}{10}=\frac{10x}{10}$
$0=x$
$\boxed{x \text{ intercept} = (0,0)}$

y: $5y=10(0)$
$\frac{5y}{5}=\frac{0}{5}$
$y=0$
$\boxed{y \text{ intercept} = (0,0)}$

94. $0.04x + 0.02y = 1400$

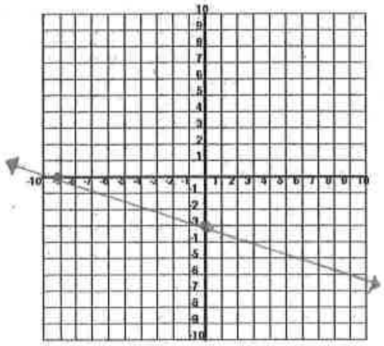
x: $0.04x+0.02(0)=1400$
$0.04x+0=1400$
$\frac{0.04x}{0.04}=\frac{1400}{0.04}$
$x=35000$
$\boxed{x \text{ intercept} = (35000,0)}$

y: $0.04(0)+0.02y=1400$
$0+0.02y=1400$
$\frac{0.02y}{0.02}=\frac{1400}{0.02}$
$y=70\,000$
$\boxed{y \text{ intercept} = (0,70\,000)}$

✓

Using and Interpreting Intercepts

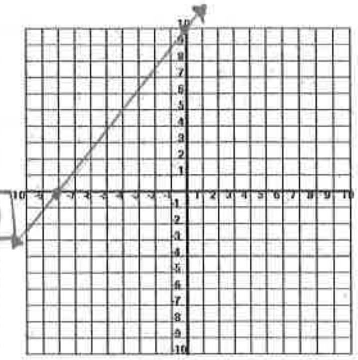
95. Find the intercepts and graph the line $2x + 6y = -18$.



x: $2x + 6(0) = -18$
$2x + 0 = -18$
$\frac{2x}{2} = \frac{-18}{2}$
$x = -9$
$\boxed{x \text{ intercept} = (-9,0)}$

y: $2(0) + 6y = -18$
$0 + 6y = -18$
$\frac{6y}{6} = \frac{-18}{6}$
$y = -3$
$\boxed{y \text{ intercept} = (0,-3)}$

96. Find the intercepts and graph the line $10x - 8y = -80$.



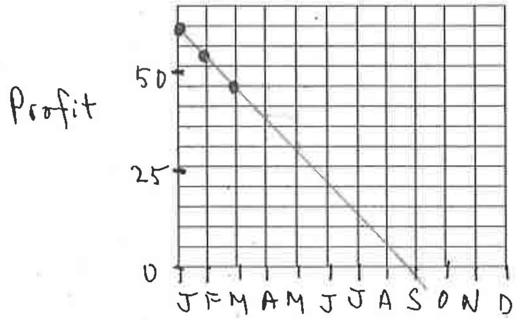
x: $10x - 8(0) = -80$
$10x - 0 = -80$
$\frac{10x}{10} = \frac{-80}{10}$
$x = -8$
$\boxed{x \text{ intercept} = (-8,0)}$

y: $10(0) - 8y = -80$
$0 - 8y = -80$
$\frac{-8y}{-8} = \frac{-80}{-8}$
$y = 10$
$\boxed{y \text{ intercept} = (0,10)}$

97. Based on the equation for the linear relation, when do you think it is most appropriate to graph the relation using intercepts?

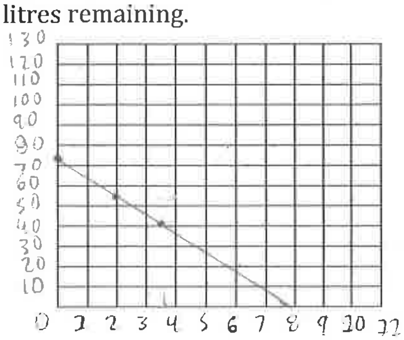
When the intercepts are integers.

98. The cost of a new pair of shoes at ShoeInc is reduced at a constant rate. The graph below shows the profit ShoeInc makes on each sale. In what month does ShoeInc "break even" on these shoes?



? $\boxed{\text{By September?}}$? ✓ but why? just guessed!

99. Use the graph below to plot the fuel consumed on Sandy's last road trip. She started out with 72 litres of fuel and drove for 2 hours. At that point she had 54 litres left. After driving another 1.5 hours she had 40.5 litres remaining.



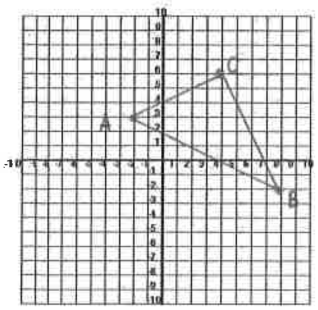
At this rate, when will she run out of fuel?

8 hours

★ ANSWER KEY: 4.5 more hours ★

Mixed Practice:

100. A triangle has vertices A(-2,3), B(8,-2), and C(4,6). Determine whether it is a right triangle.



$AB = \frac{-2-3}{8-(-2)} = \frac{-5}{20} = \frac{-1}{2}$

$BC = \frac{6-(-2)}{4-8} = \frac{8}{-4} = \frac{2}{-1}$

$CA = \frac{6-3}{4-(-2)} = \frac{3}{6} = \frac{1}{2}$

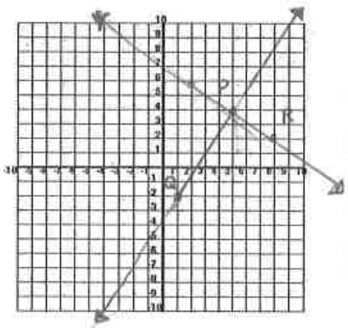
$\boxed{\text{Yes}}$

$BC = \frac{-2}{1}$

$CA = \frac{1}{2}$

ANSWER KEY:
★ AC is perpendicular to BC ★

101. P (5,4) and Q(1,-2) are points on a line. Find the coordinates of a point, R, so that PR is perpendicular to PQ.



$PQ = \frac{-2-4}{1-5} = \frac{-6}{-4} = \frac{-3}{-2} = \boxed{\frac{3}{2}}$

$PR = \frac{-2}{3}$

$\boxed{(8,2)}$ ✓

better way to get this?

102. Find the value of k so that the two slopes are perpendicular.

$m_1 = \frac{k}{2}$ and $m_2 = \frac{1}{4}$

$\frac{-2}{K} = \frac{1}{4}$

$K = -8$

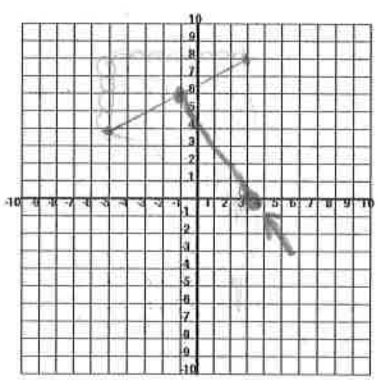
$\boxed{K = -8}$

103. Two vertices of an isosceles triangle are A(-5,4) and B(3,8). The third vertex is on the x-axis. What are the possible coordinates of the third vertex, C ?

Isoceles:

2 sides same length

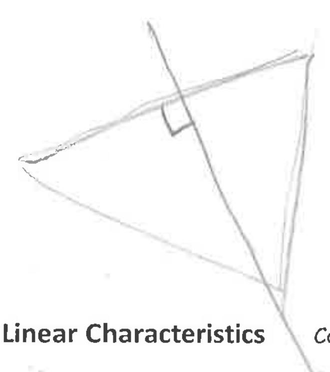
2 angles same



$\boxed{(3,0)}$

$\text{Slope} = \frac{1}{2}$

perp. slope = -2



✓

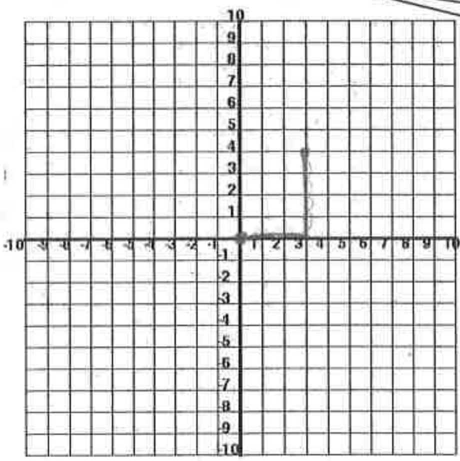
Additional Material

Distance between points (length of line segments)

104. **Challenge Question:**

How can you find the EXACT distance between (0,0) and (3,4)?

EXACT length means find the answer as a radical or fraction if it cannot be expressed as a *whole number.*



$\boxed{\frac{4}{3}}$ = slope

$4^2 + 3^2 = c^2$

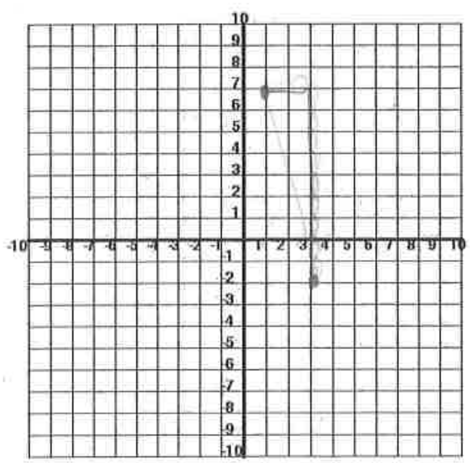
$16 + 9 = c^2$

$\sqrt{25} = \sqrt{c^2}$

$5 = c$

105. **Challenge Question:**

How can you find the EXACT distance between (3, −2) and (1,7)?



$\frac{9}{2}$

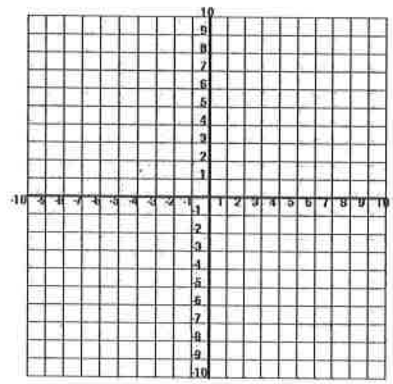
$9^2 + 2^2 = c^2$

$81 + 4 = c^2$

$\sqrt{85} = \sqrt{c^2}$

$c = 9.219544457...$

Finding the Distance Between Points



The distance between two points can be found using The Pythagorean Theorem $a^2 + b^2 = c^2$.

106. Plot points A(0,0) and B(3,4). Connect the endpoints of segment AB.

107. Draw a vertical line down from B. Draw a horizontal line right from A.

Notice that a right triangle is formed.

The distance between A and B is the hypotenuse of the right triangle.

$$a^2 + b^2 = c^2$$

$3^2 + 4^2 = d^2$

$$25 = d^2$$
$$5 = d$$

The distance between A and B is 5 units.

From this method we develop the formula for the distance between two points:

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Eg.1. Consider points A(0,0) and B(3,4). from above. To use the distance formula, we substitute values from the ordered pairs into the formula.

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ Put both x-coordinates into the first bracket.
$d = \sqrt{(3 - 0)^2 + (4 - 0)^2}$ Put both y-coordinates into the second bracket (same order).
$d = \sqrt{(3)^2 + (4)^2}$
$d = \sqrt{25}$
$d = 5$

Eg.2. Use the distance formula to find the lenth of the line segment connecting point C(3, −2) and D(1,7).

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ Put both x-coordinates into the first bracket.
$d = \sqrt{(3 - 1)^2 + ((-2) - 7)^2}$ Put both y-coordinates into the second bracket (same order as above).
$d = \sqrt{(2)^2 + (-9)^2}$
$d = \sqrt{4 + 81}$
$d = \sqrt{85}$

108. Can you see how the formula above is derived from the Pythagorean Theorem?
(some of you will often find it easier to draw a right triangle and use the Pythagorean Th.)
Explain…

Find the distance between the pairs of points.

109. $(4,2)$ and $(6,8)$

110. $(-3,-2)$ and $(-3,-5)$

111. $(0,0)$ and $(-5,2)$

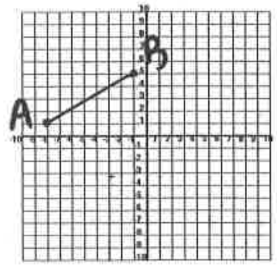
112. $(-3,0)$ and $(8,-4)$

113. $(1,-6)$ and $(5,2)$

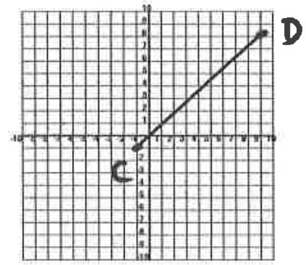
114. $(2,0)$ and $(-3,2)$

115. Find the radius of a circle with the centre at (-3, 4) and a point on the circumference at (-4, -6).

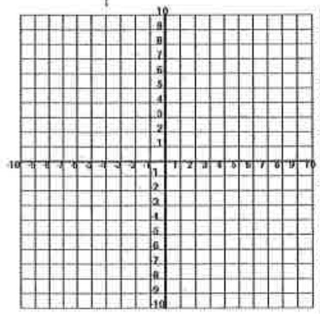
116. Find the length of AB.



117. Find the length of CD.



118. On the grid draw the rectangle with vertices at $A(-3,3)$, $B(0,-6)$, $C(3,-5)$, $D(0,4)$.

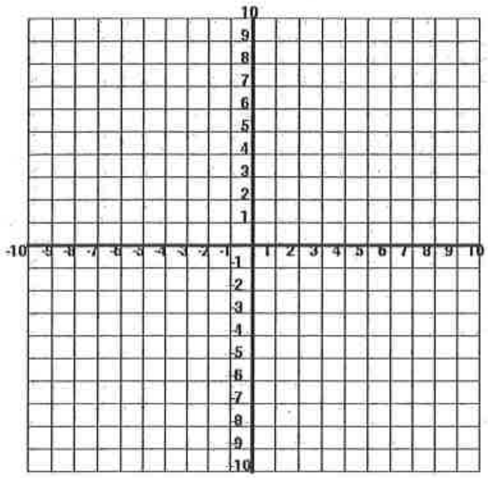


Find the side lengths and the perimeter. Leave answers in simplest radical form if necessary.

119. A Coast Guard patrol boat is located 5 km east and 8 km north of the entrance to St. John's harbour. A tanker is 9 km east and 6 km south of the entrance. Find the distance between the two ships to the nearest tenth of a km.

120. Challenge Question:

Draw a line connecting the two points (0,0) and (6,4). What are the coordinates of the *midpoint* of that line segment?

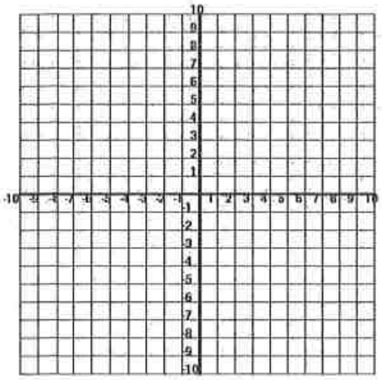


121. Challenge Question:

Given that the midpoint of a line segment is M(-1,3) and one endpoint is E(-4,5). Find the coordinates of the other endpoint

Finding the Midpoint of A Line Segment

The midpoint of a line segment can be found by plotting the points on a grid.



STEP 1: Plot points A(0,0) and B(6,4).

STEP 2: Draw a vertical line down from B.
STEP 3: Draw a horizontal line right from A.

STEP 4: Find the middle of each line you just drew.

STEP 5: Find the intersection of the two middles. This will be the midpoint.

Midpoint is at (3,2)

Using the Midpoint (X_M, Y_M) Formula:

$\left(\frac{x_1+x_2}{2}\right), \left(\frac{y_1+y_2}{2}\right)$ Notice that you are finding the average *x* value and average *y* value.

Eg.1. Find the midpoint of the line connecting points A(0,0) and B(6,4).

$\left(\frac{x_1+x_2}{2}\right), \left(\frac{y_1+y_2}{2}\right)$
$\left(\frac{0+6}{2}\right), \left(\frac{0+4}{2}\right)$
$\left(\frac{6}{2}\right), \left(\frac{4}{2}\right)$ Midpoint is at (3,2)

Eg.2. The midpoint of a line segment is M(-1,3) and one endpoint is E(-4,5). Find the coordinates of the other endpoint.

Using the formula.

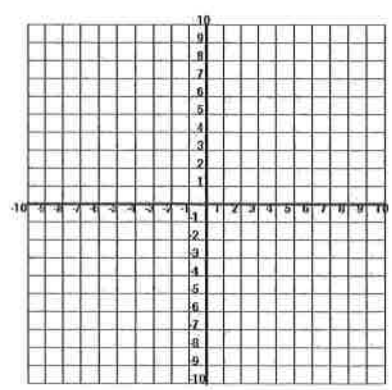
$\left(\frac{x_1+x_2}{2}\right) = x_M$
$\left(\frac{-4+x_2}{2}\right) = -1$
$(-4 + x_2) = -2$
$x_2 = 2$

$\left(\frac{y_1+y_2}{2}\right) = y_M$
$\left(\frac{5+y_2}{2}\right) = 3$
$(5 + y_2) = 6$
$y_2 = 1$

The other endpoint is at (2,1)

Graphically.
Plot the endpoint and the midpoint.

Count grid lines to locate the other endpoint.



Find the midpoint of each line segment from the given endpoints.

122. $(5,7)$ and $(3,9)$

123. $(4,2)$ and $(6,8)$

124. $(-1,0)$ and $(1,-6)$

125. $(0.2, 1.5)$ and $(3.6, 0.3)$

126. $\left(\frac{1}{2}, \frac{5}{2}\right)$ and $\left(\frac{3}{2}, \frac{-7}{2}\right)$

127. $(200,125)$ and $(403, 174)$

128. A diameter of a circle has endpoints at (-7,-4) and (-1,10). What are the coordinates of the centre of the circle?

129. One endpoint is at (6,10) and the midpoint is at (0,0). Find the other endpoint.

130. One endpoint is at (4,0) and the midpoint is at (7,2). Find the other endpoint.

131. The centre of a circle has coordinates (-1,-3). One endpoint of a diameter is at (-3,0). What are the coordinates of the other endpoint?

The endpoints of a diameter of a circle are (-3,4) and (6,0).

132. Find the centre of the circle.

133. What is the length of the radius? Answer to two decimal places.

Line segment JK has endpoints at $J(-5,5)$ and $K(7,2)$.

134. What is the slope of a line segment perpendicular to JK.

135. Calculate the exact length of segment JK.

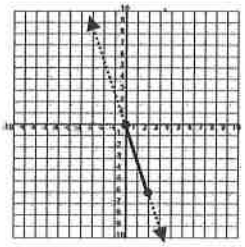
136. Find the value of q if (p, q) is the midpoint of JK.

137. Give the coordinates of a point $L(x, y)$ such that JL has an undefined slope.

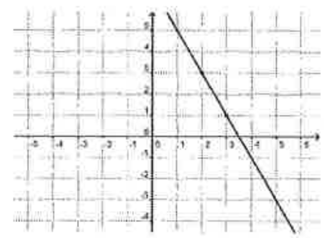
Marissa
Blk.4

Characteristics of Lines: Answers

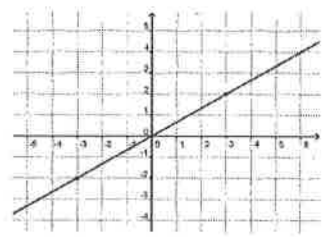
1.



2. On graph above.
3. $(0,0)$ *and* $(2,-6)$
4. There are none.
5. Many answers. Eg. Definite endpoints, straight line, no gaps, decline to the right, etc.
6. Eg. Same "direction", that is decline to the right.
7. $rise = 4$
8. $run = 2$
9. $\frac{4}{2} = \frac{2}{1} = 2$
10. Twice as fast.
11. $m = \frac{3}{4}$
12. $m = \frac{3}{4}$
13. Choose any two points on the line and calculate or count rise/run.
14. A line segment has definite endpoints.
15. Rise: 3
    Run: 4
    Slope: $\frac{3}{4}$
16. Rise: 7
    Run:0
    Slope: undefined
17. Rise: 0
    Run: 8
    Slope: $\frac{0}{8} = 0$
18. Rise: -1
    Run:3
    Slope: $-\frac{1}{3}$
19. Rise: 2
    Run: 2
    Slope: $\frac{2}{2} = 1$
20. Rise: -4
    Run: 4
    Slope: $-\frac{4}{4} = -1$
21. Rise: 4
    Run: 7
    Slope: $\frac{4}{7}$
22. Rise: -6
    Run: 3
    Slope: $\frac{-6}{3} = -2$
23. $\frac{3}{2}$
24. 4
25. -9
26. Undefined
27. (5,6), many other answers.
28. 0
29. (3,5), many other answers.
30. $-\frac{20}{3}$
31. $(5,-26)$, many other answers.
32. $t = 1$
33. $b = 9$
34. Answered on next page in booklet.
35.



36.



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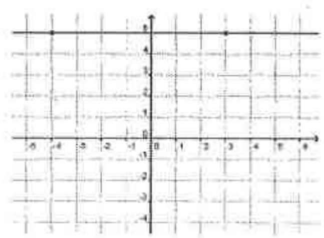

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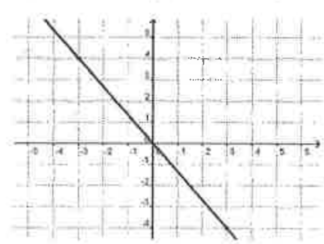

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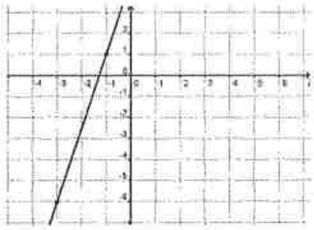

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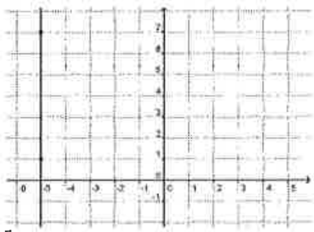
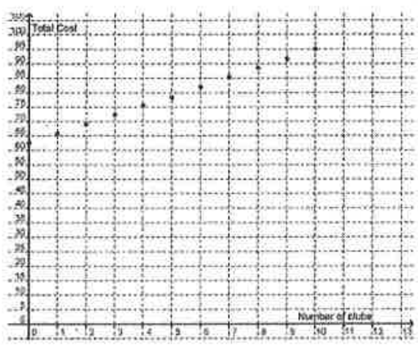
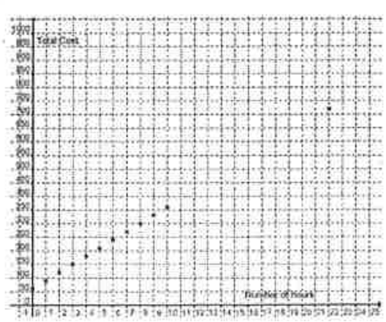


41. $\frac{5}{3}$ Use the positive slope in these applications.
42. 13 Use the positive slope in these applications.
43. \$10
44. (1,10), (2,20), ...
45. $97.9\frac{cents}{litre}$
46. $105\frac{km}{h}$
47. Speed
48. \$3.25 per club and \$62.50 for green fees
49. \$30/*hour*,
    Fixed cost: \$60
50.


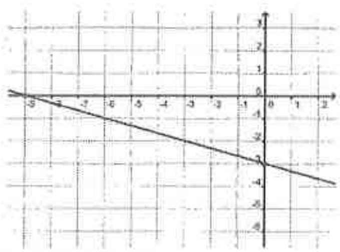
51.


52. $\frac{3\ cm}{4\ cm} = \frac{3}{4}$ or 0.75
53. $\frac{0.8\ cm}{6.9\ cm} = 0.116$
54. 4
55. $\frac{1}{2}$ or 0.5
56. 5 feet
57. Three times as wide.
58. Speed, km/h, or m/s, or mph
59. Salary, \$/hour
60. Density, g/ml
61. Yes. Lines through those points will never intersect.
62. Slopes are equal. Both segments have a slope of $\frac{1}{2}$.
63. Yes. Slope:2
64. No. Slope $\frac{5}{4}$ and Slope $\frac{5}{2}$.
65. No. Slope $\frac{1}{6}$ and Slope 1.
66. $k = 6$
67. $k = -12$
68. $k = \frac{3}{2}$
69. $(7,-7)$. $(-1,5)$ and $(-3,-3)$ also produce a parallelogram but he naming would then be out of order.
70. (0,6)
71. On graph
72. $\frac{5}{3}$
73. $-\frac{3}{5}$
74. Opposite signs, reciprocated.
    "Negative reciprocals"
75. $\frac{5}{3} \times -\frac{3}{5} = -\frac{15}{15} = -1$
76. Yes. Two of the side lengths have slopes that are negative reciprocals.
77. $\frac{1}{3}$
78. $\frac{3}{2}$
79. $-\frac{5}{4}$
80. $k = -\frac{27}{2}$
81. $k = \frac{4}{3}$
82. $k = -6$
83. No. $\frac{2}{3}$ and $\frac{1}{9}$
84. Perpendicular. $\frac{4}{3}$ and $-\frac{3}{4}$
85. y-intercept: (0,4)
    x-intercept: $(-2,0)$
86. y-intercept: (0,3)
    x-intercept: (4,0)
87. y-intercept: (0,4)
    x-intercept: (2,0)
88. y-intercept: (0,0)
    x-intercept: (0,0)

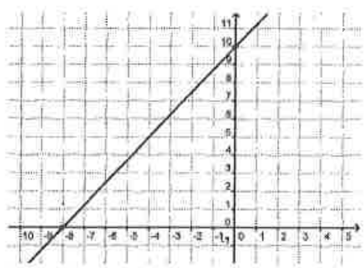
We can choose to state the intercepts without coordinates.

89. y-intercept: 4
    x-intercept: 6
90. y-intercept: 6
    x-intercept: 10
91. y-intercept: 6
    x-intercept: -8

92. y-intercept: 2
x-intercept: $\frac{5}{2}$
93. y-intercept: 0
x-intercept: 0
94. y-intercept: 70 000
x-intercept: 35 000
95. y-intercept: -3
x-intercept: -9



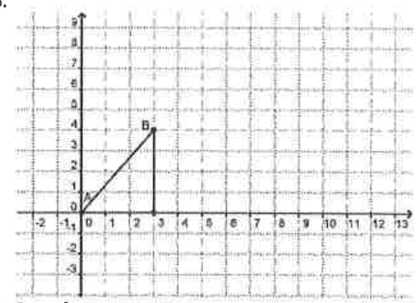
96. y-intercept: 10
x-intercept: -8



97. When the intercepts are integers (which results if the constant is divisible by the coefficients of the x and y terms).
98. September
99. 4.5 more hours.
100. Yes. AC is perpendicular to BC.
101. R could be an infinite number of points. Eg. $(-1,8)$ or $(2,6)$ or $(8,2)$...
102. $k = -8$
103. $(7,0), (-1,0), (3,0), (2,0)$

Additional Material

104. Create a right triangle using the points and intersecting grid lines then use Pythagoras Theorem to calculate a distance of 5 units.
105. Create a right triangle using the points and intersecting grid lines then use Pythagoras Theorem to calculate a distance of 5 units.
106.



107. See above.
108. $a^2 + b^2 = c^2 \rightarrow$
$c = \sqrt{a^2 + b^2}$
This is the distance formula where "a" represents the horizontal difference and "b" the vertical difference.
109. $2\sqrt{10}$
110. 3
111. $\sqrt{29}$
112. $\sqrt{137}$
113. $4\sqrt{5}$
114. $\sqrt{29}$
115. $\sqrt{101}$
116. $\sqrt{65}$
117. $\sqrt{181}$
118. Length: $3\sqrt{10}$
Width: $\sqrt{10}$
Perimeter: $8\sqrt{10}$
119. 14.6 km
120. (3,2)
121. (2,1)
122. (4,8)
123. (5,5)
124. $(0,-3)$
125. (1.9,0.9)
126. $\left(1,-\frac{1}{2}\right)$
127. (301.5,149.5)
128. $(-4,3)$
129. $(-6,-10)$
130. (10,4)
131. $(1,-6)$
132. $\left(\frac{3}{2},2\right)$ or (1.5,2)
133. 4.92
134. $m = 4$
135. $3\sqrt{17}$
136. $\left(1,\frac{7}{2}\right)$ or (1,3.5)
137. Infinite possibilities. The x-coordinate must be -5 .Eg. $(-5,0), (-5,1), \ldots$

