Linear Relations: Solutions

This booklet belongs to:_______________Period_____

LESSON #	DATE	QUESTIONS FROM NOTES	Questions that I find difficult
		Pg.	
		Pg.	
		Pg.	
		Pg.	
		Pg.	
		Pg.	
		Pg.	
		Pg.	
		Pg.	
		Pg.	
		REVIEW	
		TEST	

Your teacher has important instructions for you to write down below.

Linear Relations

Linear Relations SPECIFIC OUTCOMES		TOPIC	REVIEW EXAMPLE
C4. Describe and represent linear relations using: • Words • Ordered pairs • Tables • Graphs • Equations	4.2	Determine whether a situation represents a linear relation.	
	4.3	Determine whether a graph represents a linear relation.	
	4.4	Determine whether a table of values or set of ordered pairs represents a linear relation.	
	4.5	Draw a graph from a set of ordered pairs and determine if the relationship is linear.	
	4.6	Determine if an equation represents a linear relation.	
	4.7	Match corresponding representations of linear relations.	
C5. Determine the characteristics of graphs of linear relations, including the: • Intercepts • Slope • Domain • Range	5.1	Find the intercepts of the graph of a linear relation. State the intercepts as values or as ordered pairs.	
	5.2	Determine the slope of a graph.	
	5.4	Sketch different types of linear relations based on different information regarding intercepts.	
	5.5	Match a graph to its slope and y-intercept.	
	5.6	Identify the slope and y-intercept from a graph.	
	5.7	Solve problems that involve intercepts, slope, domain, or range of a linear relation.	

[C] Communication **[PS]** Problem Solving, **[CN]** Connections **[R]** Reasoning, **[ME]** Mental Mathematics **[T]** Technology, and Estimation, **[V]** Visualization

Key Terms

Term	Definition	Example
Linear Relation		
Linear Function		
Ordered pair		
Slope		
y-intercept		
x-intercept		
Slope-intercept form of a linear equation		
Point-slope form of a linear equation		
General form of a linear equation		
Parallel Lines		
Perpendicular Lines		
Dependent Variable		
Independent Variable		
Linear Function		

Introduction to Linear Relations

We have examined relations between two quantities earlier in this course. Now we will narrow our focus to examine only linear relations.

Linear relations are straight line relationships. Each output value is proportionate to the input value. That is, the change occurs at a constant rate.

Eg. An employee that works for an hourly wage ($10 per hour).

This is a linear relationship because the employees earnings increase at a constant rate.
The equation that relates the **Earnings** and the **hours worked** is $\boldsymbol{E = 10h}$.

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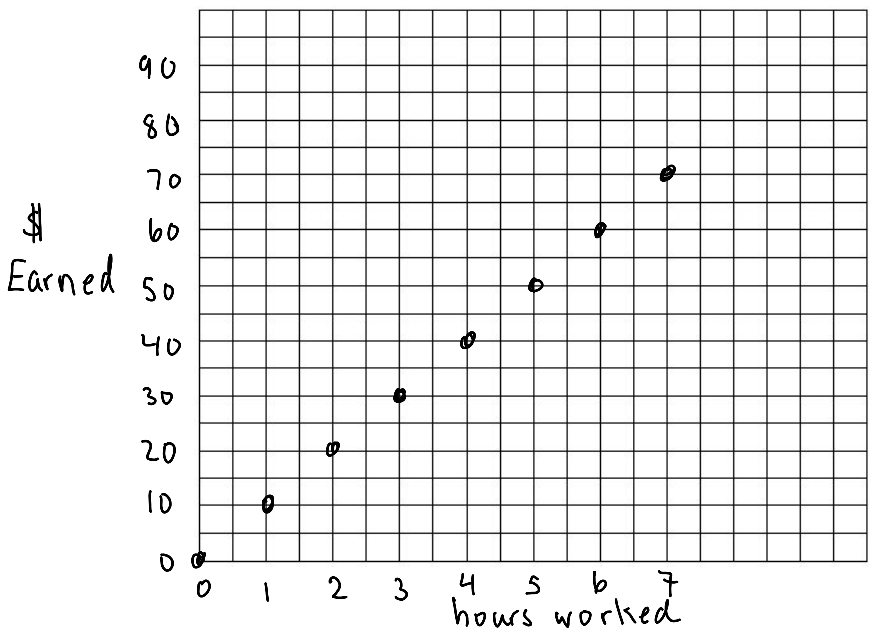
1. Plot the relationship described above if the domain is {0,1,2,3,4,5,6,7}.



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2. What is the shape of the graph you just plotted?

dots form a line

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3. Is the relation $E = 10h$ a function?

Yes, no input produces more than one output.

4. Which variable in the relation $E = 10h$ is the dependent variable

'E' depends on h.

5. Challenge #1:

If $y = 3x$, find the missing values of y.

$y = 3x$	
x	y
-2	-6
-1	-3
0	0
1	3
2	6

6. What name do we give the pairs of numbers in each row?

ordered pairs

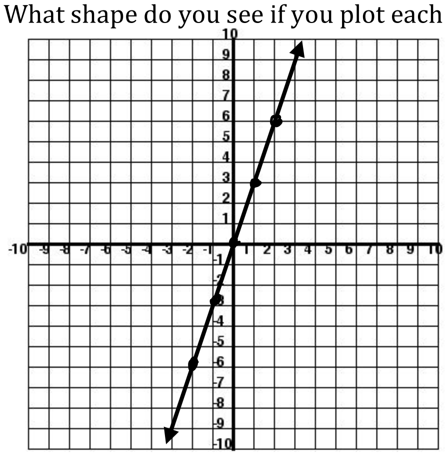
7. Does $(-8, -24)$ satisfy the equation above.

Yes. $3(-8) = -24$

8. How many pairs of numbers are there that satisfy that equation?

infinite number

9. What shape do you see if you plot each of the pairs of numbers in the table above?



a straight line

Finding coordinates from an equation:

A **Table of Values** is a tool used to find ordered pairs from an equation.

$y = 3x$	
x	y

This is a table of values set up to find 5 ordered pairs for the equation $y = 3x$.

Step1: Select 5 *input* values of ***x*** and write them in the *x* column. Eg. -2,-1,0,1,2

Step2: Substitute them into the equation and solve to find the ***y*** value.

$y = 3x$	
x	y
-2	-6
-1	-3
0	0
1	3
2	6

This is the completed table from above.

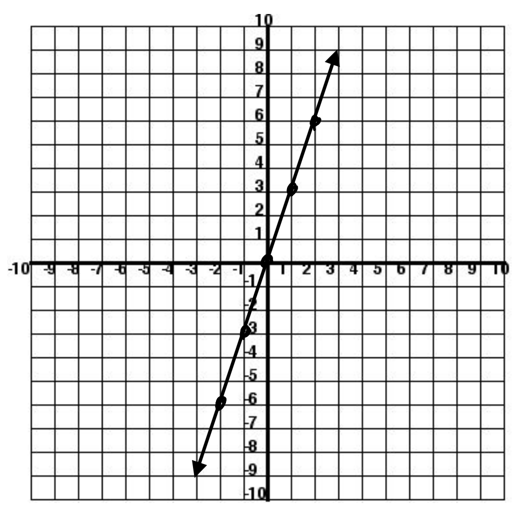
This gives us 5 ordered pairs:
$(-2,-6), (-1,-3)(0,0)(1,3)(2,6)$

**I chose to input values of *x*, but I could have selected values of *y* and solved for *x (although I find that more difficult in this case)*.

10. Challenge #2:

Using the *table of values*, graph the equation $y = 3x$ on the graph provided.

$y = 3x$	
x	y
-2	-6
-1	-3
0	0
1	3
2	6



Some Algebra Review:

When working with a table of values and linear equations, it is most useful to have 'y' isolated on the left.

Example:

$$2x+3y=12$$
$$3y=-2x+12$$
$$y=-\frac{2}{3}x+4$$

11. Isolate y.

$2x-4y=16$

$-2x \qquad -2x$

$-4y=-2x+16$

$\frac{}{-4} \qquad \frac{}{-4} \qquad \frac{}{-4}$

$y=\frac{1}{2}x-4$

12. Isolate y.

$4y-8x+12=0$

$+8x \qquad +8x$

$4y+12=8x$

$-12 \qquad -12$

$4y=8x-12$

$\frac{}{4} \qquad \frac{}{4} \qquad \frac{}{4}$

$y=2x-3$

13. Isolate y.

$\frac{20}{5}+\frac{3x}{5}=\frac{5y}{5}$

$4+\frac{3}{5}x=y$

$y=\frac{3}{5}x+4$

14. Isolate y.

$6\left(\frac{1}{3}x-\frac{3}{2}y=1\right)$

$\frac{6}{3}x-\frac{18}{2}y=6$

$2x-9y=6$

$-9y=-2x+6$

$y=\frac{2}{9}x-\frac{6}{9}$

$y=\frac{2}{9}x-\frac{2}{3}$

15. Isolate y.

$4\left(x-\frac{3y}{4}+9=0\right)$

$4x-\frac{12y}{4}+36=0$

$4x-3y=-36$

$-3y=-4x-36$

$y=\frac{4}{3}x+12$

16. Isolate y.

$30\left(\frac{2x}{3}+\frac{y}{2}-\frac{3}{5}=1\right)$

$\frac{60x}{3}+\frac{30y}{2}-\frac{90}{5}=30$

$20x+15y-18=30$

$20x+15y=48$

$15y=-20x+48$

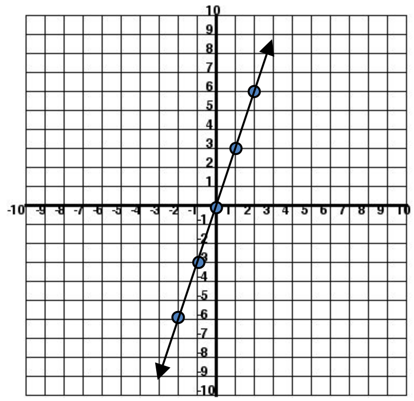
$y=\frac{-20}{15}x+\frac{48}{15}$

$y=-\frac{4}{3}x+\frac{16}{5}$

Graphing from a Table of Values.

Using the *table of values*, graph the equation $y = 3x$ on the graph provided.

$y = 3x$	
x	y
-2	-6
-1	-3
0	0
1	3
2	6



Step1: From the table of values we get the following ordered pairs. $(-2,-6), (-1,-3), (0,0), (1,3), (2,6)$

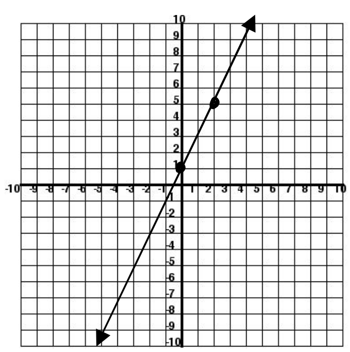
Step2: Plot each of the ordered pairs.

Step3: Draw a line through the points with arrows on each end.

Use the table of values, if necessary, to graph each of the following equations.

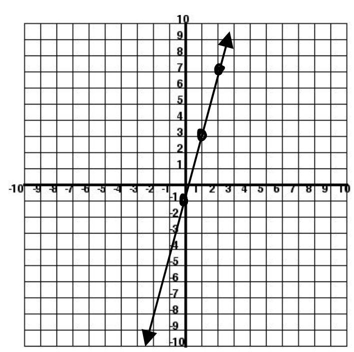
17. $y = 2x + 1$

$y = 2x + 1$	
x	y
-2	$-4+1=-3$
-1	$-2+1=-1$
0	1
1	3
2	5



18. $y = 4x - 1$

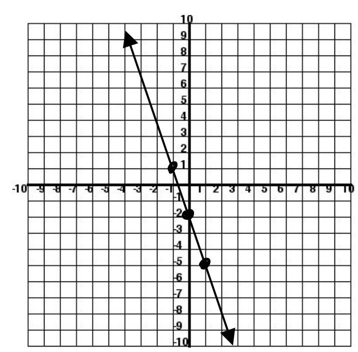
$y = 4x - 1$	
x	
-2	
-1	
0	-1
1	3
2	7



19. $3x + y = -2$

$3x + y = -2$	
x	y
-2	
-1	
0	
1	
2	

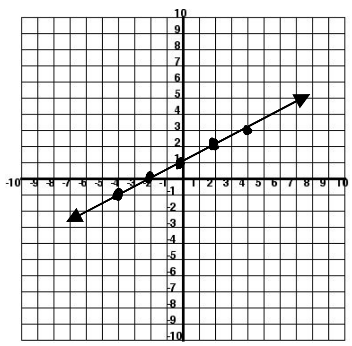
$y = -3x - 2$



Use the table of values**, if necessary**, to graph each of the following equations.

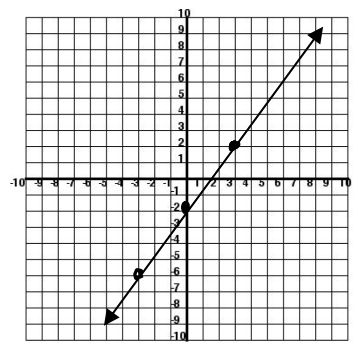
20. $y = \frac{x}{2} + 1$

$y = \frac{x}{2} + 1$	
x	y
-2	
-1	
0	
1	
2	



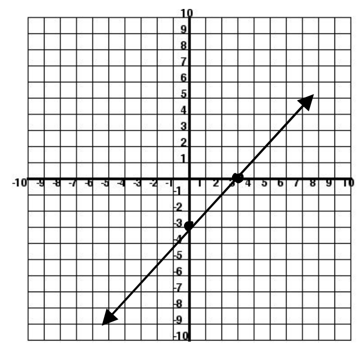
21. $y = \frac{4}{3}x - 2$

$y = \frac{4}{3}x - 2$	
x	y
-2	
-1	
0	
1	
2	



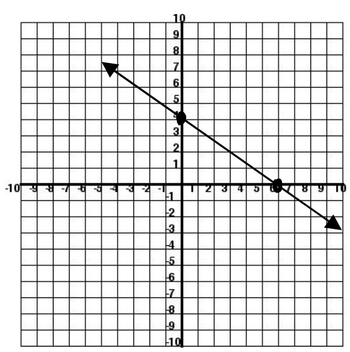
22. $y + 3 = x$

$y + 3 = x$	
x	y
-2	
-1	
0	
1	
2	



23. $2x + 3y = 12$

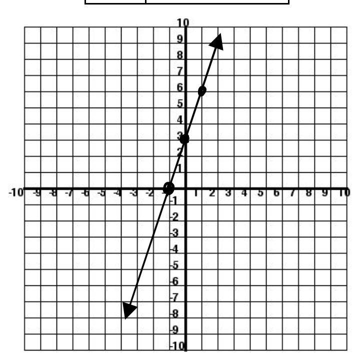
$2x + 3y = 12$	
x	y



24. $\frac{1}{3}y - x = 1$ $y - 3x = 3$

$y = 3x + 3$

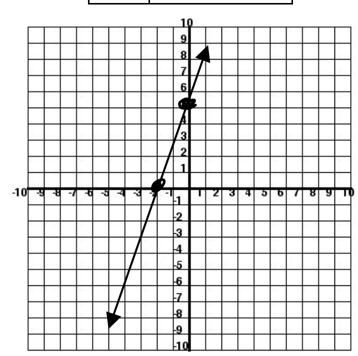
$y = 3x + 3$	
x	y



25. $\frac{2y}{5} - 2 = x$

$2y - 10 = 5x$

$y = \frac{5}{2}x + 5$	
x	y



Graphing Equations: A review from above.

Using a Table of Values:
Step 1: Choose appropriate values of 'x' to put in the table.
Step 2: Input each 'x' into the equation to find the corresponding 'y'.
Step 3: Plot the new-found 'ordered pairs'.
Step 4: Draw a line through the points. (be careful of the shape…not all are lines)

In this unit, we will be studying graphs of straight lines and their equations.

We call these **LINEAR EQUATIONS.**

An equation is said to be ***linear*** if it forms a straight line when graphed.

Equation of a Line Property:

The coordinates of every point on the line will satisfy the equation of the line.

You should REALLY memorize this!

26. How many points do you need to graph a line?

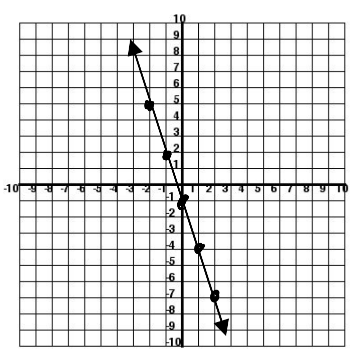
At least 2 but more points will help eliminate errors.

27. To be safe, at least how many should you have?

3 or more.

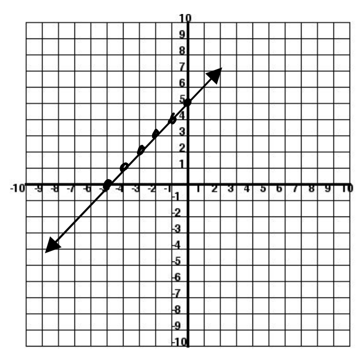
Graph these equations…

28. $y = -3x - 1$



x	y

29. $y = 5 + x$



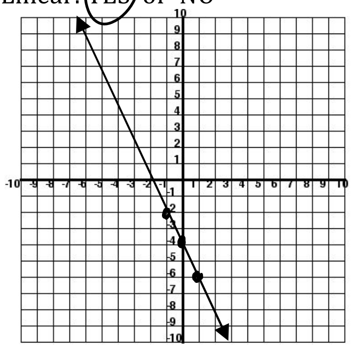
x	y

Graph the following equations, then determine if they are linear or not.

30. $y = -2x - 4$

$y = -2x - 4$	
x	y
-2	
-1	
0	
1	
2	

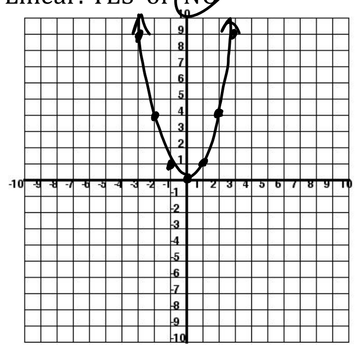
Linear: YES or NO



31. $y = x^2$

$y = x^2$	
x	y
-2	
-1	
0	
1	
2	

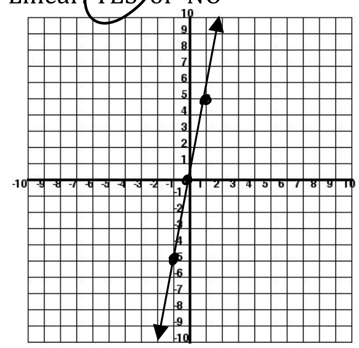
Linear: YES or NO



32. $y = 5x$

$y = 5x$	
x	y
-2	
0	
1	
4	
9	

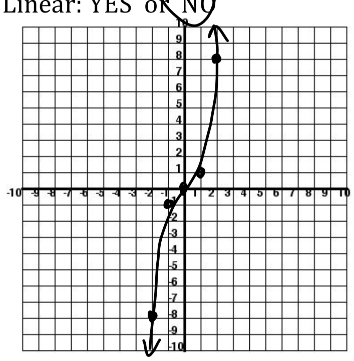
Linear: YES or NO



33. $y = x^3$

$y = x^3$	
x	y
-2	
-1	
0	
1	
2	

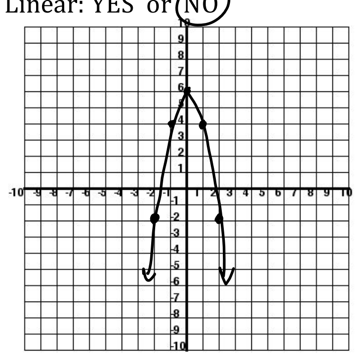
Linear: YES or NO



34. $y = -2x^2 + 6$

$y = -2x^2 + 6$	
x	y
-2	
-1	
0	
1	
2	

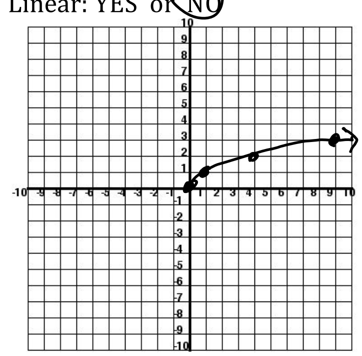
Linear: YES or NO



35. $y = \sqrt{x}$

$y = \sqrt{x}$	
x	y
-2	
-1	
0	
1	
2	

Linear: YES or NO

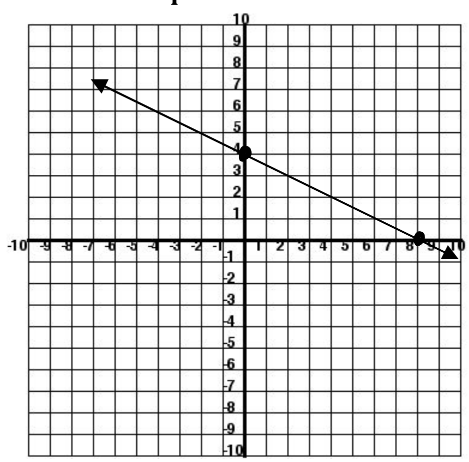


36. Can you describe a "rule of thumb" that will enable you to tell if an equation represents a linear equation or not?

exponent on the 'x' is a 1 when equation is isolated in terms of y.

Challenge #3:

The equation $2x + 4y = 16$ is a **linear equation.**



37. Find the coordinates of the point where the line crosses the y-axis. (Think…what would be the value of 'x' here?)

$2(0) + 4y = 16$
$4y = 16$
$y = 4$

$\boxed{(0,4)}$

38. What is the value of 'x' where the line crosses the y-axis?

$\boxed{\text{zero}}$

39. Find the coordinates of the point where the line crosses the x-axis.

$\boxed{(8,0)}$

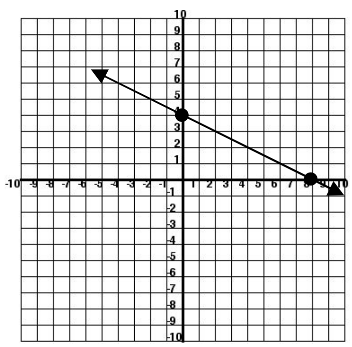
40. What is the value of "y'" where the line crosses the x-axis?

$\boxed{\text{zero}}$

Intercepts

The location where a line passes through the x-axis is called the ***x*-intercept**. This point will have the coordinates $(x, 0)$

The location where a line passes through the y-axis is called the **y-intercept**. This point will have the coordinates $(0, y)$
Consider: $2x + 4y = 16$



This line has an x-intercept at (8, 0).
And a y-intercept at (0, 4).

You may see this written as:
x-intercept is 8
y-intercept is 4

Calculating intercepts from an equation:

The x-intercept will have coordinates $(x, 0)$. This means we can substitute 0 in for y and solve to find the x-intercept. The y-intercept will have coordinates $(0, y)$.

Eg. Find the x-intercept for

$$2x + 4y = 16$$
$$2x + 4(0) = 16$$
$$2x = 16$$
$$x = 8$$

Find the y-intercept:

$$2x + 4y = 16$$
$$2(0) + 4y = 16$$
$$4y = 16$$
$$y = 4$$

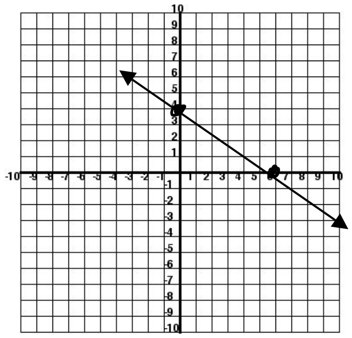
Intercepts can be expressed as ordered pairs or simply as values.
For the example above, the x-intercept is 8 or the x-intercept is (8,0).

Some notes here…

Calculate the intercepts and graph each equation using them. Fractions can be estimated on the grid.

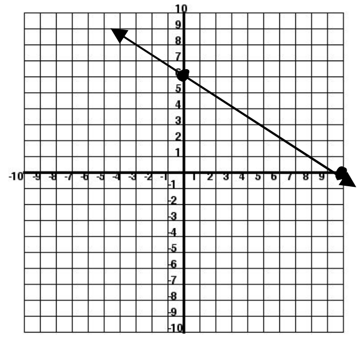
41. $2x + 3y = 12$

(0,4) (6,0)



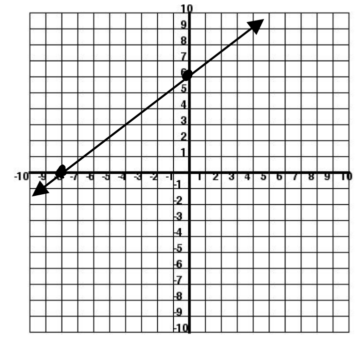
42. $3x + 5y = 30$

(10,0) (0,6)



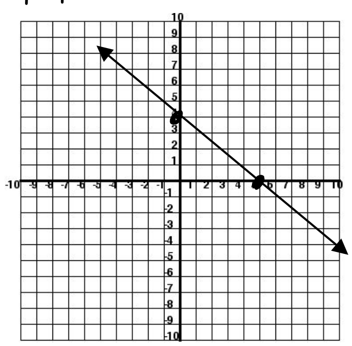
43. $3x - 4y + 24 = 0$

(-8,0) (0,6)



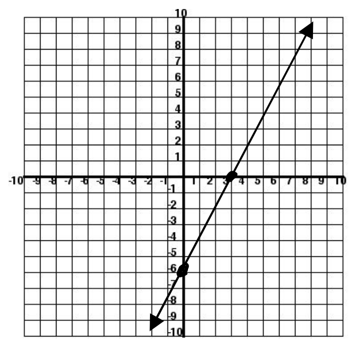
44. $4x + 5y = 20$

(5,0) (0,4)



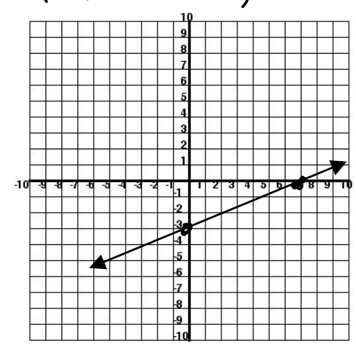
45. $6x - 3y - 18 = 0$

(3,0) (0,-6)



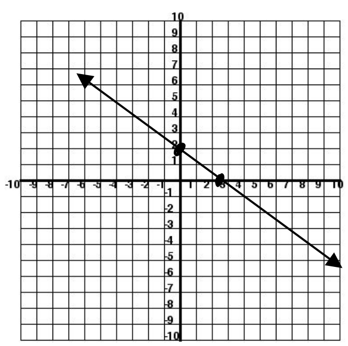
46. $3x - 7y = 21$

(7,0) (0,-3)



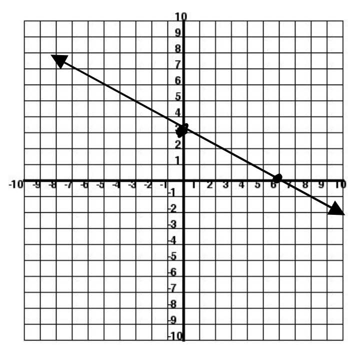
47. $4x + 5y = 10$

(2.5,0) (0,2)



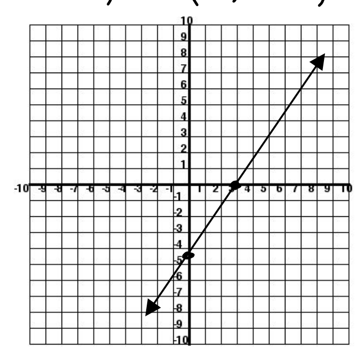
48. $9x + 3y - 18 = 0$

(2,0) (0,6)



49. $3x - 2y = 9$

(3,0) (0,-4.5)

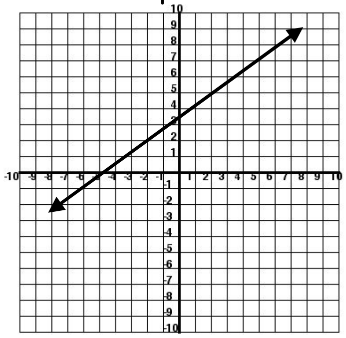


50. When do you think it would be appropriate (or the best scenario) to graph a line using the intercepts as opposed to using some other technique?

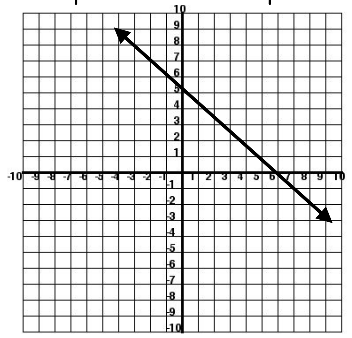
When the coefficients of x and y are factors of the constant term. This creates intercepts that are integers.

Answer the following questions about intercepts and linear relations. For these questions the domain is all real numbers.

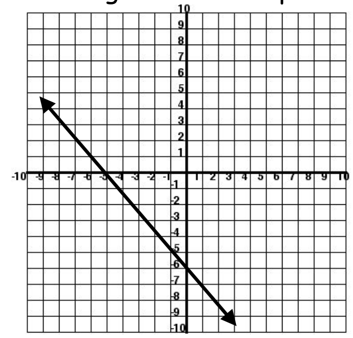
51. Draw a graph of a linear relation that has two intercepts.



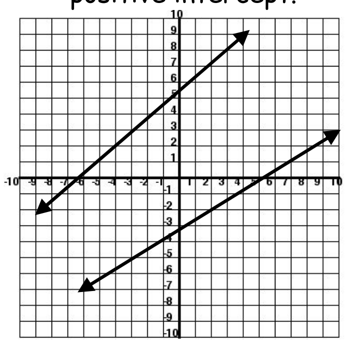
52. Draw a graph of a linear relation that has two positive intercepts.



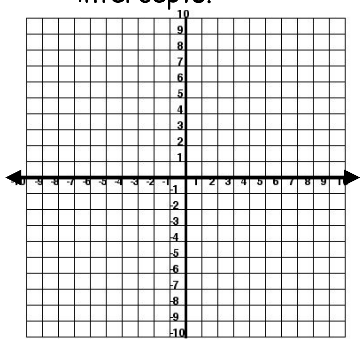
53. Draw a graph of a linear relation that has two negative intercepts.



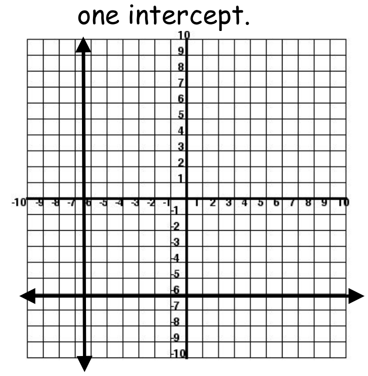
54. Draw a graph of a linear relation that has one negative and one positive intercept.



55. Draw a graph of a linear relation that as an infinite number of intercepts.



56. Draw a graph of a linear relation that has only one intercept.



57. Consider your answer to the previous question. What other **type** of line could you draw that would satisfy the problem?

Must be either horizontal or vertical.

58. Find the intercepts of the following linear equation.

$\frac{x}{2}+\frac{y}{3}=1$

y-int.

$\frac{0}{2}+\frac{y}{3}=1$

$\frac{y}{3}=1$

$y=3$

$(0,3)$

x-int.

$\frac{x}{2}+\frac{0}{3}=1$

$\frac{x}{2}=1$

$x=2$

$(2,0)$

59. Find the intercepts of the following **non**-linear relation.

$y=x^2-4$

y-int

$y=(0)^2-4$

$y=-4$

$(0,-4)$

x-int

$0=x^2-4$

$4=x^2$

$\sqrt{4}=\sqrt{x^2}$

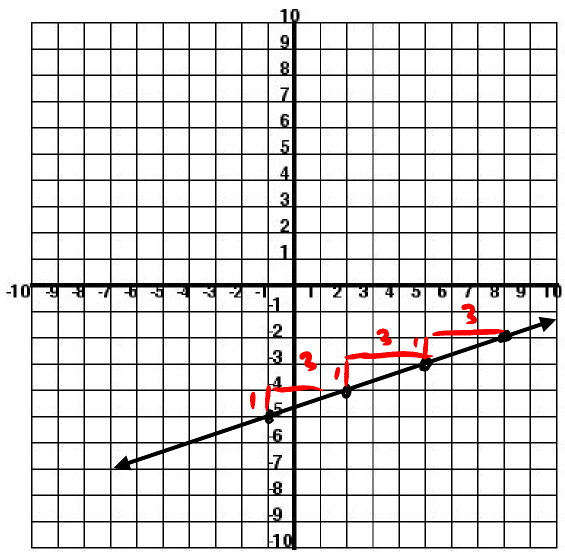
$\pm 2=x$

$(2,0)$ and $(-2,0)$

Slope of a Line

Challenge #4:

60. Plot the following points:
$(-1,-5),(2,-4),(5,-3),(8,-2)$



61. Draw a line through the points you plotted.

62. Choose three sections of the line you just plotted and find their slopes.

Slope of section 1:
$m = \frac{rise}{run} = \frac{1}{3}$

Slope of section 2:
$m = \frac{1}{3}$

Slope of section 3:
$m = \frac{1}{3}$

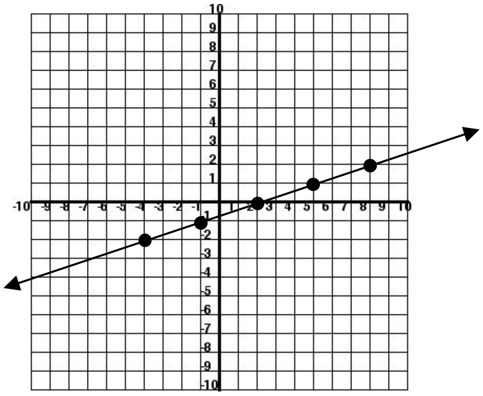
63. What do you notice?
all sections have same slope

Some notes here...

Slope of a Line

Recall from our discussion of line segments that slope can be calculated using: $\boldsymbol{m} = \frac{y_2 - y_1}{x_2 - x_1}$ or $\frac{rise}{run}$

For a straight line, the slopes of all segments on the line are equal. That is, if you find the slope of any two parts of the line, they will be equal.



Pick any three segments of the line and calculate the slope.

Slope will always be $\frac{1}{3}$.

The equations discussed earlier in this booklet result in lines that continue in two directions. Working with slope allows us to extend the line if we need to.

Remember:
- Parallel lines have equal slopes.
- Perpendicular lines have slopes that are negative reciprocals.

* You may need to find 2 points using table of values then use slope formula. As you move forward, you will find other means.

64. Find the slope of the line represented by the equation $y = 3x - 5$.

$(0,-5)$
$(1,-2)$
$\frac{-2--5}{1-0} = \frac{3}{1}$ $\boxed{m=3}$

65. Find the slope of the line represented by the equation $2x + 5y = 20$.

$5y = -2x + 20$
$y = -\frac{2}{5}x + 4$ $\boxed{m = -\frac{2}{5}}$

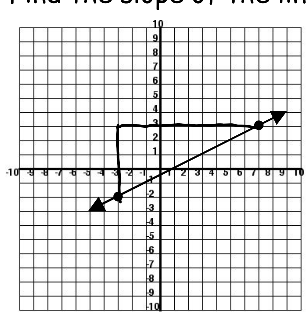
66. Find the slope of the line represented by the equation $y - 4 = 3(x - 5)$.

$y - 4 = 3x - 15$
$y = 3x - 11$ $\boxed{m=3}$

67. Find the slope of the line represented by the equation $\frac{1}{3}(x + 2) = y - 1$.

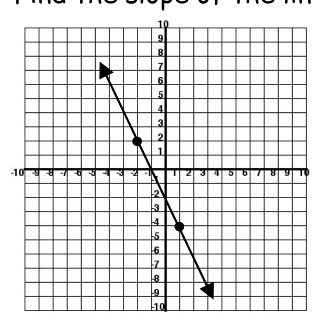
$m = \frac{1}{3}$
$(-2,1)$
$(1,2)$
$\frac{2-1}{1--2} = \boxed{\frac{1}{3}}$

68. Find the slope of the line below.



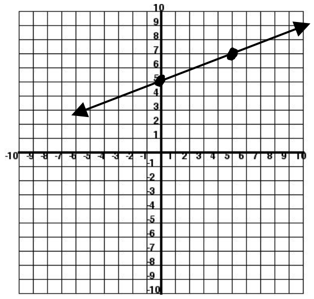
Slope is $\frac{5}{10} = \frac{1}{2}$

69. Find the slope of the line below.

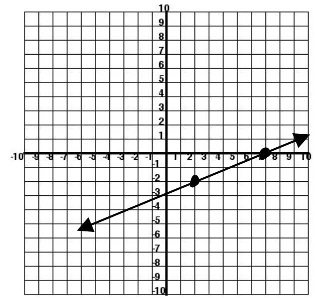


Slope is $\frac{-6}{3} = -2$

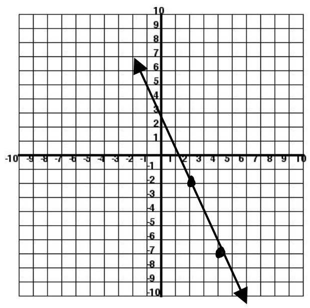
70. Draw a line through T(5,7) with slope $\frac{2}{5}$.



71. Draw a line through U(2, -2) parallel to the line in the previous question.



72. Draw a line through U(2, -2) perpendicular to the line in the question above.



73. If you were given a triangle with its vertices drawn as coordinates on an x-y coordinate plane, how could you determine if the triangle was a right triangle?

Calculate slopes of sides and look for "negative reciprocals" to prove perpendicular.

Do you know another way?

Do the side lengths satisfy Pythagoras's?

74. The slope of a line is $\frac{3}{2}$. If the line passes through point $B(5,2)$, find the coordinates of another point.

$\frac{3}{2} = \frac{y-2}{x-5}$

① Substitute slope + the point.
② Pick a value for x (or y).
③ Cross-multiply

$\frac{3}{2} = \frac{y-2}{1-5}$

$\frac{3}{2} = \frac{y-2}{-4}$

$-12 = 2y-4$

$-8 = 2y$

$-4 = y$

$\therefore (1,-4)$ is on the line.

75. The slope of a line is -2.5. If the line passes through point $C(-1,2)$, find the coordinates of another point. $-2.5 = -\frac{5}{2}$

$-\frac{5}{2} = \frac{y-2}{x--1}$

$-\frac{5}{2} = \frac{y-2}{x+1}$

$-\frac{5}{2} = \frac{y-2}{1+1}$

$-10 = 2y-4$

$-6 = 2y$

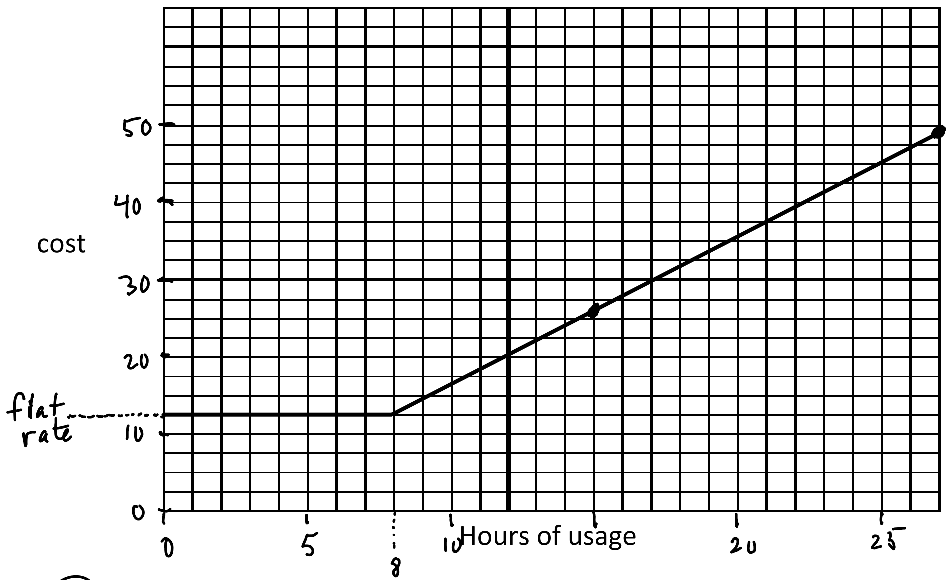
$-3 = y$

$\therefore (1,-3)$ is on the line.

Visualizing a graph would be another (perhaps more simple) method of finding other points. Think $m = \frac{-5}{2}$ $\frac{\text{rise}}{\text{run}}$

$(-1,2)$ down 5, right 2 $\Rightarrow$ $(1,-3)$

76. Julanya's internet provider charges a flat fee for the first 8 hr of access per month, plus an hourly rate for additional access. One month, 15 hr of usage cost her $25.88. The next month, 27 hr of access cost her $49.76.
a) Graph the data.



77. Find the hourly **rate** for access above 8 hr/month.

slope $\frac{49.76 - 25.88}{27 - 15} = \frac{23.88}{12} = \$1.99/\text{hr}$

78. What word is synonymous with rate in this unit?

slope is a measure of rate of change

79. Find the flat fee for the first 8 hours. (Where will you find this value on the graph?)

slope formula $m = \frac{y_2 - y_1}{x_2 - x_1}$

$\frac{49.76 - y}{27 - 8} = 1.99$

$\frac{49.76 - y}{19} = 1.99$

$49.76 - y = 19(1.99)$

$49.76 - y = 37.81$

$-y = -11.95$

$y = 11.95$

Alternate Method: Work backwards from (15, 25.88)

$\underbrace{15 \text{ hrs to } 8}_{7 \text{ hrs}}$ hrs

15 hours subtract 7 hrs cost

$25.88 - 1.99(7)$

$= 11.95$ } the cost for 8 hrs useage ∴ the flat fee

Find the slope of the line passing through the points:

80. $(2,1)$ and $(6,6)$

$$m = \frac{6-1}{6-2} = \frac{5}{4}$$

81. $(-5,2)$ and $(4,2)$

$$m = \frac{2-2}{4--5} = 0$$

82. $(-3,0)$ and $(3,-4)$

$$m = \frac{-4-0}{3--3} = \frac{-4}{6} = -\frac{2}{3}$$

83. The slope of a line is -2. The line passes through (0,0) and (-3,y). Find the value of y.

$$-\frac{2}{1} = \frac{y-0}{-3-0} \quad \text{\} substitute}$$

$$-\frac{2}{1} = \frac{y}{-3} \quad \text{\} cross - multiply}$$

$$6 = y$$

84. A line has a slope of 1.5. It passes through (-2,1) and (x,7). Find the value of x.

$$1.5 = \frac{3}{2} = \frac{7-1}{x--2}$$

$$\frac{3}{2} = \frac{6}{x+2}$$

$$3(x+2) = 12$$

$$3x+6 = 12$$

$$3x = 6$$

$$x = 2$$

85. **Challenge#5:** Show that (7, -1) is on the line $y = 2x - 15$

Algebraically:

$$y = 2x - 15$$

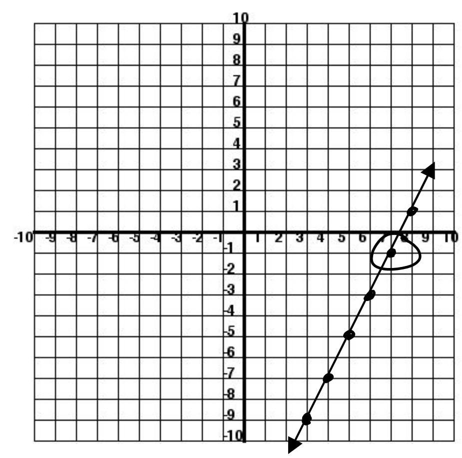
$$-1 = 2(7) - 15 \quad \text{\} substitute (7,-1)}$$

$$-1 = 14 - 15$$

$$\underbrace{-1 = -1}_{\text{equation satisfied}}$$

$\therefore$ (7,-1) is on the line.

Graphically:



The Equation of a Line

As you have seen, equations such as $2x + 3y = 12$ or $3y = x + 9$ or $y = \frac{5}{6}x - 4$ produce straight lines when graphed. They are **linear equations**.

Linear Equations may be written in several forms:

Slope-Intercept Form	$y = 3x + 2$
Point-Slope Form	$m(x - 3) = (y - 2)$
General Form	$3x - y + 2 = 0$

Recall the ***Equation of a Line Property:***

The coordinates of every point on the line will satisfy the equation of the line.

Eg.1. Show that (7, -1) is on the line $y = 2x - 15$

$y = 2x - 15$	If (7, -1) is on the line, it will satisfy the equation.
$(-1) = 2(7) - 15$	Substitue the ordered pair into the equation.
$-1 = 14 - 15$	Does the left side = right side?
$-1 = -1$	Yes. The point IS on the line.

Do you recall the "text box" like this on page 10?

Determine if the following points lie on the line $y = 2x + 4$

86. (-10, 24)

$24 = 2(-10) + 4$
$24 = -20 + 4$
$24 = -16$
NO

87. (5,14)

$14 = 2(5) + 4$
$14 = 10 + 4$
$14 = 14$
YES

88. (-7,-10)

$-10 = 2(-7) + 4$
$-10 = -14 + 4$
$-10 = -10$
YES

Determine if the following points lie on the line $3x - 2y + 6 = 0$

89. (10, 18)

$3(10) - 2(18) + 6 = 0$
$30 - 36 + 6 = 0$
$-6 + 6 = 0$
$0 = 0$
YES

90. (0,-3)

$3(0) - 2(-3) + 6 = 0$
$0 + 6 + 6 = 0$
$12 = 0$
NO

91. (-6,-6)

$3(-6) - 2(-6) + 6 = 0$
$-18 + 12 + 6 = 0$
$-6 + 6 = 0$
$0 = 0$
YES

92. Determine if the point $(2, -3)$ is on the line $y = 3x - 9$.

$-3 = 3(2) - 9$

$-3 = 6 - 9$

$-3 = -3$

Explain why or why not:

Yes, it is on the line because when the coordinates 2,-3 are substituted into the equation, left side and right side are equal.

93. Determine if the point $(-1, -4)$ is on the line $3x - 2y - 11 = 0$.

$3(-1) - 2(-4) - 11 = 0$

$-3 + 8 - 11 = 0$

$-6 = 0$] False

Explain why or why not:

No, (-1,-4) does not satisfy the equation.

94. Determine if the point $(2, -3)$ is on the line $y + 1 = \frac{3x}{2}$.

$-3 + 1 = \frac{3(2)}{2}$

$-2 = \frac{6}{2}$

$-2 = 3$ NO

Explain why or why not:

(2,-3) does not satisfy the equation

95. Determine if the set of ordered pairs represents a linear relation.

(2,3), (3,4), (4,5), (5,6)

increase by 1

2 , 3
3 , 4
4 , 5
5 , 6

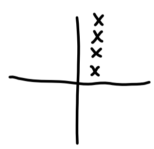
+1

Explain why or why not:

YES, rate of change (slope) is constant. Graphing would show this clearly.

96. Determine if the set of ordered pairs represents a linear relation.

(1,1), (1,2), (1,3), (1,4)



Explain why or why not:

YES, it is a vertical line

97. Determine if the set of ordered pairs represents a linear relation.

(2,1), (3,0), (4, −1), (5, −2)

x-coordinates : +1 constant

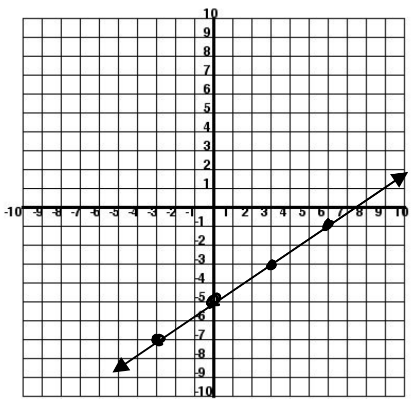
y-coord : -1 constant

Explain why or why not:

YES, constant rate of change

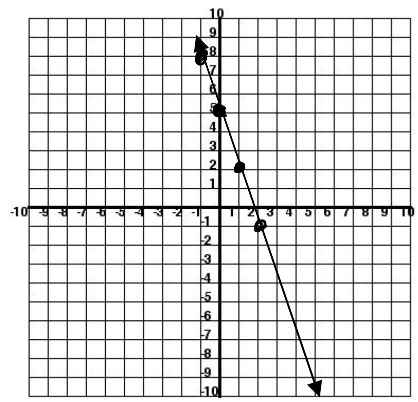
Equation of a Line: Slope-Intercept Form

98. Graph the line $y = \frac{2}{3}x - 5$ using a table of values.



x	y
-3	-7
0	-5
3	-3
6	-1

99. Graph the line $y = -3x + 5$ using a table of values.



x	y
-1	8
0	5
1	2
2	-1

100. What is the slope of the line above?

$\frac{2}{3}$

101. What is the slope of the line above?

-3

102. What is the y-intercept of the line above?

-5

103. What is the y-intercept of the line above?

5

104. Compare these values to the equation. What do you notice?

$y = \frac{2}{3}x - 5$

slope y-int

105. Compare these values to the equation. What do you notice?

$y = -3x + 5$

slope y-int

We say the equations above are written in ***slope-intercept form***. A general formula for an equation in slope intercept form is $y = mx + b$

The slope is the coefficient of *x*.

The *y*-intercept. (Make note of the sign)

Remember, *x* and *y* are the coordinates of ANY point on the line. When substituted, they will satisfy the equation. See your work on the previous page!

State the slope and y-intercept for the line represented by each equation.

106. $y = -3x + 2$

slope : -3
y-int : 2

107. $y = -\frac{3}{5}x - 7$

slope : $-\frac{3}{5}$
y-int : -7

108. $y = \frac{9}{2}x - \frac{3}{2}$

slope : $\frac{9}{2}$
y-int : $-\frac{3}{2}$

Write the equation of each line given the slope and y-intercept.

109. $m = 2, b = -5$

$y = 2x - 5$

110. $m = \frac{7}{3}, b = \frac{2}{3}$

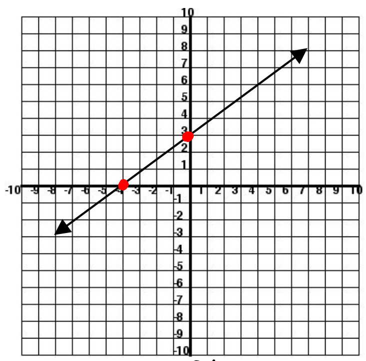
$y = \frac{7}{3}x + \frac{2}{3}$

111. $m = -3, b = -2$

$y = -3x - 2$

For each line below, state the slope, y-intercept, and equation.

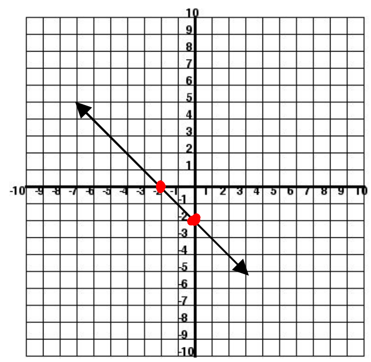
112.



slope $\underline{\ 3/4\ }$
y-intercept $\underline{\ 3\ }$

equation:
$y = \frac{3}{4}x + 3$

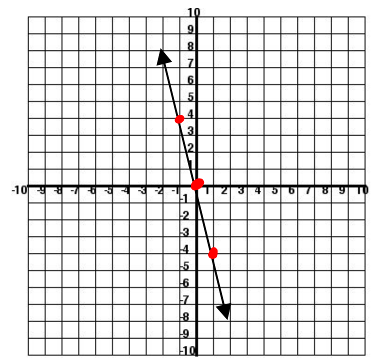
113.



slope $\underline{\ -1\ }$
y-intercept $\underline{\ -2\ }$

equation:
$y = -x - 2$

114.

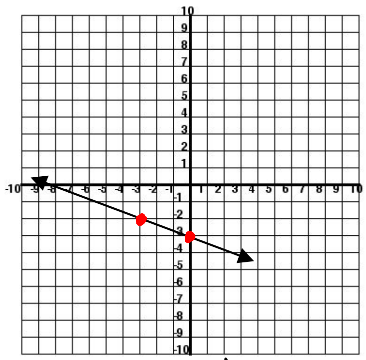


slope $\underline{\ -4\ }$
y-intercept $\underline{\ 0\ }$

equation:
$y = -4x$

For each line below, state the slope, y-intercept, and equation.

115.

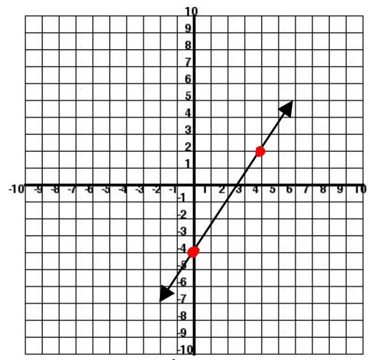


slope $-\frac{1}{3}$

y-intercept -3

equation: $y = -\frac{1}{3}x - 3$

116.



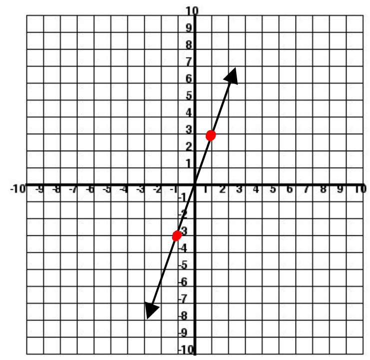
slope $\frac{6}{4} = \frac{3}{2}$

y-intercept -4

equation:

$y = \frac{3}{2}x - 4$

117.



slope 3

y-intercept 0

equation:

$y = 3x$

118. What do you notice about the **equation** of the lines passing through the origin?

There is no constant term.

$y = ax$

119. When is *b* positive?

line crosses y-axis above x-axis

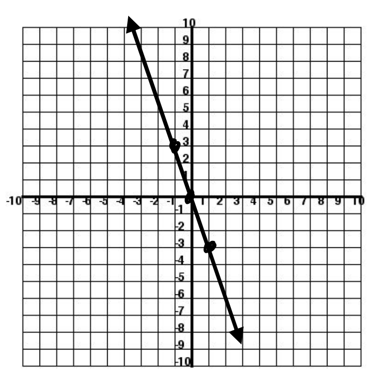
120. When is *b* negative?

line crosses y-axis below x-axis

Graph the equations below by finding the slope and y-intercept from the equation.

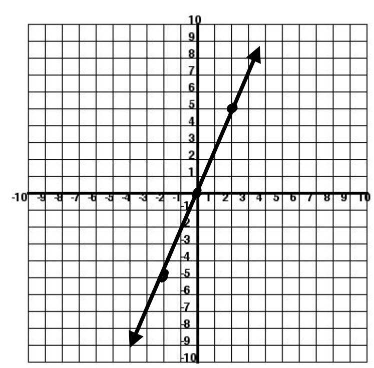
121.

$y = -3x$



122.

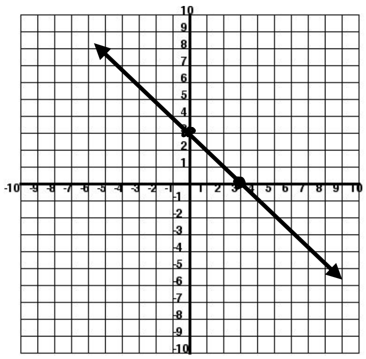
$y = \frac{5}{2}x$



Graph the equations below by finding the slope and y-intercept from the equation.

123.

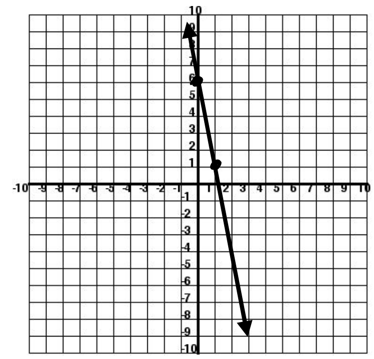
$$y = -x + 3$$



124.

$$2y = -10x + 12$$

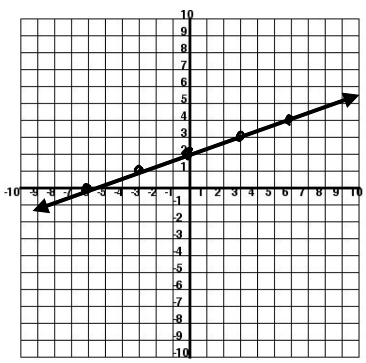
$y = -5x + 6$



125.

$$y - 5 = \frac{1}{3}x - 3$$

$y = \frac{1}{3}x + 2$

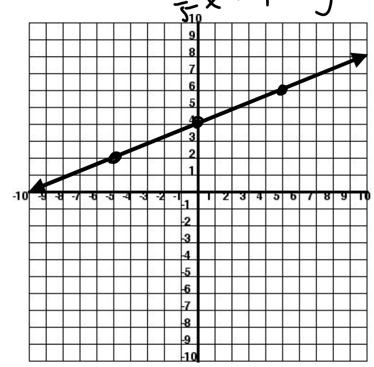


126.

$$2x - 5y + 20 = 0$$

$2x + 20 = 5y$

$\frac{2}{5}x + 4 = y$



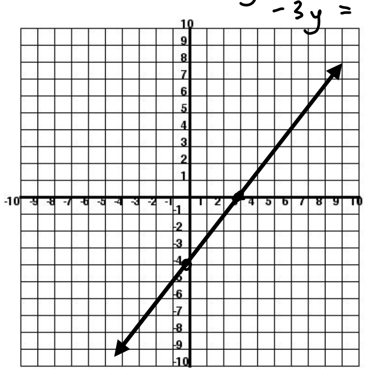
127.

$$12\left(\frac{x}{3} - \frac{y}{4} = 1\right)$$

$4x - 3y = 12$

$-3y = -4x + 12$

$y = \frac{4}{3}x - 4$



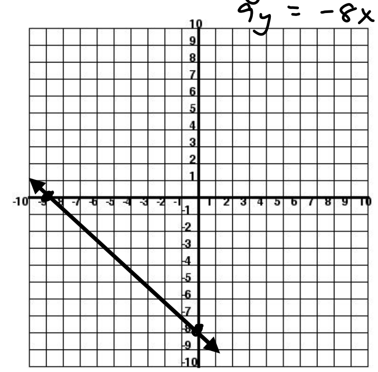
128.

$$12\left(\frac{2x}{3} + \frac{3y}{4} = -6\right)$$

$8x + 9y = -72$

$9y = -8x - 72$

$y = -\frac{8}{9}x - 8$



Determine the value of *b* for the equation $y = 3x + b$ if the line passes through the following points. Then write the equation in slope-intercept form.

129. $R(2,1)$

$y=3x+b$
$1=3(2)+b$
$1=6+b$
$-5=b$
Therefore: $y=3x-5$

130. $K(-1,4)$

$y = 3x + b$
$4 = 3(-1) + b$
$4 = -3 + b$
$7 = b$
$\therefore y = 3x + 7$

131. $A(3,-2)$

$y = 3x + b$
$-2 = 3(3) + b$
$-2 = 9 + b$
$-11 = b$
$\therefore y = 3x - 11$

132. $J(2,1)$

$y = 3x + b$
$1 = 3(2) + b$
$1 = 6 + b$
$-5 = b$
$\therefore y = 3x - 5$

133. $T\left(-2,\frac{1}{2}\right)$

$y = 3x + b$
$\frac{1}{2} = 3(-2) + b$
$\frac{1}{2} = -6 + b$
$6\frac{1}{2} = b$
$\therefore y = 3x + \frac{13}{2}$

134. $L\left(\frac{2}{3},1\right)$

$y = 3x + b$
$1 = 3\left(\frac{2}{3}\right) + b$
$1 = \frac{6}{3} + b$
$1 = 2 + b$
$-1 = b$
$\therefore y = 3x - 1$

Determine the value of *m* for the equation $y = mx + 2$ if the line passes through the following points. Then write the equation in slope-intercept form.

135. $R(12,5)$

$y = mx + 2$
$5 = m(12) + 2$
$3 = 12m$
$\frac{1}{4} = m$
$\therefore y = \frac{1}{4}x + 2$

136. $K(1,-3)$

$y = mx + 2$
$-3 = m(1) + 2$
$-5 = m$
$\therefore y = -5x + 2$

137. $A(-5,1)$

$y = mx + 2$
$1 = m(-5) + 2$
$-1 = -5m$
$\frac{1}{5} = m$
$\therefore y = \frac{1}{5}m + 2$

What you just did above is one way that you will be able to find the equation of a line. **IF** you have the __slope__ or the __y-intercept__, you can input the __coordinates__ of a point on the line to solve for the unknown part of the equation.

Then you will write the full equation with __slope__ and __y-intercept__ in place of *m* and *b*.

The following is another method.

The Equation of a Line

The three forms

Slope-Intercept Form	Point-Slope Form	General Form
$y = mx + b$	$y - y_1 = m(x - x_1)$	$Ax + By + C = 0$
m is the slope b is the y-intercept	Derived from $m = \frac{y_1 - y_1}{x_2 - x_1}$ Cross multiply to get point-slope form. Need one point and slope	A must be positive. A,B,C are integers.

Write in general form.

138. $y = 3x - 5$ $3x - y - 5 = 0$	139. $y - 5 = x + 7$ $x - y + 12 = 0$	140. $5 - 2x = -4y + 2$ $2x - 4y - 3 = 0$
141. $3\left(-\frac{1}{3}x - 4y = 2\right)$ $-x - 12y = 6$ $x + 12y + 6 = 0$	142. $3\left(y - 5 = \frac{2}{3}x + 7\right)$ $3y - 15 = 2x + 21$ $2x - 3y + 36 = 0$	143. $12\left(5 = \frac{2}{3}y + \frac{3}{4}x\right)$ $60 = 8y + 9x$ $9x + 8y - 60 = 0$

144. **Challenge #6**

Write the equation of the line that passes through A(2,5) and has slope 3. Express your answer in general from and in slope intercept form.

Method 1

$y = mx + b$

$y = 3x + b$

$5 = 3(2) + b$

$5 = 6 + b$

$-1 = b$

$\therefore\ y = 3x - 1$ or $3x - y - 1 = 0$

Method 2

$m = \frac{y_2 - y_1}{x_2 - x_1}$

$\frac{3}{1} = \frac{y - 5}{x - 2}$

$3(x-2) = y - 5$ } Point-Slope form

$3x - 6 = y - 5$

$3x - 1 = y$

$\boxed{y = 3x - 1 \quad \text{or} \quad 3x - y - 1 = 0}$

The Equation of a Line

IMPORTANT!!! There is only one line that passes through a given point with a given slope.

Given the slope and a point:

Eg.1. A line passes through A(2,5) and has slope 3. Write the equation of the line.

Use the slope formula :

$\boldsymbol{m} = \frac{y_2 - y_1}{x_2 - x_1}$ Cross-Multiply. This creates the Point-Slope form of an equation.

$\boldsymbol{m(x_2 - x_1) = y_2 - y_1}$ Fill in what you know. m = 3. Substitute the given point in for x_1 and y_1.

$3(x-2) = (y-5)$ This is our equation in **point-slope form.**
We no longer need the subscripts on x and y

$3x - 6 = y - 5$ Expanded.

$3x - y - 1 = 0$ Collecting the terms to the left side is called writing the equation in **general form.**

Or

$y = 3x - 1$ Isolate for 'y' to get the equation in **slope-intercept form**.

Write the equation of the line that passes through the given point and has the given slope. Express the equation in a) point-slope form b) general form c) slope-intercept form.

145. (-2,3), -2

$y-3=-2(x- -2)$

$y-3 = -2(x+2)$ point-slope

$y-3=-2x-4$

$y=-2x-1$ slope-intercept

$2x+y+1=0$ general

a) $y-3=-2(x+2)$

b) $2x+y+1=0$

c) $y=-2x-1$

146. (-5,2),2

$y-2 = 2(x--5)$

a) $y-2 = 2(x+5)$

$y-2 = 2x+10$

b) $y = 2x+12$

c) $2x-y+12=0$

a)

b)

c)

147. (-5,-1),-2

$y--1 = -2(x--5)$

a) $y+1 = -2(x+5)$

$y+1 = -2x-10$

b) $y = -2x-11$

c) $2x+y+11=0$

a)

b)

c)

148. (-3,4),$-\frac{1}{3}$

$y-4 = -\frac{1}{3}(x--3)$

a) $y-4 = -\frac{1}{3}(x+3)$

$y-4 = -\frac{1}{3}x-1$

b) $y = -\frac{1}{3}x+3$

$3y = -x+9$

c) $x+3y-9=0$

a)

b)

c)

149. (2,4),$\frac{1}{2}$

a) $y-4 = \frac{1}{2}(x-2)$

$y-4 = \frac{1}{2}x-1$

b) $y = \frac{1}{2}x+3$

$2y = x+6$

c) $x-2y+6=0$

a)

b)

c)

150. (0,7), -1

a) $y-7 = -1(x-0)$

$y-7 = -x$

b) $y = -x+7$

c) $x+y-7=0$

a)

b)

c)

Write the equation of the line that passes through the given point and has the given slope. Express the equation in a) point-slope form b) slope-intercept form c) general form.

151. $(3,-6), m = -3$

Start with Point-Slope formula:

$y_2 - y_1 = m(x_2 - x_1)$

$y - -6 = -3(x-3)$

$y + 6 = -3(x-3)$

$y + 6 = -3x + 9$

$y = -3x + 3$

$3x + y - 3 = 0$

a) $y + 6 = -3(x-3)$

b) $y = -3x + 3$

c) $3x + y - 3 = 0$

152. $(4,6), m = 5$

a) $y - 6 = 5(x-4)$

$y - 6 = 5x - 20$

b) $y = 5x - 14$

c) $5x - y - 14 = 0$

a)

b)

c)

153. $(-2,-1), m = \frac{1}{2}$

a) $y + 1 = \frac{1}{2}(x+2)$

$y + 1 = \frac{1}{2}x + 1$

b) $y = \frac{1}{2}x$

$2y = x$

c) $x - 2y = 0$

a)

b)

c)

154. $(5,-6), m = -\frac{3}{4}$

a) $y + 6 = -\frac{3}{4}(x-5)$

$y + 6 = -\frac{3}{4}x + \frac{15}{4}$

$y + \cancel{\frac{24}{4}} = -\frac{3}{4}x + \frac{15}{4} - \frac{24}{4}$

b) $y = -\frac{3}{4}x - \frac{9}{4}$

$4y = -3x - 9$

c) $3x + 4y + 9 = 0$

a)

b)

c)

155. $\left(\frac{1}{2}, 6\right), m = \frac{4}{3}$

a) $y - 6 = \frac{4}{3}\left(x - \frac{1}{2}\right)$

$y \cancel{-6} = \frac{4}{3}x - \frac{4}{6} + \frac{6}{1}$

$y = \frac{4}{3}x - \frac{2}{3} + \frac{18}{3}$

b) $y = \frac{4}{3}x + \frac{16}{3}$

$3y = 4x + 16$

c) $4x - 3y + 16 = 0$

a)

b)

c)

156. $(-2,1), m = 1.5$ $\frac{3}{2}$

a) $y - 1 = \frac{3}{2}(x+2)$

$y - 1 = \frac{3}{2}x + 3$

b) $y = \frac{3}{2}x + 4$

$2y = 3x + 8$

c) $3x - 2y + 8 = 0$

a)

b)

c)

157. **Challenge #7:**

Write the equation of a line in general form given that the line passes through $(3,4)$ and $(4,6)$.

$m = \frac{6-4}{4-3} = \frac{2}{1}$ // $y - 4 = 2(x-3)$

$y - 4 = 2x - 6$

$\boxed{2x - y - 2 = 0}$

Given two points:

When given two points we must first find the slope of the line. Then we will follow the same process as above.

Write the equation of the line that passes through (3,4) and (4,6).

$m = \frac{y_2 - y_1}{x_2 - x_1}$	Find the slope.
$m = \frac{6-4}{4-3} = \frac{2}{1} = 2$	The slope is 2.
$2 = \frac{y-4}{x-3}$	Substitute slope and **ONE** of the points.
$2(x-3) = y-4$	Cross-multiply. **Point-slope form**
$2x - 6 = y - 4$	Expand and simplify.
$2x - y - 2 = 0$	Write in general form.
$y = 2x - 2$	And in slope-intercept form if necessary.

Write the equation of the line that passes through the following two points in general form.

158. (3,4) and (4,6)

Answered above.

Explain your reasoning

First find slope. Then use the slope and ONE of the given points with the point-slope formula to write the equation.

159. (-2, -4) and (0, 6)

$m = \frac{6 - -4}{0 - -2} = \frac{10}{2} = 5$

$y - 6 = 5(x - 0)$

$y - 6 = 5x$

$y = 5x - 6$

$5x - y - 6 = 0$

Explain your reasoning

First find slope. Then use the slope and ONE of the given points with the point-slope formula to write the equation.

Write the equation of the line that passes through the following two points in general form.

160. (-5, -8) and (-7, -9)

$\frac{-9 - -8}{-7 - -5} = \frac{-1}{-2} = \frac{1}{2}$

$y + 8 = \frac{1}{2}(x + 5)$

$y + 8 = \frac{1}{2}x + \frac{5}{2}$

$2y + 16 = x + 5$

$x - 2y - 11 = 0$

161. (-1, -2) and (3,0)

$\frac{0 - -2}{3 - -1} = \frac{2}{4} = \frac{1}{2}$

$y - 0 = \frac{1}{2}(x - 3)$

$y = \frac{1}{2}x - \frac{3}{2}$

$2y = x - 3$

$x - 2y - 3 = 0$

162. (0,4) and (5, 0)

$\frac{0 - 4}{5 - 0} = \frac{-4}{5}$

$y - 0 = -\frac{4}{5}(x - 5)$

$y = -\frac{4}{5}x + \frac{20}{5}$

$y = -\frac{4}{5}x + 4$

$5y = -4x + 20$

$4x + 5y - 20 = 0$

163. (8, -7) and (-6, -7)

$\frac{-7 - -7}{-6 - 8} = \frac{0}{-14} = 0$

$y + 7 = 0(x + 6)$

$y + 7 = 0$

164. $\left(\frac{2}{3}, \frac{1}{4}\right)$ *and* $\left(\frac{1}{3}, \frac{1}{3}\right)$

$m = \frac{\frac{1}{3} - \frac{1}{4}}{\frac{1}{3} - \frac{2}{3}} = \frac{\frac{4}{12} - \frac{3}{12}}{-\frac{1}{3}} = \frac{\frac{1}{12}}{-\frac{1}{3}}$

$m = \frac{1}{12} \div -\frac{1}{3}$

$m = \frac{1}{12} \times -\frac{3}{1} = -\frac{3}{12} = -\frac{1}{4}$

$y - \frac{1}{3} = -\frac{1}{4}\left(x - \frac{1}{3}\right)$

$12\left(y - \frac{1}{3} = -\frac{1}{4}x + \frac{1}{12}\right)$

$12y - 4 = -3x + 1$

$3x + 12y - 5 = 0$

165. (0.3, 0.4) and (0.5, 0.7)

$\frac{0.7 - 0.4}{0.5 - 0.3} = \frac{0.3}{0.2} = \frac{\frac{3}{10}}{\cancel{\frac{2}{10}}} \times \frac{10}{2} = \frac{3}{2}$

$y - 0.4 = \frac{3}{2}(x - 0.3)$

$y - \frac{4}{10} = \frac{3}{2}\left(x - \frac{3}{10}\right)$

$y - \frac{4}{10} = \frac{3}{2}x - \frac{9}{20}$

$20y - 8 = 30x - 9$

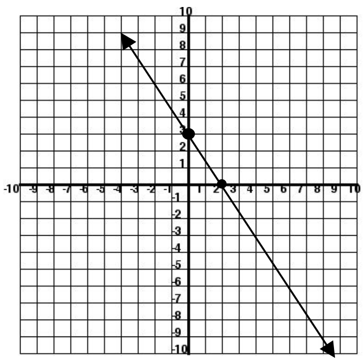
$30x - 20y - 1 = 0$

Working With Linear Equations:

- Be able to convert equations between general form and slope-intercept form.
- Be able to graph equations given to you in either form.
- Be able to make comparisons based on parallel and perpendicular lines.

Eg.1. Graph the line $3x + 2y - 6 = 0$.

Your Options:
1) use intercepts 2)make a table of values 3)convert to slope-intercept form



I chose **option 1** because this equation allows for easy calculations to find both intercepts.

$3(0) + 2y - 6 = 0 \qquad 2y - 6 = 0 \qquad 2y = 6 \qquad y = 3$

The y-intercept is 3.

$3x + 2(0) - 6 = 0 \qquad 3x - 6 = 0 \qquad 3x = 6 \qquad x = 2$

The x-intercept is 2.

Plot the two points & draw the line through them.

My second choice would have been option 3, conversion to slope-intercept form.

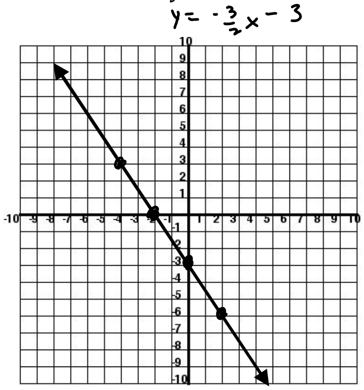
$3x + 2y - 6 = 0 \qquad 2y = -3x + 6 \qquad y = \frac{-3}{2}x + 3$

Plot the y-intercept then use the slope to plot another point, draw a line through the two points.

Graph the lines represented by each of the following equations. Use any method.

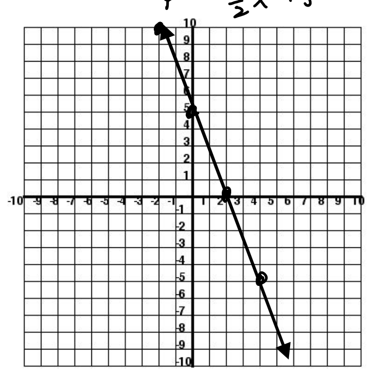
166. $3x + 2y + 6 = 0$

$y = -\frac{3}{2}x - 3$

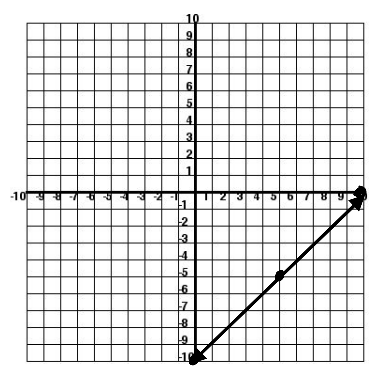


167. $5x + 2y - 10 = 0$

$y = -\frac{5}{2}x + 5$



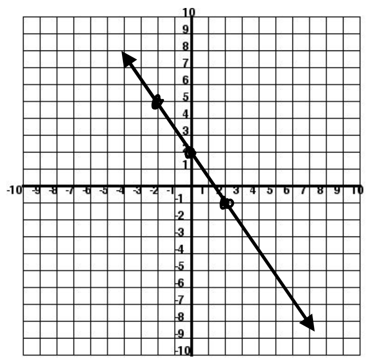
168. $x - y = 10$



Graph the lines represented by each of the following equations. Use any method.

169. $3x + 2y - 4 = 0$

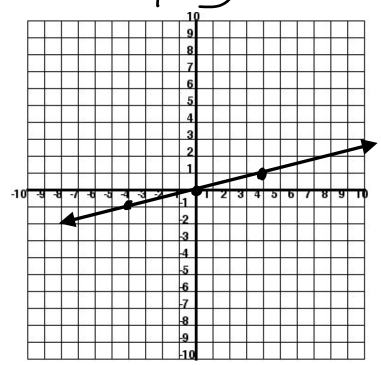
$y = -\frac{3}{2}x + 2$



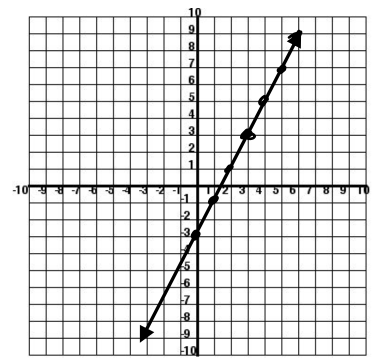
171. $x - 4y = 0$

$x = 4y$

$\frac{1}{4}x = y$



172. $2(x - 3) = y - 3$



170. Explain your strategy:

isolate 'y' to see slope / y-intercept

Explain your strategy:

Isolate 'y' : slope is $\frac{1}{4}$, y-int is 0

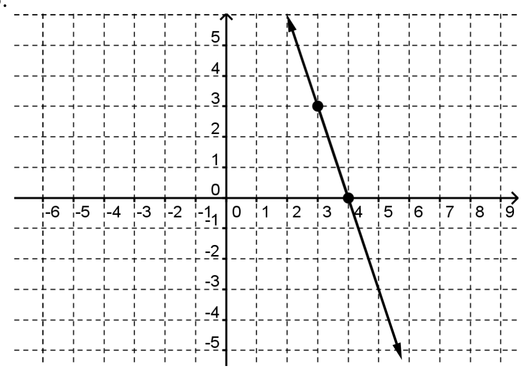
Explain your strategy:

I saw that the equation was in point-slope form $y - y_1 = m(x - x_1)$

slope 2 through (3, 3)

Match the following graphs to their corresponding equations. Choose the best match.

173.



a) $x - 3y + 3 = 0$

b) $3x - y - 12 = 0$

c) $3x + y - 12 = 0$

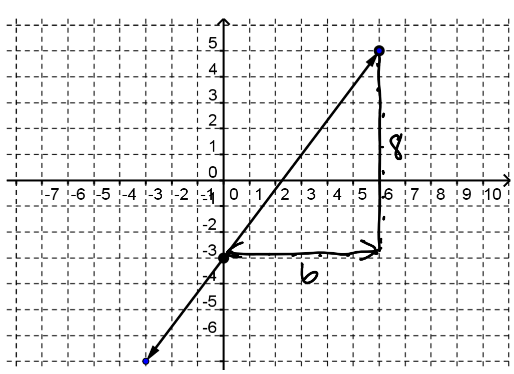
d) None of the above

$y - 3 = -3(x - 3)$

$y - 3 = -3x + 9$

$3x + y - 12 = 0$

174.



a) $4x - 3y + 9 = 0$

b) $3x - 4y + 9 = 0$

c) $3x + 4y - 9 = 0$

d) None of the above

$m = \frac{8}{6} = \frac{4}{3}$

y-int : -3

$y = \frac{4}{3}x - 3$

$3y = 4x - 9$

$4x - 3y - 9 = 0$

OR

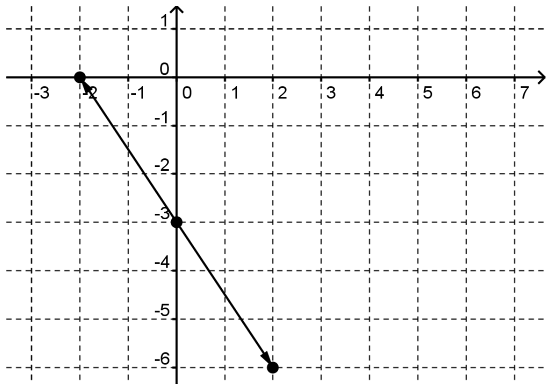
$y - 5 = \frac{4}{3}(x - 6)$

$y - 5 = \frac{4}{3}x - 8$

$3y - 15 = 4x - 24$

$4x - 3y - 9 = 0$

175. Which equation on the right represents the graph below?



a) $2x - 3y + 6 = 0$

b) $3x - 2y + 6 = 0$

c) $3x + 2y + 6 = 0$

d) None of the above

$m = -\frac{3}{2}$, y-int : -3

$y = -\frac{3}{2}x - 3$ // $2y = -3x - 6$

$3x + 2y + 6 = 0$

176. Which of the following equations represents the word statement "each element of the range is equal to one less than double an element in the domain." $y = 2x - 1$

a. $2x - y - 1 = 0$
b. $x - 2y = -1$
c. $2x + y + 1 = 0$

177. Which of the following equations represents the word statement "each element of the range is equal to two more than one third an element in the domain." $y = \frac{1}{3}x + 2$ $3y = x + 6$

a. $3x - y = 6$
b. $x - 3y = -6$
c. $x + 3y + 6 = 0$

178. Which of the following equations represents the word statement "triple each element of the range is equal to one less than double an element in the domain." $3y = 2x - 1$

a. $2x - 3y = -1$
b. $2x - 3y = 1$
c. $2x + 3y = 1$

179. Write a "word statement" to describe the following equation.
$y = 3x - 2$

range is two less than triple an element in the domain

180. Write a "word statement" to describe the following equation.
$2x + 4y - 8 = 0$

$x + 2y - 4 = 0$
$x + 2y = 4$
The sum of an element in the domain and double its range is four.

181. Write a "word statement" to describe the following equation.
$3x - 5y = 20$

$y = \frac{3}{5}x - 4$
The range is four less than three-fifths an element in the domain

182. Which of the following equations represent the same line as $y = 3x - 2$?
Circle all that apply.

a. $3x = y + 2$
b. $3x - y - 2 = 0$
c. $y - 3x = -2$
d. none

183. Which of the following equations represent the same line as $5x - 2y + 10 = 0$?
Circle all that apply. $-2y = -5x - 10$ $y = \frac{5}{2}x + 5$

e. $y = \frac{5}{2}x + 5$
f. $\frac{2}{5}(x - 4) = y - 15$
g. $x = \frac{2}{5}y - 2$
h. none

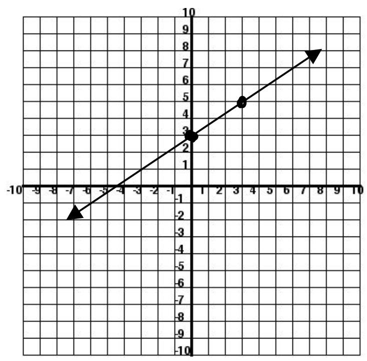
184. Which of the following equations represent the same line as $y - 4 = 2(x + 1)$?
Circle all that apply.

i. $2x - y + 6 = 0$
j. $y = 2x + 6$
k. $2x + y = 6$
l. none

$y - 4 = 2x + 2$
$y = 2x + 6$

Find the slope and *y*-intercept, write the equation in slope-intercept form, then in general form.

185.



$m = \frac{2}{3}$

$b = 3$

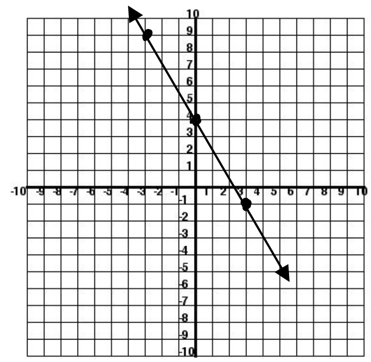
m $\frac{2}{3}$ *b* 3

slope-intercept form $y = \frac{2}{3}x + 3$

$3y = 2x + 9$

general form $2x - 3y + 9 = 0$

186.



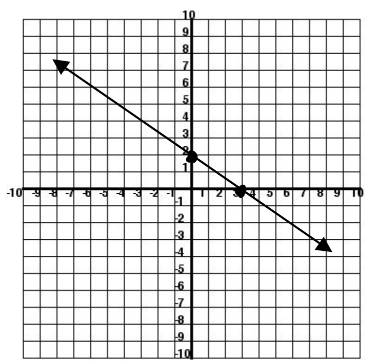
m $-\frac{5}{3}$ *b* 4

slope-intercept form $y = -\frac{5}{3}x + 4$

$3y = -5x + 12$

general form $5x + 3y - 12 = 0$

187.



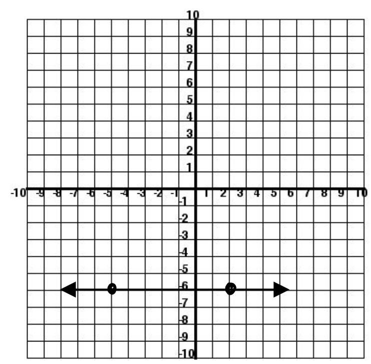
m $-\frac{2}{3}$ *b* 2

slope-intercept form $y = -\frac{2}{3}x + 2$

$3y = -2x + 6$

general form $2x + 3y - 6 = 0$

188.



m 0 *b* -6

slope-intercept form $y = -6$

general form $y + 6 = 0$

Parallel and Perpendicular Lines

Recall:

- Parallel lines have equal slopes.
- Perpendicular lines have slopes that are negative reciprocals.

For each line below, state the slope of a line that would be (a) parallel (b) perpendicular.

189. $y = 3x - 5$

a) 3

b) $-\frac{1}{3}$

190. $y - 5 = -\frac{2}{3}x$

a) $-\frac{2}{3}$

b) $\frac{3}{2}$

191. $5x - 3y = 14$

a) $\frac{5}{3}$

b) $-\frac{3}{5}$

Eg.1. Write the equation of the line parallel to $5x - 8y + 12 = 0$ and through the point (-2,3).

Parallel means same slope. So we need to find slope of $5x - 8y + 12 = 0$.

$5x - 8y + 12 = 0$ $-8y = -5x - 12$ $y = \frac{5}{8}x + \frac{12}{8}$	Convert to slope intercept form. This gives us the slope. $m = \frac{5}{8}$
	Use the slope, $m = \frac{5}{8}$ and the point (-2,3) to write the equation.
$m = \frac{y_2 - y_1}{x_2 - x_1}$	Fill in what you know. $m = \frac{5}{8}$. Substitute point (-2,3)
$\frac{5}{8} = \frac{y-3}{x--2}$	Cross-Multiply.
$5(x+2) = 8(y-3)$ $5x + 10 = 8y - 24$	Simplify.
$5x - 8y + 34 = 0$	General Form
$y = \frac{5}{8}x + \frac{17}{4}$	Slope-Intercept Form

192. Write the equation of the line parallel to $4x - 6y + 12 = 0$ and through the point (5,7).

Find m: $4x - 6y + 12 = 0$

$-6y = -4x - 12$

$y = \frac{-4}{-6}x + 2$ $\quad m = \frac{2}{3}$

$y = \frac{2}{3}x + 2$

$\frac{2}{3} = \frac{y-7}{x-5}$

$3(y-7) = 2(x-5)$

$3y - 21 = 2x - 10$

$2x - 3y + 11 = 0$

or $y = \frac{2}{3}x + \frac{11}{3}$

Explain your reasoning

If written in $Ax+By+C=0$, slope will be $\frac{-A}{B}$. Parallel means same slope.

I will use slope formula, then cross multiply and simplify.

Eg.2. Write the equation of the line perpendicular to $3x + 2y - 4 = 0$ and through the point (2,3).

Perpendicular means slopes are negative reciprocals.

Step 1: Find the slope of $3x + 2y - 4 = 0$.

$3x + 2y - 4 = 0$ Convert to slope-intercept form.

$2y = -3x + 4$

$y = \frac{-3}{2}x - \frac{4}{2}$ This line has a slope, $m = \frac{-3}{2}$.

Negative reciprocal!

The perpendicular line will have a slope of $m = \frac{2}{3}$

Use: $m = \frac{y_2 - y_1}{x_2 - x_1}$

$\frac{2}{3} = \frac{y-3}{x-2}$ Fill in what you know. $m = \frac{2}{3}$. Substitute point (2,3)

$2(x-2) = 3(y-3)$ Cross-Multiply.

$2x - 4 = 3y - 9$ Simplify.

$2x - 3y + 5 = 0$ General Form

$y = \frac{2}{3}x + \frac{5}{3}$ Slope-Intercept Form

193. Write the equation of the line perpendicular to $4x + 3y - 24 = 0$ and through the point (1,4).

$m = -\frac{A}{B} = \frac{-4}{3}$

$\therefore m_{\perp} = \frac{3}{4}$

$\frac{3}{4} = \frac{y-4}{x-1}$

$3(x-1) = 4(y-4)$

$3x - 3 = 4y - 16$

$3x - 4y + 13 = 0$

or

$y = \frac{3}{4}x + \frac{13}{4}$

Eg.3.Write an equation for the line through C(2,4) that is perpendicular to the line through A(1,2) and B(4,8).

First find slope AB. $m = \frac{8-2}{4-1} = \frac{6}{3} = 2$ Therefore, the perpendicular line has slope, $m = \frac{-1}{2}$.

$\boldsymbol{m = \frac{y_2 - y_1}{x_2 - x_1}}$ Fill in what you know: $m = \frac{-1}{2}$. & substitute point (2,4)

$\boldsymbol{\frac{-1}{2} = \frac{y-4}{x-2}}$ Cross-Multiply.

$-1(x-2) = 2(y-4)$ Simplify.

$-x + 2 = 2y - 8$

$x + 2y - 10 = 0$ General Form

$y = -\frac{1}{2}x + 5$ Slope-Intercept Form

Know which of these forms you are being asked to answer in. If it is not specified, you can choose.

Both describe the same line.

194. Write an equation for the line through C(1,2) that is perpendicular to the line through A(2,4) and B(5,5).

$m = \frac{5-4}{5-2} = \frac{1}{3}$

$m_{\perp} = -\frac{3}{1}$

Explain your reasoning

$-\frac{3}{1} = \frac{y-2}{x-1}$

$-3(x-1) = 1(y-2)$

$-3x+3 = y-2$

$y = -3x+5$

$3x+y-5=0$

195. Write an equation for the line through Q(1,2) that is perpendicular to the line through R(-2,0) and S(3,5).

$m = \frac{5-0}{3--2} = \frac{5}{5} = 1$

$m_{\perp} = -1$

I will use point-slope form this time.

$y-2 = -1(x-1)$

$y-2 = -x+1$

$y = -x+3$ or $x+y-3=0$

Determine the equation of the following lines. Answer in general form.

196. The line parallel to $2x - 3y + 1 = 0$ and passing through the point (1, 2).

$m = \frac{-A}{B} = \frac{-2}{-3} = \frac{2}{3}$

$\frac{2}{3} = \frac{y-2}{x-1}$

$2(x-1) = 3(y-2)$

$2x-2 = 3y-6$

$2x-3y+4=0$

197. The line perpendicular to $x - 5y + 2 = 0$ and passing through the point (-2, 5).

$m = \frac{-A}{B} = \frac{-1}{-5} = \frac{1}{5}$

$m_{\perp} = -\frac{5}{1}$

$-\frac{5}{1} = \frac{y-5}{x--2}$

$-5(x+2) = 1(y-5)$

$-5x-10 = y-5$

$5x+y+5=0$

198. The line perpendicular to $3x - 12y + 16 = 0$ and having the same y-intercept as $14x - 13y - 52 = 0$.

$m = \frac{-A}{B} = \frac{-3}{-12} = \frac{1}{4}$ || $y\text{-int} = \frac{-C}{B} = \frac{52}{-13} = -4$

$\therefore m_{\perp} = -\frac{4}{1}$ through $(0, -4)$

$-\frac{4}{1} = \frac{y--4}{x-0}$

$-4(x-0) = 1(y+4)$

$-4x = y+4$

$4x+y+4=0$

199. Two perpendicular lines intersect on the x-axis. An equation of one line is $y = 3x + 9$ Find the equation of the other line.

need x-intercept → subst. 0 in for 'y'.

$0 = 3x+9$

$-9 = 3x$ $(-3, 0)$

$-3 = x$

$m_{\perp} = -\frac{1}{3}$ through $(-3, 0)$

$-\frac{1}{3} = \frac{y-0}{x--3}$

$-1(x+3) = 3(y-0)$

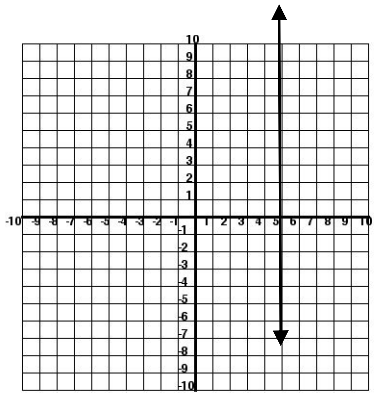
$-x-3 = 3y$

$x+3y+3=0$

Horizontal & Vertical Lines:

The equation of a horizontal line that is 3 units above the x-axis will be $\boldsymbol{y = 3}$ or $\boldsymbol{y - 3 = 0}$.
The equation of a horizontal line that is 12 units below the x-axis will be $y = -12$ or $y + 12 = 0$.

The equation of a vertical line 7 units to the right of the y-axis will be $\boldsymbol{x = 7}$ or $\boldsymbol{x - 7 = 0}$.
The equation of a vertical line 2 units to the left of the y-axis will be $x = -2$ or $x + 2 = 0$.

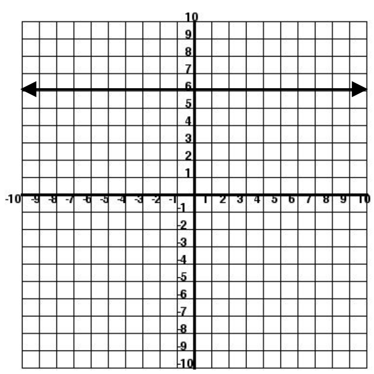


The equation of this line is $\boldsymbol{x = 5}$.

Think to yourself…
Every point on this line
has an x-coordinate of 5.
It makes sense for the
equation to be x=5.

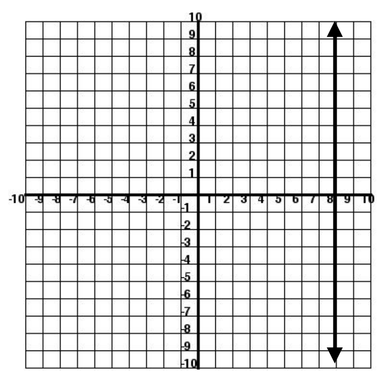
Write the equation of the following lines.

200.



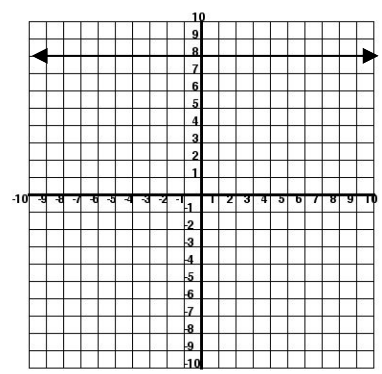
$y = 6$
$y - 6 = 0$

201.



$x = 8$
$x - 8 = 0$

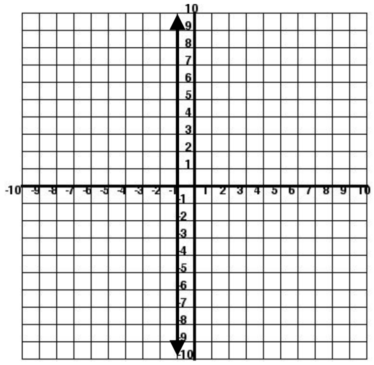
202.



$y = 8$
$y - 8 = 0$

Write the equation of the following lines.

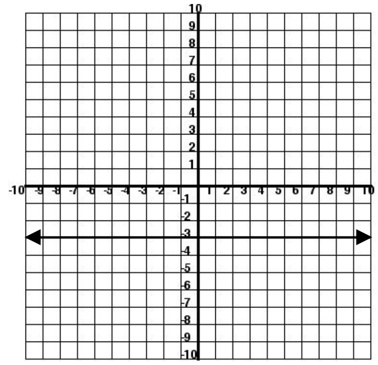
203.



$x = -1$

$x + 1 = 0$

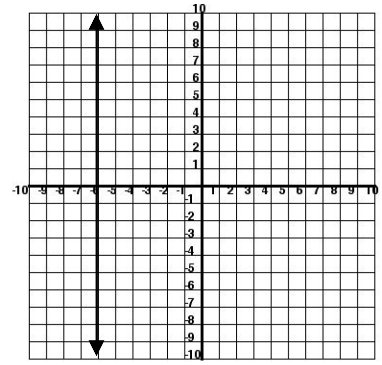
204.



$y = -3$

$y + 3 = 0$

205.

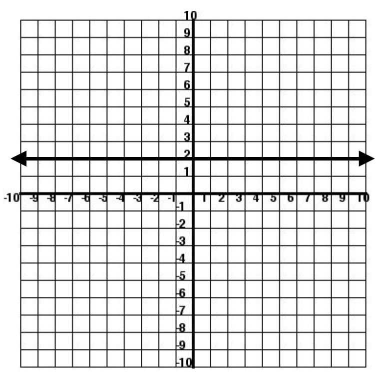


$x = -6$

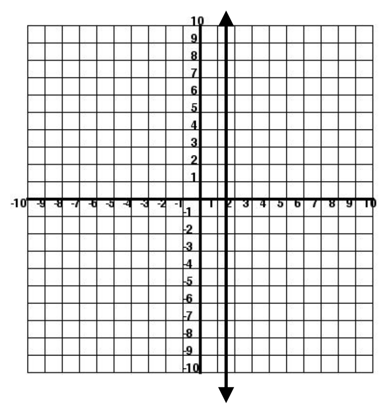
$x + 6 = 0$

206. Graph the line represented by the equation $2y - 4 = 0$.

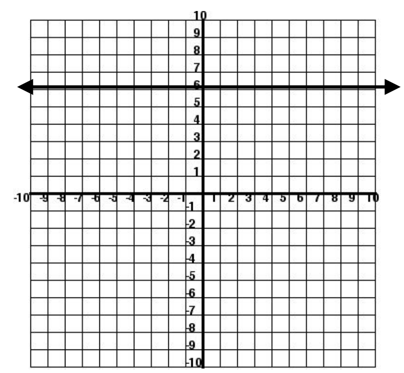
$y = 2$



207. Graph the line represented by the equation $3x - 2 = 0$.



208. Graph the line represented by the equation $y - 4 = 2y - 10$.



Mixed Practice:

209. Which of the following equations represents the steepest line?

slope $= -\frac{A}{B}$

a. $5x + 4y - 12 = 0$ $\quad -\frac{5}{4}$
b. $6x + 2y = 14$ $\quad -\frac{6}{2} = -3$
c. $-3x - 7y - 21 = 0$ $\quad \frac{3}{-7}$
d. $12x + 24y + 64 = 0$ $\quad \frac{-12}{24} = -\frac{1}{2}$

210. Which of the following passes through $(9, -8)$ and has an x-intercept of -3?

$(-3, 0)$

a. $3x + 2y + 9 = 0$
b. $5x + 9y + 27 = 0$
c. $2x + 3y + 6 = 0$
d. $4x + 3y + 12 = 0$

$m = \frac{0 - -8}{-3 - 9} = \frac{8}{-12} = \frac{-2}{3}$

$m = \frac{-A}{B} = \frac{-2}{3}$

$A = 2,\ B = 3$

211. What is unique about lines that are written in the form $x = a$.

vertical lines

212. What is unique about lines that are written in the form $y = b$.

horizontal lines

213. What is the equation, in general form, of the line that passes through the point (6, -3) and is parallel to $y = \frac{2}{3}x + 4$.

$\frac{2}{3} = \frac{y+3}{x-6}$

$2x - 12 = 3y + 9$
$2x - 3y - 21 = 0$

214. Determine the slope of the line perpendicular to $x - 2y - 3 = 0$.

$m = -\frac{A}{B} = \frac{-1}{-2} = \frac{1}{2}$

$\therefore m_{\perp} = -2$

215. Determine the equation of the line that contains the diameter of the following circle.
Centre $(-4,3)$
Point on circumference $(2, -1)$

Answer in general form.

$m = \frac{-1-3}{2+4} \quad \frac{-4}{6} \quad -\frac{2}{3}$

$-\frac{2}{3} = \frac{y+1}{x-2}$

$-2x + 4 = 3y + 3$

$2x + 3y - 1 = 0$

216. The slope of the line represented by the equation $8x - ky + 2 = 0$ is $\frac{2}{3}$.
Determine the value of k.

$m = -\frac{A}{B}$

$\frac{-8}{-k} = \frac{2}{3}$

$-24 = -2k$

$12 = k$

217. What is the equation of a line with undefined slope and an x-intercept of 5. Write your answer in general form.

vertical $\therefore x = 5$

$$\boxed{x - 5 = 0}$$

218. Write the equation $y = \frac{1}{5}x - 4$ in the form $Ax + By + C = 0$ where A is positive and all coefficients are rational numbers?

$5\left(y = \frac{1}{5}x - 4\right)$

$5y = x - 20$

$x - 5y - 20 = 0$

219. Find the value of k if $2x + ky + 7 = 0$ is parallel to $3x - 6y + 12 = 0$.

$m = \frac{-A}{B} = \frac{-3}{-6} = \frac{1}{2}$

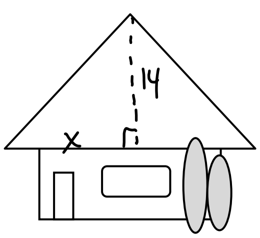
$m = \frac{-2}{k}$

$\therefore \frac{-2}{k} = \frac{1}{2}$ $\qquad k = -4$

220. Find all of the following points that are on the line $3x = 2y + 24$?

a. (8,0)
b. (6,-3)
c. (4,6)
d. (-2,9)
e. (0,-12)

221. The slope of the roof on Mr. J's hidden surf shack is $\frac{4}{3}$. If the roof is 14m tall, how wide is it?



$\frac{4}{3} = \frac{14}{x}$

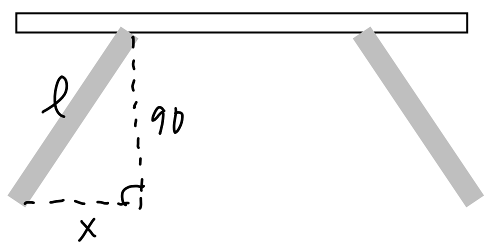
$4x = 42$

$x = 10.5$

width $= 2x$

$= 21$ m

222. Anya is building a picnic table for her backyard. The slope of the table legs is 2 and the table height is 90cm. Find the length of a table leg to the nearest cm.



$\frac{2}{1} = \frac{90}{x}$

$2x = 90$

$x = 45$

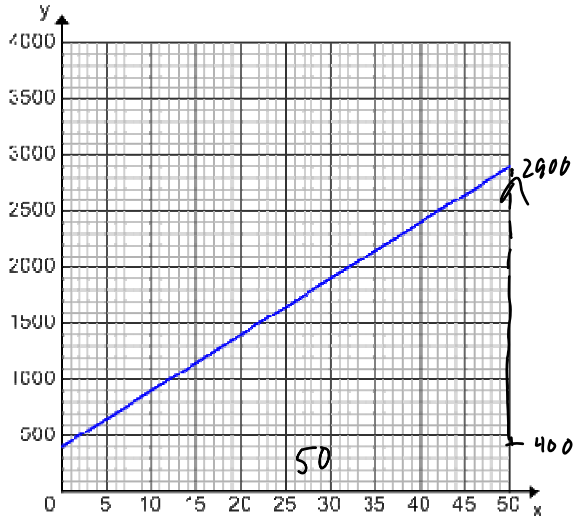
$90^2 + 45^2 = l^2$

$10125 = l^2$

$100.6 = l$

≈ 101 cm

223. Write an equation that represents the graph below.



$\boxed{y = 50x + 400}$

$m = \frac{2500}{50}$

$m = 50$

224. What is a possible relationship for the graph (and equation) above?

Many possibilities.
Something that has a fixed cost (400) & a rate. (50)

Eg. Tune-up + additional work at $50 per hour.

Eg. Banquet hall rental plus $50 per guest for food.

225. Challenge#8

The equation $y = 75x + 1500$ represents the cost of a wedding reception. The total cost consists of $1500 fee to rent the hall plus $75 per guest. Express the equation of this relation using function notation.

$C(n) = 75n + 1500$

Linear Functions

Function notation is used to show the relationship between two quantities.
The use of function notation allows the reader to identify the dependent and independent variable. Also, the letters chosen often identify what the variables represent.

Eg. The equation $y = 75x + 1500$ represents the cost of a wedding reception. The total cost consists of $1500 fee to rent the hall plus $75 per guest. Express the equation of this relation using function notation.

$C(n) = 75n + 1500$ Cost is a function of the number of guests.

226. The cost of a taxi ride in Victoria is $5.25 plus $0.35 per kilometer. Write an equation using function notation for this relation.

$C(d) = 0.35d + 5.25$

227. J-Tees Pedi-Cabs provide tours for visitors to Victoria. The cost is 25 cents per minute. Write an equation using function notation for this relation (in dollars).

$C(t) = 0.25t$

228. JLA-Skuterz rent gas-powered scooters. The cost is $40 per day plus 25 cents per kilometre ridden. Write an equation using function notation for this relation.

$C(k) = 0.25k + 40$

229. The skating rink at the recreation centre charges students $5.00 admission. Write an equation for the cost (C) as a function of the number of students (s).

$C(s) = 5s$

230. The skating rink will let a group of students book the entire rink for $500. Write an equation for the cost (C) as a function of the number of students (s).

$C(s) = 500$

231. At the same skating rink, another option is to reserve the rink for $200 and then pay $4 per student. Write an equation for the cost (C) as a function of the number of students (s).

$C(s) = 4s + 200$

Find the range value for each of the following.

232. $C(n) = 25n + 12$

Find $C(12)$.

$C(12) = 25(12) + 12$

$= 312$

233. $f(x) = \frac{1}{2}x - 3$

Find $f(-3)$.

$f(-3) = \frac{1}{2}(-3) - 3$

$= -\frac{3}{2} - 3$ or $-\frac{3}{2} - \frac{6}{2}$

$= -1.5 - 3$ $= -\frac{9}{2}$

$= -4.5$

234. $h(t) = -250t + 1200$

Find $h(20)$.

$h(20) = -250(20) + 1200$

$= -3800$

Find the domain value for each of the following.

235. $C(n) = 25n + 12$

Find n if $C(n) = 24$.

$24 = 25n + 12$

$12 = 25n$

$\frac{12}{25} = n$

236. $f(x) = \frac{1}{2}x - 3$

Find x if $f(x) = 12$.

$12 = \frac{1}{2}x - 3$

$15 = \frac{1}{2}x$

$30 = x$

237. $h(t) = -250t + 1200$

Find t if $h(t) = 1000$.

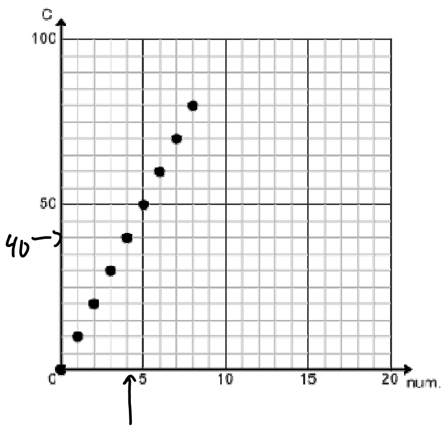
$1000 = -250t + 1200$

$-200 = -250t$

$\frac{-200}{-250} = t$

$\frac{4}{5} = t$

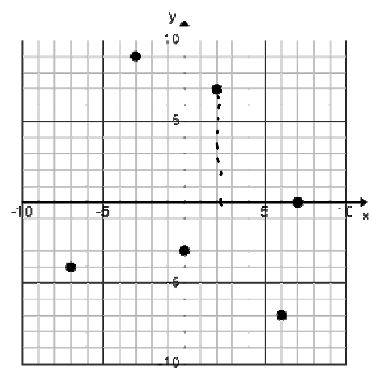
238. Below is a graph of C(n).



Find C(4).

$C(4) = 40$

239. Below is a graph of f(x).



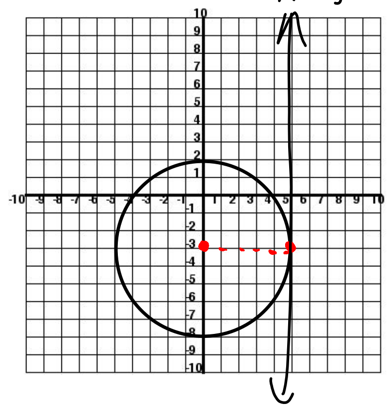
Find x if f(x) = 7.

$x = 2$

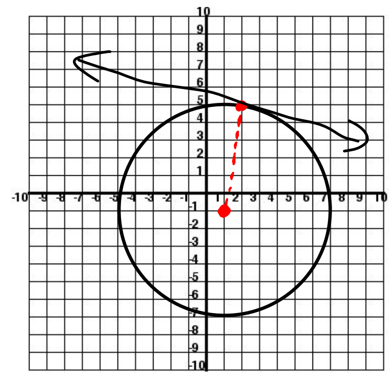
Extended Practice

240. The centre of a circle is located at (-3,0). Draw a tangent at (5,-3). What is the equation of the tangent?

$x = 5$



241. The centre of a circle is located at (1,-1). Draw a tangent at (2,5). What is the equation of the tangent?



$m_{radius} = \frac{6}{1}$

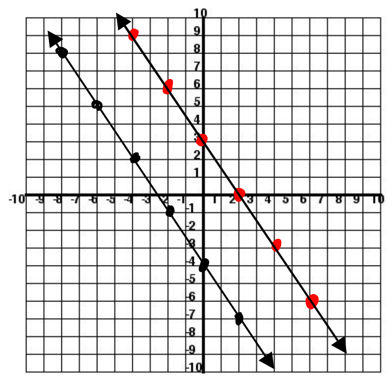
$m_{tangent} = \frac{-1}{6}$

$\frac{-1}{6} = \frac{y-5}{x-2}$

$-x+2 = 6y - 36$

$x + 6y - 32 = 0$

242. Are the lines below parallel?

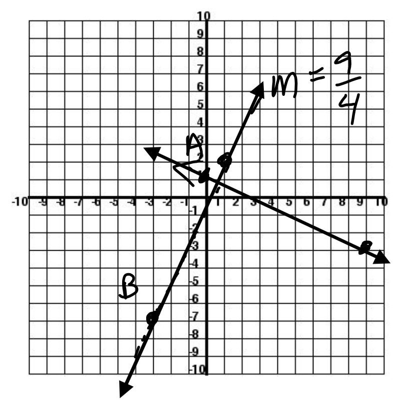


$m_1 = -\frac{3}{2}$

$m_2 = -\frac{3}{2}$

Explain how you know slopes are equal.

243. Draw a line through A(1,2) and B(-3,-7). Now draw a perpendicular line through C(9,-3).



$m_\perp = \frac{-4}{9}$

What is the equation of the perpendicular line?

$\frac{-4}{9} = \frac{y+3}{x-9}$

$-4x + 36 = 9y + 27$

$4x + 9y - 9 = 0$