HW Mark: 10 9 8 7 6 RE-Submit

Polynomials

This booklet belongs to: Marissa Louie Period 4

LESSON #	DATE	QUESTIONS FROM NOTES	Questions that I find difficult
1	Feb. 26/15	Pg. 4-11	
2	March 2/15	Pg. 12-20	
3	March 4/15	Pg. 21-27	
4	March 25/15	Pg. 28-34	
5	April 1/15	Pg. 35-38	211, 214, 215, 216
6	April 2/15	Pg. 39-42	231,
7	April 8/15	Pg. 43-51	247, 248, 249, 258, 259, 260, 263, 264, 265 266 267 269 270 271 272 273 282
		Pg.	
		Pg.	
		Pg.	
		REVIEW	
		#1: March 30/15 #2: Monday, April, 13/15	TEST

Your teacher has important instructions for you to write down below.

HW Mark: 10 9 8 7 6 RE-Submit

Polynomials 2

This booklet belongs to:________________Period___

LESSON #	DATE	QUESTIONS FROM NOTES	Questions that I find difficult
		Pg.	
		Pg.	
		Pg.	
		Pg.	
		Pg.	
		Pg.	
		Pg.	
		Pg.	
		Pg.	
		Pg.	
		REVIEW	
		TEST	

Your teacher has important instructions for you to write down below.

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POLYNOMIALS

Algebra & Number SPECIFIC OUTCOMES		TOPICS	REVIEW Note or Example
Demonstrate an understanding of the multiplication of polynomial expressions (limited to monomials, binomials and trinomials), concretely, pictorially and symbolically.	4.1	Model the multiplication of two given binomials, concretely or pictorially, and record the process symbolically.	
	4.2	Relate the multiplication of two binomial expressions to an area model.	
	4.3	Explain, using examples, the relationship between the multiplication of binomials and the multiplication of two-digit numbers.	
	4.4	Verify a polynomial product by substituting numbers for the variables.	
	4.5	Multiply two polynomials symbolically, and combine like terms in the product.	
	4.6	Generalize and explain a strategy for multiplication of polynomials.	
	4.7	Identify and explain errors in a solution for a polynomial multiplication.	
Demonstrate an understanding of common factors and trinomial factoring, concretely, pictorially and symbolically.	5.1	Determine the common factors in the terms of a polynomial, and express the polynomial in factored form.	
	5.2	Model the factoring of a trinomial, concretely or pictorially, and record the process symbolically.	
	5.3	Factor a polynomial that is a difference of squares, and explain why it is a special case of trinomial factoring where b =0	
	5.4	Identify and explain errors in a polynomial factorization.	
	5.5	Factor a polynomial, and verify by multiplying the factors.	
	5.6	Explain, using examples, the relationship between multiplication and factoring of polynomials.	
	5.7	Generalize and explain strategies used to factor a trinomial.	
	5.8	Express a polynomial as a product of its factors.	

[C] Communication **[PS]** Problem Solving, **[CN]** Connections **[R]** Reasoning, **[ME]** Mental Mathematics **[T]** Technology, and Estimation, **[V]** Visualization

Polynomials: Key Terms

Term	Definition	Example
Term		
Coefficient	$\in \mathbb{R}$	
Variable		
Constant		
Monomial		
Binomial		
Trinomial		
Polynomial	1 or more terms	
Degree of a term		
Degree of a Polynomial		
Algebra Tiles		
Combine like-terms		
Area Model		
Distribution (Expanding)		
FOIL		
GCF vs LCM		
Factoring using a GCF		
Factoring by Grouping		
Factoring $ax^2 + bx + c$ when $a = 1$		
Factoring $ax^2 + bx + c$ when $a \neq 1$		
Difference of Squares		
Perfect Square Trinomial		

What is a Polynomial?

What is a Term?

A **term** is a number and/or variable connected by multiplication or division. One term is also called a **monomial**.

The following are terms: 5, x, $3x$, $5x^2$, $\frac{3x}{4}$, $-2xy^2z^3$

Each term may have a **coefficient, variable(s) and exponents**. One term is also called a **monomial.**

If there is no variable present…we call the term a **constant.**

Answer the questions below.

1. What is/are the coefficients below?	2. What is/are the constant(s) below?	3. What is/are the variable(s) below?
$5xy^2 - 7x + 3$	$12x^2 - 5x + 13$	$5xy^2 + 3$
5, -7	13	x, y

A **polynomial** is an expression made up of **one or more terms** connected to the next by addition or subtraction.

We say a polynomial is any expression where the **coefficients are real** numbers and all **exponents are whole** numbers. That is, no variables under radicals (rational exponents), no variables in denominators (negative exponents).

Degree to constant term = 0.

The following are polynomials:

x, $2x-5$, $5+3x^2-12y^3$, $\frac{x^2+3x+2}{2}$, $\sqrt{3}x^2+5y-z$

Degree

$17x^0$

Term

$4x^2y^2z$

$2+2+1=\boxed{5}$

Polynomial

$4x^2y^2-3x^3-17+8x$

★ $\boxed{5}$ ★ $\boxed{3}$ $\boxed{0}$ $\boxed{1}$

$=ⓢ$

The following are **NOT** polynomials:

x^{-2}, $3\sqrt{x}$, $4xy+3xy^{-3}$, $12xz+3^x$

- Variable can't be exponents (2^x)

$=\cancel{5}$

- Polynomial: 1 or more terms
- Coefficient $\in \mathbb{R}$
- Exponents $\in \mathbb{W}$

✓

Which of the following are not polynomials? Indicate why.

4. $3xyz - \frac{2}{x}$ no, variable in denominator.	5. $\frac{1}{-5}x^3 - 5y$ yes	6. $2x - 4y^{-2}$ no, negative exponent.
7. $(3x+2)^{\frac{1}{3}}$ no, exponent not a whole number	8. $\sqrt{3} + x^2 - 5$ yes	9. $\frac{5}{3}x - 2^x$ no, variable is exponent.

Classifying polynomials:

By Number of Terms:

- **Monomial**: one term. Eg. $7x$, 5, $-3xy^3$
- **Binomial**: two terms Eg. $x + 2$, $5x - 3y, y^3 + \frac{5x}{3}$
- **Trinomial**: three terms Eg. $x^2 + 3x + 1$, $5xy - 3x + y^2$
- **Polynomial**: four terms Eg. $7x + y - z + 5$, $x^4 - 3x^3 + x^2 - 7x$

By Degree:
To find the degree of a *term*, add the exponents within that term.

Eg. $-3xy^3$ is a 4th degree term because the sum of the exponents is 4.
$5z^4y^2x^3$ is a 9th degree term because the sum of the exponents is 9.

To find the degree of a polynomial first calculate the degree of each term. The highest degree amongst the terms is the degree of the polynomial.

Eg. $x^4 - 3x^3 + x^2 - 7x$ is a 4th degree polynomial. The highest degree term is x^4.
$3xyz^4 - 2x^2y^3$ is a 6th degree binomial. The highest degree term is $3xyz^4$ (6th degree)

Classify each of the following by degree and by number of terms.

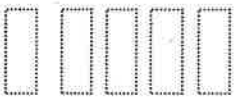
10. $2x + 3$ Degree: 1 Name: Binomial	11. $x^3 - 2x^2 + 7$ Degree: 3 Name: trinomial	12. $2a^3b^4 + a^2b^4 - 27c^5 + 3$ Degree: 7 Name: polynomial
13. 7 Degree: 0 Name: monomial	14. Write a polynomial with one term that is degree 3. $2x^2y$	15. Write a polynomial with three terms that is degree 5. $5x^3y^2 + 2x - 2$

Algebra Tiles

The following will be used as a legend for algebra tiles in this guidebook.

Write an expression that can be represented by each of the following diagrams.

16.

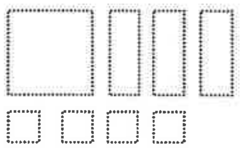


$5x$

17.

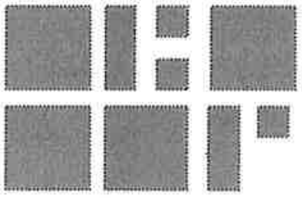
$-3x^2$

18.



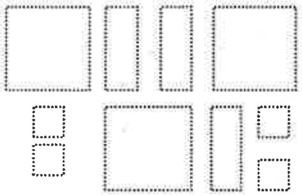
x^2+3x+4

19.



$-4x^2-2x-3$

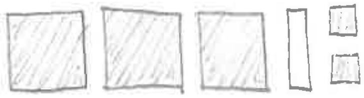
20.



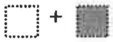
$3x^2+3x+4$

21. Draw a diagram to represent the following polynomial.
$3x^2-5x+6$

22. Draw a diagram to represent the following polynomial.
$-3x^2+x-2$



23. What happens when you add the following?



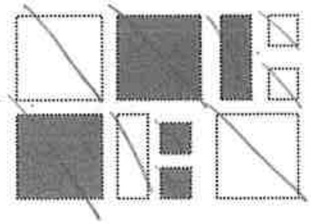
$=0$
(zero principle)

24. What happens when you add the following?

$=0$
(zero principle)

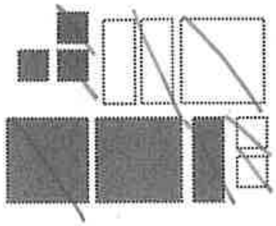
✓

25. Simplify by cancelling out tiles that add to zero. Write the remaining expression.



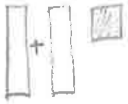
$\boxed{0}$

26. Simplify by cancelling out tiles that add to zero. Write the remaining expression.



$-x^2+x-1$

27. Represent the following addition using algebra tiles. Do not add.
$x + (x - 1)$

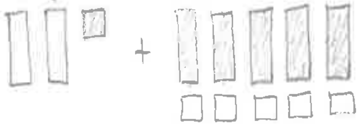


28. Represent the following addition using algebra tiles. Do not add.

$(5x + 3) + (2x + 1)$

29. Use Algebra tiles to add the following polynomials. (collect like-terms)

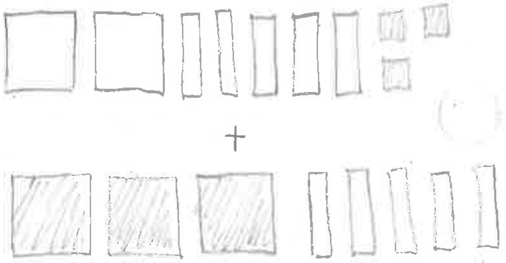
$(2x - 1) + (-5x + 5)$



$-3x+4$

30. Use Algebra tiles to add the following polynomials. (collect like-terms)

$(2x^2 + 5x - 3) + (-3x^2 + 5)$

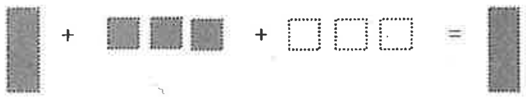


$-x^2+5x+2$

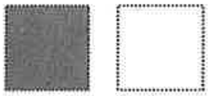
The Zero Principle:

The idea that opposites cancel each other out and the result is zero.

Eg. $x + 3 + (-3) = x$ The addition of opposites did not change the initial expression.

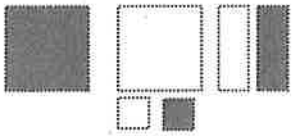


31. What is the sum of the following tiles?



Sum 0

32. If you add the following to an expression, what have you increased the expression by?



0

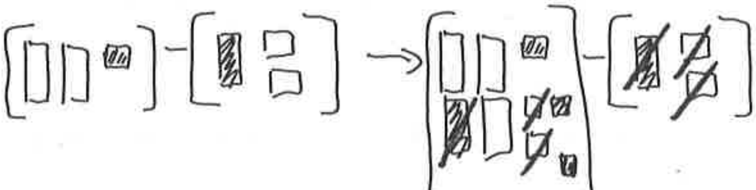
33. Represent the following subtraction using algebra tiles.

$(2x - 1) - (-x + 2)$

34. Why can you not simply "collect like-terms" when subtracting the two binomials in the previous question?

b/c the subtraction outside the bracket changes signs

35. When asked to subtract $(2x - 1) - (-x + 2)$, Raj drew the following diagram:



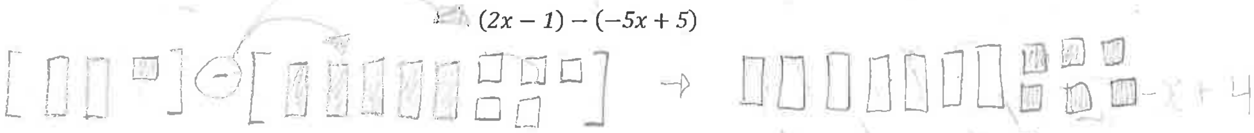
Explain how Raj applied the zero principle to subtract the polynomials.

~~He added the opposite~~

He added a zero pair.

36. Use Algebra tiles to subtract the following polynomials.

$(2x - 1) - (-5x + 5)$

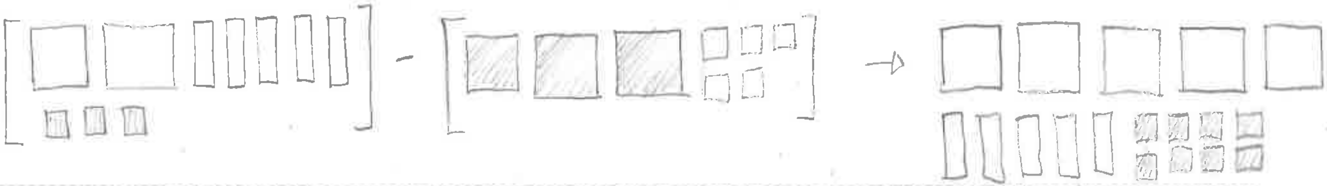


$\boxed{7x-6}$

37. Use Algebra tiles to subtract the following polynomials.

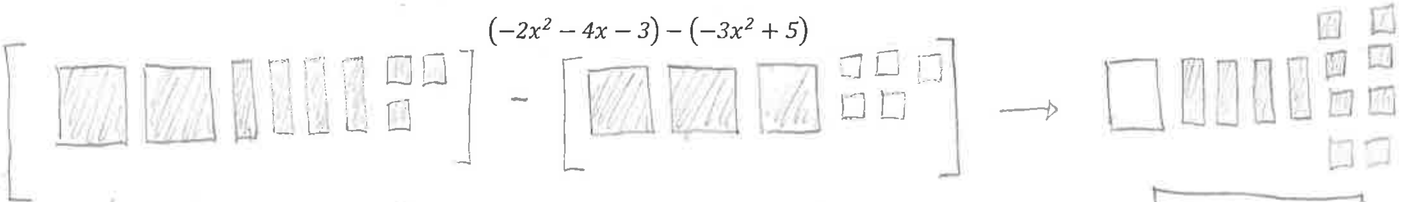
$\boxed{5x^2+5x-8}$

$(2x^2 + 5x - 3) - (-3x^2 + 5)$



38. Use Algebra tiles to subtract the following polynomials.

$(-2x^2 - 4x - 3) - (-3x^2 + 5)$



$\boxed{x^2-4x-8}$

✓

Like Terms

39. When considering algebra tiles, what makes two tiles "alike"?

if they have the same variable and degree.

★same shape★

40. What do you think makes two algebraic terms alike? (Remember, tiles are used to represent the parts of an expression.)

Collecting Like Terms without tiles:

Exactly the same variable & exponents.

You have previously been taught to combine like terms in algebraic expressions.

Terms that have the same variable factors, such as 7*x* and 5*x*, are called ***like terms.***

Simplify any expression containing like terms by adding their coefficients.

Eg.1. Simplify:

$7x + 3y + 5x - 2y$

$7x + 5x + 3y - 2y$

$= 12x + y$

Eg.2. Simplify

$3x^2 + 4xy - 6xy + 8x^2 - 3yx$

$3x^2 + 8x^2 + 4xy - 6xy - 3xy$

$= 11x^2 - 5xy$

Remember … $3yx$ is the same

Simplify by collecting like terms. Then evaluate each expression for $x = 3, y = -2$.

41. $3x + 7y - 12x + 2y$

$\boxed{-9x + 9y}$

$-9(3) + 9(-2)$

$-27 - 18$

$= \boxed{-45}$

42. $2x^2 + 3x^3 - 7x^2 - 6$

$\boxed{3x^3 - 5x^2 - 6}$

$3(3)^3 - 5(3)^2 - 6$

$3(27) - 5(9) - 6$

$81 - 45 - 6$

$= \boxed{30}$

43. $5x^2y^3 - 5 + 6x^2y^3$

$\boxed{11x^2y^3 - 5}$

$11(3)^2(-2)^3 - 5$

$11(9)(-8) - 5$

$= \boxed{-797}$

Adding & Subtracting Polynomials without TILES.

ADDITION

To add polynomials, collect like terms.

Eg.1. $(x^2+4x-2)+(2x^2-6x+9)$

Horizontal Method:

$=x^2+4x-2+2x^2-6x+9$

$=x^2+2x^2+4x-6x-2+9$

$=3x^2-2x+7$

Vertical Method:

$$\begin{array}{l} x^2+4x-2 \\ \underline{2x^2-6x+9} \\ \\ =3x^2-2x+7 \end{array}$$

SUBTRACTION

It is important to remember that the subtraction refers to all terms in the bracket immediately after it.

To subtract a polynomial, determine the opposite and add.

Eg.2. $(4x^2-2x+3)-(3x^2+5x-2)$

This question means the same as:

Multiplying each term by -1 will remove the brackets from the **second** polynomial.

$(4x^2-2x+3)-\mathbf{1}(3x^2+5x-2)$

$=4x^2-2x+3-3x^2-5x+2$

$=4x^2-3x^2-2x-5x+3+2$

$=x^2-7x+5$

We could have used vertical addition once the opposite was determined if we chose.

Add or subtract the following polynomials as indicated.

44. $(4x+8)+(2x+9)$

$4x+8+2x+9$

$\boxed{6x+17}$

45. $(3a+7b)+(9a-3b)$

$3a+7b+9a-3b$

$\boxed{12a+4b}$

46. $(7x+9)-(3x+5)$

$7x+9-3x-5$

$\boxed{4x+4}$

47. Add.

$$\begin{array}{r} (4a-2b) \\ \underline{+(3a+2b)} \end{array}$$

$$\begin{array}{r} 4a-2b \\ +\ 3a+2b \\ \hline 7a+0b \end{array}$$

$\boxed{7a}$

48. Subtract.

$$\begin{array}{r} (7x-3y) \\ \underline{-(-5x+2y)} \end{array}$$

$7x-3y+5x-2y$

$\boxed{12x-5y}$

49. Subtract.

$$\begin{array}{r} (12a-5b) \\ \underline{-(-7a-2b)} \end{array}$$

$12a-5b+7a+2b$

$\boxed{19a-3b}$

✓

Add or subtract the following polynomials as indicated.

50. $(5x^2 - 4x - 2) + (8x^2 + 3x - 3)$

$5x^2 - 4x - 2 + 8x^2 + 3x - 3$

$\boxed{13x^2 - x - 5}$

51. $(3m^2n + mn - 7n) - (5m^2n + 3mn - 8n)$

$3m^2n + mn - 7n - 5m^2n - 3mn + 8n$

$\boxed{-2m^2n - 2mn + n}$

52. $(8y^2 + 5y - 7) - (9y^2 + 3y - 3)$

$8y^2 + 5y - 7 - 9y^2 - 3y + 3$

$\boxed{-y^2 - 2y - 4}$

53. $(2x^2 - 6xy + 9) + (8x^2 + 3x - 3)$

$2x^2 - 6xy + 9 + 8x^2 + 3x - 3$

$\boxed{10x^2 - 6xy + 3x + 6}$

Your notes here…

✓

Multiplication and the Area Model

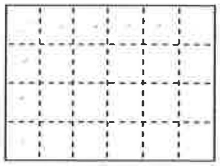
Sometimes it is convenient to use a tool from one aspect of mathematics to study another.

To find the product of two numbers, we can consider the numbers as side lengths of a rectangle.

How are side lengths, rectangles, and products related? The Area Model

The product of the two sides is the area of a rectangle. $A = lw$

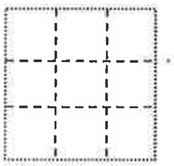
Consider:



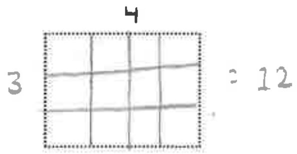
Length = 4 Width = 6

54. Show why $3 \times 3 = 9$ using the area model.

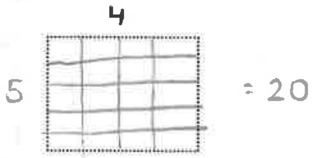
Solution:



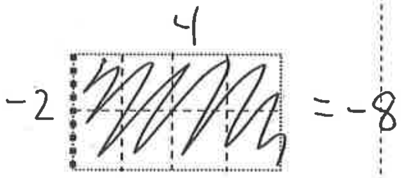
55. Show why $3 \times 4 = 12$ using the area model.



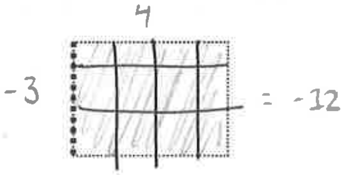
56. Calculate 5×4 using the area model.



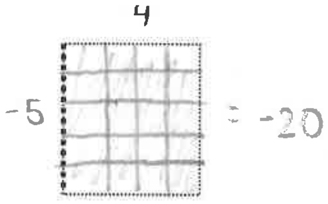
57. How might we show $-2 \times 4 = -8$ using the area model?



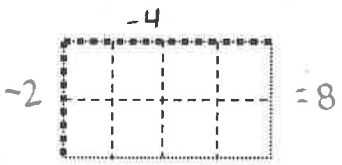
58. Calculate -3×4 using the area model.



59. Calculate -5×4 using the area model.

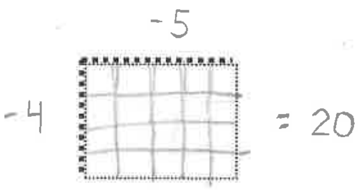


60. How might we show $-2 \times -4 = 8$ using the area model?



61. Calculate -3×-4 using the area model.

62. Calculate -5×-4 using the area model.



There are some limitations when using the area model to show multiplication. The properties of multiplying integers (+,+), (+,-), (-,-) need to be interpreted by the reader.

63. Show how you could break apart the following numbers to find the product.

$21 \times 12 =$

$= (20 + 1) \times (10 + 2)$
$= 200 + 40 + 10 + 2$
$= 252$

64. Show how you could break apart the following numbers to find the product.

$32 \times 14 =$

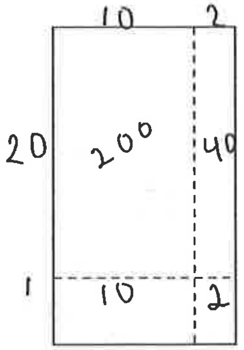
$(30+2) \times (10+4)$
$= 300+120+20+8$
$= 448$

65. Show how you could break apart the following numbers to find the product.

$17 \times 24 =$

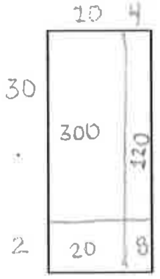
$(10+7) \times (20+4)$
$= 200 + 40 + 140 + 28$
$= 408$

66. Draw a rectangle with side lengths of 21 units and 12 units. Model the multiplication above using the rectangle.



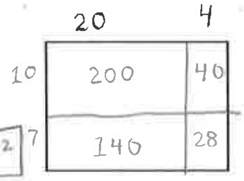
= 252

67. Draw a rectangle with side lengths of 32 units and 14 units. Model the multiplication above using the rectangle.



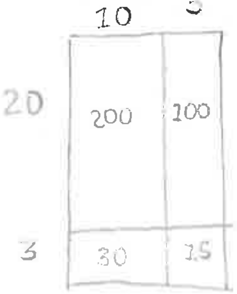
$= 300 + 120 + 20 + 8 = \boxed{448 \text{units}^2}$

68. Draw a rectangle with side lengths of 17 units and 24 units. Model the multiplication above using the rectangle.



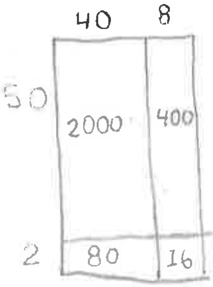
$200 + 40 + 140 + 28 = \boxed{408 \text{ units}^2}$

69. Use an area model to multiply the following without using a calculator.
23×15



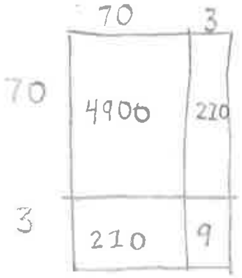
$= 200 + 100 + 30 + 15 = \boxed{345 \text{ units}^2}$

70. Use an area model to multiply the following without using a calculator.
52×48



$2000 + 400 + 80 + 16 = \boxed{2496 \text{ units}^2}$

71. Use an area model to multiply the following without using a calculator.
73×73

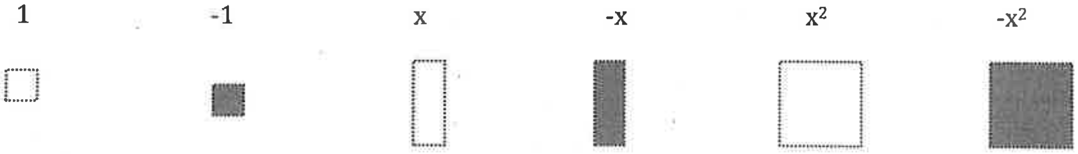


$4900 + 210 + 210 + 9 = \boxed{5329 \text{ units}^2}$

✓

Algebra tiles and the area model: Multiplication/Division of algebraic expressions.

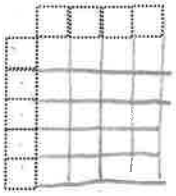
First we must agree that the following shapes will have the indicated meaning.



We must also remember the result when we multiply:

- Two positives = Positive
- Two negatives = Positive
- One positive and one negative = Negative

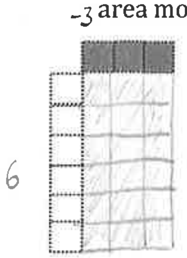
72. Write an equation represented by the diagram below and then multiply the two monomials using the area model.



$(5)(4)$

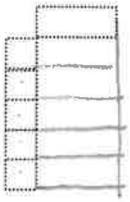
$= \boxed{20}$

73. Write an equation represented by the diagram below and then multiply the two monomials using the area model.



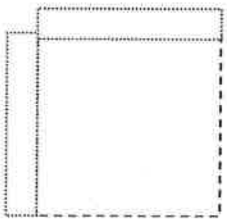
$(6)(-3)$

$= \boxed{-18}$

74. Write an equation represented by the diagram below and then multiply the two monomials using the area model.

$(5)(x)$

$= \boxed{5x}$

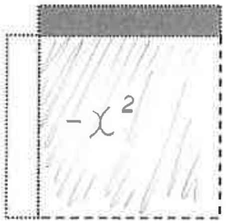
75. Write an equation represented by the diagram below and then multiply the two monomials using the area model.



$(x)(x)$

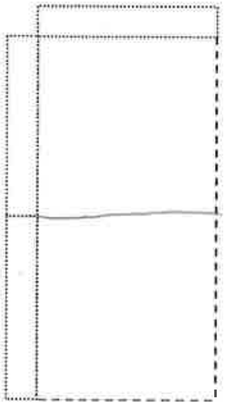
$= \boxed{x^2}$

76. If the shaded rectangle represents a negative value, find the product of the two monomials.



$\boxed{-x^2}$

77. Write an equation represented by the diagram below and then multiply the two monomials using the area model.

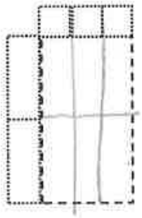


$(2x)(x)$

$= \boxed{2x^2}$

✓

78. Write an equation represented by the diagram below and then multiply the two polynomials using the area model.



$\boxed{(2x)(3)}$

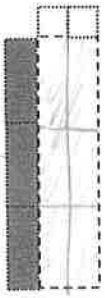
$\rightarrow \boxed{6x}$

79. Write an equation represented by the diagram below and then multiply the two polynomials using the area model.

$(2x)(-3)$

$\boxed{-6x}$

80. Write an equation represented by the diagram below and then multiply the two polynomials using the area model.



$\boxed{(-3x)(2)}$

$\rightarrow \boxed{-6x}$

81. Write a quotient that can be represented by the diagram below and then find the missing side length using the area model.

Area $= 6x$

$\frac{6x}{x} = \boxed{6}$

Length: 6

82. Write a quotient that can be represented by the diagram below and then find the missing side length using the area model.

Area $= 6x^2$

$\frac{\overset{2}{\cancel{6}}x^{\cancel{2}}}{\cancel{3}\cancel{x}} = \boxed{2x}$

Length: $2x$

83. Write a quotient that can be represented by the diagram below and then find the missing side length using the area model.

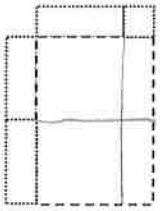
Area $= -6x^2$

$\frac{\overset{-2}{\cancel{-6}}x^{\cancel{2}}}{\cancel{3}\cancel{x}} = \boxed{-2x}$

Length: $-2x$

✓

84. Write an equation represented by the diagram below and then multiply the two polynomials using the area model.



$\boxed{(2x)(x+1)}$

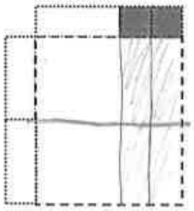
→ $\boxed{2x^2+2x}$

85. Write an equation represented by the diagram below and then multiply the two polynomials using the area model.

$\boxed{(2x)(-x+1)}$

→ $\boxed{-2x^2+2x}$

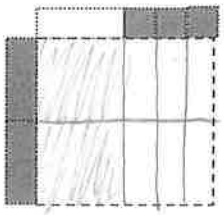
86. Write an equation represented by the diagram below and then multiply the two polynomials using the area model.



$\boxed{(2x)(x-2)}$

→ $\boxed{2x^2-4x}$

87. Write an equation represented by the diagram below and then multiply the two expressions using the area model.



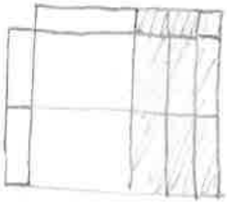
$\boxed{(-2x)(x-3)}$

→ $\boxed{-2x^2+6x}$

88. Draw and use an area model to find the product:
$(2)(2x+1)$

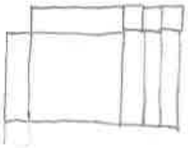
$\boxed{4x+2}$

89. Draw and use an area model to find the product:
$(2x)(x-3)$



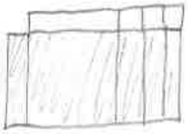
$\boxed{2x^2-6x}$

90. Draw and use an area model to find the product:
$(x)(x+3)$



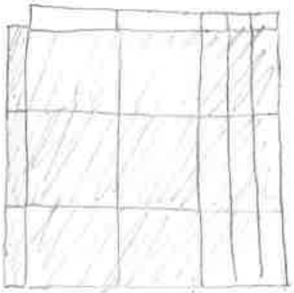
$\boxed{x^2+3x}$

91. Draw and use an area model to find the product:
$(-x)(x+3)$



$\boxed{-x^2-3x}$

92. Draw and use an area model to find the product:
$(-3x)(2x+3)$



$\boxed{-6x^2-9x}$

93. Write a quotient that can be represented by the diagram below and then find the missing side length using the area model.

Area $= x^2 + 3x$

$\frac{x^2+3x}{x}$

$= \boxed{x+3}$

Length: $x+3$

94. Write a quotient that can be represented by the diagram below and then find the missing side length using the area model.

Area $= -x^2 - 3x$

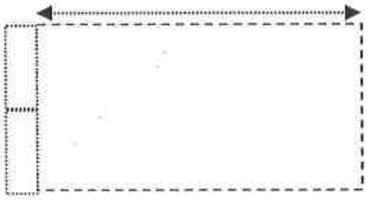
$\frac{-x^2-3x}{x}$

$\boxed{-x-3}$

Length: $-x-3$

95. Write a quotient that can be represented by the diagram below and then find the missing side length using the area model.

Area $= 2x^2 - 8x$



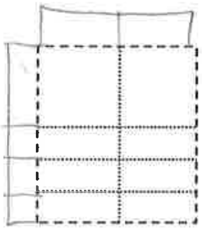
$\frac{2x^2-8x}{2x}$

$\boxed{x-4}$

Length: $x-4$

✓

96. **Find the area, length and width that can be represented by the diagram.**

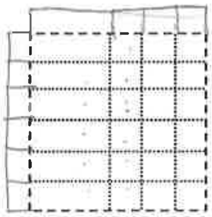


Area: $2x^2 + 6x$

Length: $x+3$

Width: $2x$

97. **Find the area, length and width that can be represented by the diagram.**

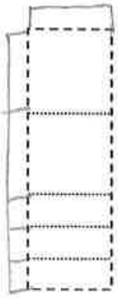


Area: $6x + 18$

Length: 6

Width: $x + 3$

98. **Find the area, length and width that can be represented by the diagram.**



Area: $2x^2 + 3x$

Length: $2x + 3$

Width: x

✓

Multiplying & Dividing Monomials without TILES

When multiplying expressions that have more than one variable or degrees higher than 2, algebra tiles are not as useful.

Multiplying Monomials:

Eg.1.

$(2x^2)(7x)$ Multiply numerical coefficients.

$= 2 \times 7 \times x \times x^2$ Multiply variables using exponent laws.

$= 14x^3$

Eg.2.

$(-4a^2b)(3ab^3)$

$= -4 \times 3 \times a^2 \times a \times b \times b^3$

$= -12a^3b^4$

Dividing Monomials:

Eg.1.

$\frac{20x^3y^4}{-5x^2y^2}$ Divide the numerical coefficients.

$= \frac{20}{-5}\frac{x^3}{x^2}\frac{y^4}{y^2}$ Divide variables using exponent laws.

$= -4xy^2$

Eg.2.

$\frac{-36m^3n^4p^2}{-9m^3np}$

$= \frac{-36}{-9}\frac{m^3}{m^3}\frac{n^4}{n}\frac{p^2}{p}$

$= 4n^3p$

Revisit the exponent laws if

(monomials)(monomials)

Multiply or Divide the following.

99. $(-2ab^3)(-3ab^5)$

$\boxed{6a^2b^8}$

100. $(5x^2y^3)(-2x^3y^5)$

$\boxed{-10x^5y^8}$

101. $4x(-3x^3)$

$\boxed{-12x^4}$

102. $\left(\frac{1}{2}ab^2\right)\left(\frac{3}{4}a^3b\right)$

$\frac{1 \times 3}{2 \times 4}$ $\frac{3}{8}$

$\boxed{\frac{3}{8}a^4b^3}$

103. $\frac{\overset{-5}{-\cancel{75}}\cancel{s^4}t^{\overset{3}{\cancel{5}}}}{\cancel{15}\cancel{s^4}\cancel{t^2}}$

$\boxed{-5t^3}$

104. $\frac{-45x^3yz^2}{-9x^2y}$ → $\frac{\overset{5}{\cancel{45}}x^{\cancel{3}}\cancel{y}z^2}{\cancel{9}\cancel{x^2}\cancel{y}}$

$\boxed{5xz^2}$

105. $\frac{\overset{4}{\cancel{24}}x^{\overset{2}{\cancel{3}}}\cancel{y^2}}{\underset{3}{\cancel{18}}\cancel{x}y^{\cancel{3}}}$

$\boxed{\frac{4x^2}{3y}}$

106. $(2cd)(-2c^2d^3)(5c)$

$\boxed{-20c^4d^4}$

107. $\frac{(3xy)(4x^3y^2)}{2x^2y}$

$\frac{\overset{6}{\cancel{12}}x^{\cancel{4}}y^{\overset{2}{\cancel{3}}}}{\cancel{2}\cancel{x^2}\cancel{y}}$

$\boxed{6x^2y^2}$

✓

Foil = binomial x binomial

Multiplying Binomials

adds to : middle 9

Challenge:

multiplies to : last 18

108. Which of the following are equal to $x^2 + 9x + 18$? $\rightarrow (x+6)(x+3)$

+ x

[✓ a)] $(x+3)(x+6)$
b) $(x+1)(x+18)$
c) $(x-3)(x-6)$
d) $(x+2)(x+9)$

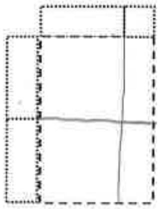
Challenge:

109. Multiply $(2x+1)(x-5)$

$2x^2 - 10x + x - 5$

$\rightarrow \boxed{2x^2 - 9x - 5}$

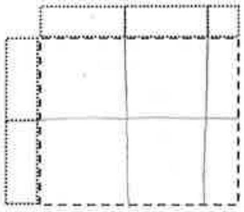
110. Write an equation represented by the diagram below and then multiply the two polynomials using the area model.



$\boxed{2x(x+1)}$

$\rightarrow \boxed{2x^2 + 2x}$

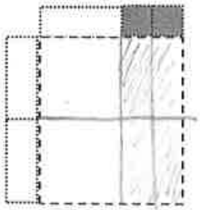
111. Write an equation represented by the diagram below and then multiply the two polynomials using the area model.



$\boxed{2x(2x+1)}$

$= \boxed{4x^2 + 2x}$

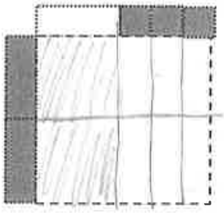
112. Write an equation represented by the diagram below and then multiply the two polynomials using the area model.



$\boxed{2x(x-2)}$

$\boxed{2x^2 - 4x}$

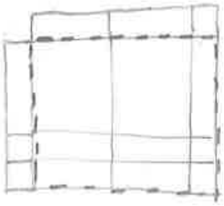
113. Write an equation represented by the diagram below and then multiply the two polynomials using the area model.



$\boxed{-2x(x-3)}$

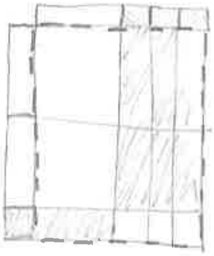
$\boxed{-2x^2 + 6x}$

114. Draw and use an area model to find the product: $(x+2)(2x+1)$



$\boxed{2x^2 + 5x + 2}$

115. Draw and use an area model to find the product: $(2x-1)(x-3)$



$\boxed{2x^2 - 7x + 3}$

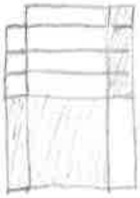
✓

116. Draw and use an area model to find the product:
$(2-x)(x+2)$

$-x^2+2x-2x+4$

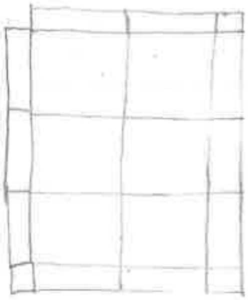
$\boxed{-x^2+4}$

117. Draw and use an area model to find the product:
$(3-x)(x-1)$



$\boxed{-x^2+4x-3}$

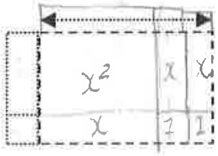
118. Draw and use an area model to find the product:
$(3x+1)(2x+1)$



$\boxed{6x^2+5x+1}$

119. Write a quotient that can be represented by the diagram below and then find the missing side length using the area model.

Area $= x^2+3x+2$



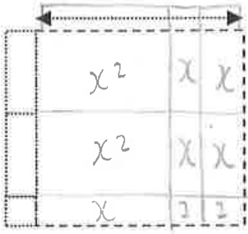
$=x+2$

$$\frac{x^2+3x+2}{x+1}$$

Length: $\boxed{x+2}$

120. Write a quotient that can be represented by the diagram below and then find the missing side length using the area model.

Area $= 2x^2+5x+2$

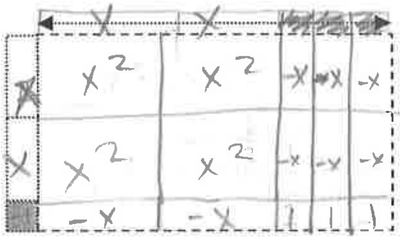


$$\frac{2x^2+5x+2}{2x+1}$$

Length: $\boxed{x+2}$

121. Write a quotient that can be represented by the diagram below and then find the missing side length using the area model.

Area $= 4x^2-8x+3$



$$\frac{(4x^2-8x+3}{2x-1)}$$

Length: $\boxed{2x-3}$

$(2x-1)(\quad)=4x^2-8x+3$

$(2x-1)(2x-3)=4x^2-8x+3$

$(2x+1)(\quad)=2x^2+5x+2$

$(2x+1)(x+2)=2x^2+5x+2$

$4x^2-8x+3$

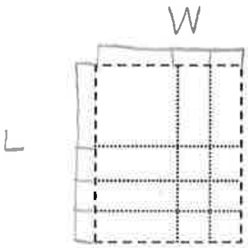
$(4x^2-2x)+(-6x+3)$

$2x(2x-1)-3(2x-1)$

$(2x-3)(2x-1)$

✓

122. Find the area, length and width that can be represented by the diagram.

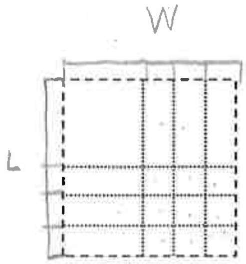


Area: x^2+5x+6

Length: $x+3$

Width: $x+2$

123. Find the area, length and width that can be represented by the diagram.

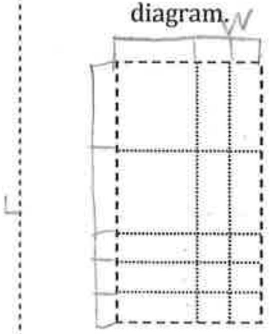


Area: x^2+6x+9

Length: $x+3$

Width: $x+3$

124. Find the area, length and width that can be represented by the diagram.

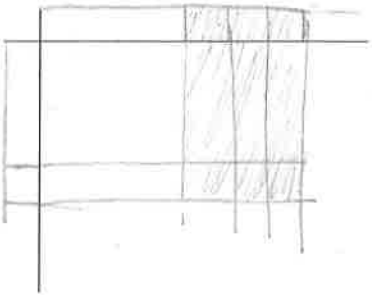


Area: $2x^2+7x+6$

Length: $2x+3$

Width: $x+2$

125. Draw tiles that represent the multiplication of $(x+1)(x-3)$.

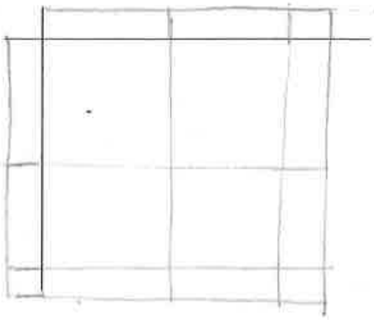


What is the product of $(x+1)(x-3)$?

$x^2-3x+x-3$

$\rightarrow \boxed{x^2-2x-3}$

126. Draw tiles that represent the multiplication of $(2x+1)(2x+1)$.

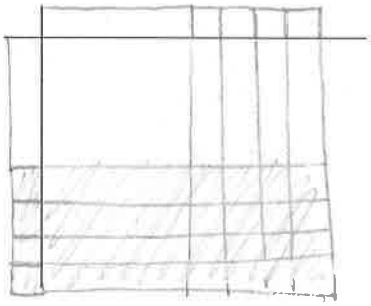


What is the product of $(2x+1)(2x+1)$?

$4x^2+2x+2x+1$

$\rightarrow \boxed{4x^2+4x+1}$

127. Draw tiles that represent the multiplication of $(x-4)(x+4)$.



What is the product of $(x-4)(x+4)$?

$x^2+4x-4x-16$

$\rightarrow \boxed{x^2-16}$

✓

Multiplying Polynomials without TILES (also called expanding or distribution)

Multiplying Binomials: *use FOIL

Eg.1. $(x+3)(x+6) = x^2 + 6x + 3x + 18 = \mathbf{x^2 + 9x + 18}$

FOIL

Firsts - Outsides-Insides-Lasts

(x)(x) + (x)(6) + (3)(x) + (3)(6)

Eg.2. $(2x+1)(x-5) = 2x^2 - 10x + x - 5 = \mathbf{2x^2 - 9x - 5}$

Multiplying a Binomial by a Trinomial:

Eg. $(y-3)(y^2-4y+7) = y^3 - 4y^2 + 7y - 3y^2 + 12y - 21 = \mathbf{y^3 - 7y^2 + 19y - 21}$

Multiply each term in the first polynomial by each term in the second.

Multiplying: Binomial × Binomial × Binomial

Eg.

$$\begin{aligned}&(x+2)(x-3)(x+4)\\&=(x^2-3x+2x-6)(x+4)\\&=(x^2-x-6)(x+4)\\&=x^3+4x^2-x^2-4x-6x-24\\&=\mathbf{x^3+3x^2-10x-24}\end{aligned}$$

Multiply the first two brackets (FOIL) to make a new trinomial.

Then multiply the new trinomial by the remaining binomial

Multiply the following as illustrated above.

128. $(x+2)(x-5)$

$x^2 - 5x + 2x - 10$

$\to \boxed{x^2 - 3x - 10}$

129. $(2x+1)(x-3)$

$2x^2 - 6x + x - 3$

$\to \boxed{2x^2 - 5x - 3}$

130. $(x-3)(x-3)$

$x^2 - 3x - 3x + 9$

$\to \boxed{x^2 - 6x + 9}$

✓

Multiply the following.

131. $(x+2)(x+2)$

$x^2+2x+2x+4$

$\rightarrow \boxed{x^2+4x+4}$

132. $(2x+1)(3x-3)$

$6x^2-6x+3x-3$

$\rightarrow \boxed{6x^2-3x-3}$

133. $(2x+1)(2x-1)$

$4x^2-2x+2x-1$

$\rightarrow \boxed{4x^2-1}$

134. $(x+2)^2$

perfect square trinomial

$(x+2)(x+2)$

$x^2+2x+2x+4$

$\rightarrow \boxed{x^2+4x+4}$

135. $(2x+5)^2$

$(2x+5)(2x+5)$

$4x^2+10x+10x+25$

$\rightarrow \boxed{4x^2+20x+25}$

136. $\overset{bin}{(x-1)}\overset{bin}{(x-1)}\overset{bin}{(x+4)}$

$(x^2-x-x+1)(x+4)$

$(x^2-2x+1)(x+4)$

$x^3+4x^2-2x^2-8x+x+4$

$\boxed{x^3+2x^2-7x+4}$ ✓

137. $(x-5)(x^2-5x+1)$

$x^3-5x^2+x-5x^2+25x-5$

$\boxed{x^3-10x^2+26x-5}$ ✓

138. $(2x-3)(3x^2+2x+1)$

$6x^3+4x^2+2x-9x^2-6x-3$

$\boxed{6x^3-5x^2-4x-3}$ ✓

139. $(x+2)^3$

$(x+2)(x+2)(x+2)$

$(x^2+2x+2x+4)(x+2)$

$(x^2+4x+4)(x+2)$

$x^3+4x^2+4x+2x^2+8x+8$

$\boxed{x^3+6x^2+12x+8}$

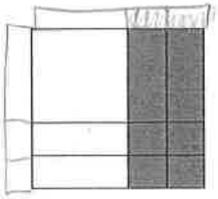
✓

difference of squares

Special Products: Follow the patterns

Conjugates: $(a+b)(a-b)$
$= a^2 + ab - ab - b^2$
$= a^2 - b^2$

140. Write an expression for the following diagram (do not simplify):



☆ $x^2+2x-2x-4$ ☆

X $(x+2)(x+2) = (x+2)^2$ X

☆ $(x+2)(x-2)$

What two binomials are being multiplied above?

X $(x+2)(x+2)$

☆ $(x+2)(x-2)$ ☆

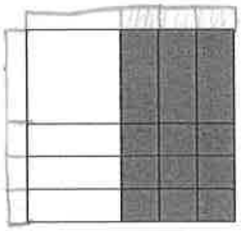
Write an equation using the binomials above and the simplified product.

$(x+2)(x-2)$

$x^2-2x+2x-4$

→ $\boxed{x^2-4}$

141. Write an expression for the following diagram (do not simplify):



☆ $x^2+3x-3x-9$ ☆

$(x+3)(x-3)$

What two binomials are being multiplied above?

$(x+3)(x-3)$

Write an equation using the binomials above and the simplified product.

$(x+3)(x-3)$

$x^2-3x+3x-9$

→ $\boxed{x^2-9}$

$(x+3)(x-3) = x^2-$

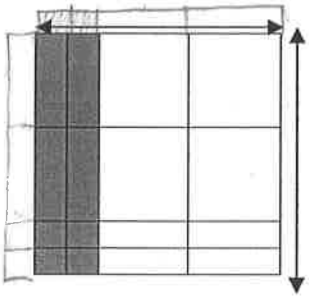
QUESTION… Describe any patterns you observe in the two questions above.

when the constants are opposite signs (inverse) the answer is a binomial.

Remember this pattern…it will be important when we factor "A Difference of Squares" later in this booklet.

✓

142. Write an expression (polynomial) for the following diagram (do not simplify):



$(2x+2)(2x-2)$

☆ $4x^2+4x-4x-4$ ☆

What two binomials are being multiplied above?

$(2x+2)(2x-2)$

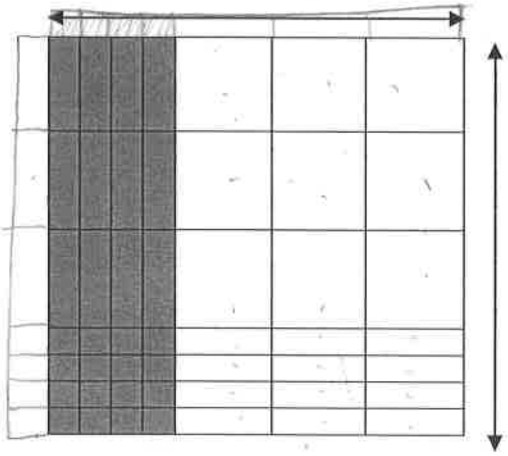
Write an equation using the binomials above and the simplified product.

$(2x+2)(2x-2)$

$4x^2-4x+4x-4$

$\rightarrow \boxed{4x^2-4}$

143. Write an expression for the following diagram (do not simplify):



$(3x+4)(3x-4)$

☆ $9x^2+12x-12x-16$ ☆

What two binomials are being multiplied above?

$(3x+4)(3x-4)$

Write an equation using the binomials above and the simplified product.

$(3x+4)(3x-4)$

$9x^2-12x+12x-16$

$\rightarrow \boxed{9x^2-16}$

Simplify the following.

144. $(x+3)(x-3)$

$x^2-3x+3x-9$

$\rightarrow \boxed{x^2-9}$

145. $(2x+3)(2x-3)$

$\boxed{4x^2-9}$

146. $(3x-1)(3x+1)$

$9x^2+3x-3x-1$

$\rightarrow \boxed{9x^2-1}$

147. $(x+\sqrt{2y})(x-\sqrt{2y})$

$x^2-x\sqrt{2y}+x\sqrt{2y}-(\sqrt{2y})^2$

$\boxed{x^2-2y}$

$(\sqrt{x})^2$

✓

Simplify the following.

148. $3(b-7)(b+7)$

$(3b-21)(b+7)$

$3b^2+21b-21b-147$

$\rightarrow \boxed{3b^2-147}$

149. $-2(c-5)(c+5)$

$(-2c+10)(c+5)$

$-2c^2-10c+10c+50$

$\rightarrow \boxed{-2c^2+50}$

150. $(x+6)(x+4)+(x+2)(x+3)$

$(x^2+4x+6x+24)+(x^2+3x+2x+6)$

$(x^2+10x+24)+(x^2+5x+6)$

$x^2+10x+24+x^2+5x+6$

$\rightarrow \boxed{2x^2+15x+30}$

151. $3(x-4)(x+3)-2(4x+1)$

$(3x-12)(x+3)-(8x+2)$

$3x^2+9x-12x-36-8x-2$

$\rightarrow \boxed{3x^2-11x-38}$

152. $5(3t-4)(2t-1)-(6t-5)$

$(15t-20)(2t-1)-(6t-5)$

$30t^2-15t-40t+20-6t+5$

$\rightarrow \boxed{30t^2-61t+25}$

153. $10-2(2y+1)(2y+1)-(2y+3)(2y+3)$

$10+(-4y-2)$

$10-((4y+2)(2y+1))-((2y+3)(2y+3))$

$10-(8y^2+4y+4y+2)-(4y^2+6y+6y+9)$

$10-8y^2-4y-4y-2-4y^2-6y-6y-9$

$\boxed{-12y^2-20y-1}$ ✓

Some key points to master about the Distributive Property...

FOIL

$(a+b)(a-b)$

$a^2-ab+ab-b^2$

$\boxed{a^2-b^2}$

$(a+b)^2$

$(a+b)(a+b)$

$a^2+ab+ab+b^2$

$a^2+b^2+2(ab)$

$\boxed{a^2+2ab+b^2}$

$(a+b)^3$

$(a+b)(a+b)(a+b)$

$(a^2+ab+ab+b^2)(a+b)$

$a^3+a^2b+a^2b+ab^2$
$+a^2b+ab^2+ab^2+b^3$

$a^3+b^3+3(ab^2)$ ✓
$+3(a^2b)$

$\boxed{a^3+3a^2b+3ab^2+b^3}$

Factoring:

Don't use × → use ·

When a number is written as a product of two other numbers, we say it is factored.

"Factor Fully" means to write as a product of **prime factors**.

Eg.1.
Write 15 as a product of its prime factors.

$$15 = 5 \times 3$$

5 and 3 are the prime factors.

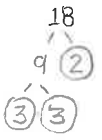
Eg.2.
Write 48 as a product of its prime factors.

$$48 = 8 \times 6$$
$$48 = 2 \times 2 \times 2 \times 3 \times 2$$
$$48 = 2^4 \times 3$$

Eg.3.
Write 120 as a product of its prime factors.

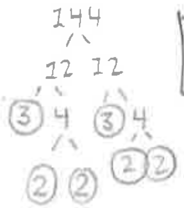
$$120 = 10 \times 12$$
$$120 = 2 \times 5 \times 2 \times 2 \times 3$$
$$120 = 2^3 \times 3 \times 5$$

154. Write 18 as a product of its prime factors.



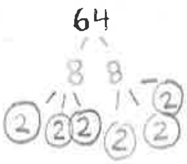
$\boxed{18 = 2 \cdot 3^2}$

155. Write 144 as a product of its prime factors.



$\boxed{144 = 2^4 \cdot 3^2}$

156. Write 64 as a product of its prime factors.



$\boxed{64 = 2^6}$

157. Find the greatest common factor (GCF) of 48 and 120.

Look at each factored form.

$$48 = 2^4 \times 3$$
$$120 = 2^3 \times 3 \times 5$$

Both contain $2 \times 2 \times 2 \times 3$, therefore this is the GCF,

GCF is 24.

158. Find the greatest common factor (GCF) of 144 and 64.

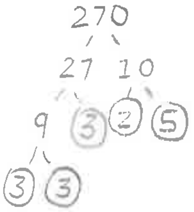
$144 = 2^4 \times 3^2$

$64 = 2^6$

$GCF = 2^4$

→ $\boxed{GCF = 16}$

159. Find the greatest common factor (GCF) of 36 and 270.



$36 = 2^2 \times 3^2$

$270 = 2 \times 3^3 \times 5$

$GCF = 2 \times 3^2$

→ $\boxed{GCF = 18}$

We can also write algebraic expressions in factored from.

Eg.4. Write $36x^2y^3$ as a product of its factors.

$$36x^2y^3 = 9 \times 4 \times x \times x \times y \times y \times y$$
$$36x^2y^3 = 3^2 \times 2^2 \times x^2 \times y^3$$

✓

160. Write $10a^2b$ as a product of its factors.

$10a^2b = 2 \times 5 \times a \times a \times b$

$\boxed{10a^2b = 2 \times 5 \times a^2 \times b}$

✓

161. Write $18ab^2c^3$ as a product of its factors.

$18ab^2c^3 = 2 \times 3 \times 3 \times a \times b \times b \times c \times c \times c$

$18 = 2 \times 3^2$

② 9

③③

$\boxed{18ab^2c^3 = 2 \times 3^2 \times a \times b^2 \times c^3}$

✓

162. Write $12b^3c^2$ as a product of its factors.

$12b^3c^2 = 2 \times 2 \times 3 \times b \times b \times b \times c \times c$

$12 = 2^2 \times 3$

③ 4

②②

$\boxed{12b^3c^2 = 2^2 \times 3 \times b^3 \times c^2}$

163. Find the greatest common factor (GCF) of $10a^2b$ and $18ab^2c^3$.

$10a^2b = 2 \times 5 \times a^2 \times b$

$18ab^2c^3 = 2 \times 3^2 \times a \times b^2 \times c^3$

$GCF = 2 \times a \times b$

$\boxed{GCF = 2ab}$

164. Find the greatest common factor (GCF) of $12b^3c^2$ and $18ab^2c^3$.

$12b^3c^2 = 2^2 \times 3 \times b^3 \times c^2$

$18ab^2c^3 = 2 \times 3^2 \times a \times b^2 \times c^3$

$GCF = 2 \times 3 \times b^2 \times c^2$

$\boxed{GCF = 6b^2c^2}$

165. Find the greatest common factor (GCF) of $10a^2b$, $18ab^2c^3$, and $12b^3c^2$.

$10a^2b = 2 \times 5 \times a^2 \times b$

$18ab^2c^3 = 2 \times 3^2 \times a \times b^2 \times c^3$

$12b^3c^2 = 2^2 \times 3 \times b^3 \times c^2$

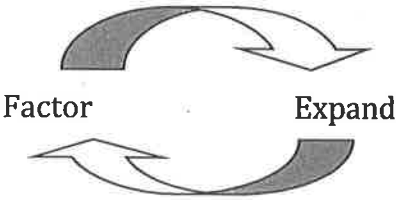
$GCF = 2 \times \cancel{a} \times b \rightarrow 2 \times b$

~~GCF = 2ab~~

$\boxed{GCF = 2b}$

✓

Factoring Polynomials:



The process of factoring "undoes" the process of expanding, and vice versa.

They are opposites.

You must be able to interchange a polynomial between these two forms.

Factoring means "*write as a product of factors.*"

The method you use depends on the type of polynomial you are factoring.

Challenge Question:

Write a multiplication that would be equal to $5x + 10$.

$\boxed{5(x+2)}$

Challenge Question:

Write a multiplication that would be equal to $3x^3 + 6x^2 - 12x$.

$\boxed{3(x^3 + 2x^2 - 4x)}$

$\boxed{☆\ 3x(x^2 + 2x - 4)\ ☆}$

The answers to the above questions are called the "FACTORED FORM".

✓

Factoring: Look for a Greatest Common Factor

Hint: Always look for a GCF first.

Ask yourself: "Do all terms have a common integral or variable factor?"

Eg.1. Factor the expression.

$5x + 10$

Think...what factor do 5x and 10 have in common?

Both are divisible by 5...that is the GCF.

$= 5(x) + 5(2)$ Write each term as a product using the GCF.

$= 5(x + 2)$ Write the GCF outside the brackets, remaining factors inside.

You should check your answer by expanding. This will get you back to the original polynomial.

Eg.2.

Factor the expression

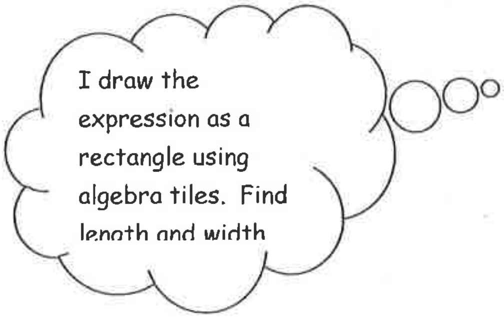
$3ax^3+6ax^2-12ax$

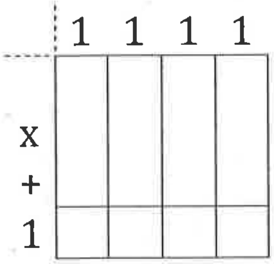
GCF = 3ax

$=3ax(x^2)+3ax(2x)+3ax(-4)$

$=3ax(x^2+2x-4)$

Eg.3. Factor the expression $4x + 4$ using algebra tiles.





$4(x + 1) = 4x + 4$

✓

Factor the following polynomials.

166. $5x + 25$

$\boxed{5(x+5)}$

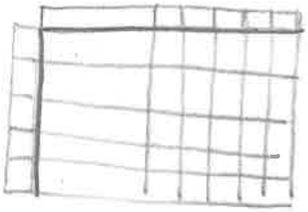
167. $4x + 13$

★not factorable★

168. $8x + 8$

$\boxed{8(x+1)}$

169. Model the expression above using algebra tiles.



170. Model the expression above using algebra tiles.

★not factorable★

171. Model the expression above using algebra tiles.

172. $4ax + 8ay - 6az$

$2(2ax+4ay-3az)$

$\boxed{2a(2x+4y-3z)}$

173. $24w^5 - 6w^3$

$6(4w^5 - w^3)$

$6w^3(4w^2 - 1)$

174. $3w^3xy + 12wxy^2 - wxy$

$wxy(3w^2+12y-1)$

175. $27a^2b^3 + 9a^2b^2 - 18a^3b^2$

$9a^2b^2(3b+1-2a)$

176. $6m^3n^2 + 18m^2n^3 - 12mn^2 + 24mn^3$

$6mn^2(m^2+3mn-2+4n)$

✓

Factoring a Binomial Common Factor: Hint: There are brackets with identical terms.

The common factor **IS** the term in the brackets!

Eg.1. Factor. $4x(w+1)+5y(w+1)$

$$\begin{aligned}&4x(w+1)+5y(w+1)\\&=(w+1)(4x)+(w+1)(5y)\\&=(w+1)(4x+5y)\end{aligned}$$

Eg.2. Factor. $3x(a+7)-(a+7)$

$$\begin{aligned}&3x(a+7)-(a+7)\\&=(a+7)(3x)-(a+7)(1)\\&=(a+7)(3x-1)\end{aligned}$$

Sometimes it is easier to understand if we substitute a letter, such as d where the common binomial is.

Consider Eg.1.
$4x(w+1)+5y(w+1)$ Substitute d for $(w+1)$.
$4xd+5yd$
$d(4x+5y)$ Now replace $(w+1)$.
$=(w+1)(4x+5y)$

Factor the following, if possible.

177. $5x(a+b)+3(a+b)$

$5xd+3d$
$d(5x+3)$
$=\boxed{(a+b)(5x+3)}$

178. $3m(x-1)+5(x-1)$

$3md+5d$
$d(3m+5)$
$=\boxed{(x-1)(3m+5)}$

179. $3t(x-y)+(x+y)$

not factorable

180. $4t(m+7)+(m+7)$

$4td+1d$
$d(4t+1)$
$=\boxed{(m+7)(4t+1)}$

181. $3t(x-y)+(y-x)$

$3t(x-y)+(-x+y)$
$3t(x-y)-(x-y)$
$=\boxed{(3t-1)(x-y)}$

182. $4y(p+q)-x(p+q)$

$4yd-xd$
$x(4y-x)$
$=\boxed{(4y-x)(p+q)}$

Challenge Question:

Factor the expression $ac+bd+ad+bc$.

$ac+ad+bd+bc$
$a(c+d)+b(d+c)$
$a(c+d)+b(c+d)$
$=\boxed{(a+b)(c+d)}$

✓

Factoring: *Factor by Grouping*. Hint: 4 terms!

Sometimes a polynomial with 4 terms but no common factor can be arranged so that grouping the terms into two pairs allows you to factor.

You will use the concept covered above...common binomial factor.

Eg.1. Factor $ac + bd + ad + bc$

$ac + bc \quad + ad + bd$ Group terms that have a common factor.

$c(a + b) + d(a + b)$ Notice the newly created binomial factor, $(a + b)$.

$= \boldsymbol{(a + b)(c + d)}$ Factor out the binomial factor.

Eg.2. Factor $5m^2t - 10m^2 + t^2 - 2t$

$5m^2t - 10m^2 \quad - t^2 + 2t$ Group.

$5m^2(t - 2) - t(t - 2)$ *Notice that I factored out a $-t$ in the second group. This made the binomials into common factors, $(t - 2)$.

$= \boldsymbol{(t - 2)(5m^2 - t)}$

183. $wx + wy + xz + yz$

$wx + wy \quad + xz + yz$

$w(x+y) + z(x+y)$

$= \boxed{(w+z)(x+y)}$

184. $[x^2 + x][-xy - y]$

$x(x+1) - y(x+1)$

$= \boxed{(x-y)(x+1)}$

185. $xy + 12 + 4x + 3y$

$xy + 3y \quad + 12 + 4x$

$y(x+3) + 4(3+x)$

$y(x+3) + 4(x+3)$

$= \boxed{(y+4)(x+3)}$

$*(x+3)(y+4)*$

186. $2x^2 + 6y + 4x + 3xy$

$2x^2 + 4x \quad + 6y + 3xy$

$2x(x+2) + 3y(2+x)$

$2x(x+2) + 3y(x+2)$

$= \boxed{(2x+3y)(x+2)}$

187. $m^2 - 4n + 4m - mn$

$m^2 + 4m \quad - 4n - mn$

$m(m+4) - n(4+m)$

$m(m+4) - n(m+4)$

$= \boxed{(m-n)(m+4)}$

188. $3a^2 + 6b^2 - 9a - 2ab^2$

$3a^2 - 9a \quad + 6b^2 - 2ab^2$

$3a(a-3) + 2b^2(3-a)$

$3a(a-3) + 2b^2(-a+3)$

$3a(a-3) - 2b^2(a-3)$

$= \boxed{(3a-2b^2)(a-3)}$

✓

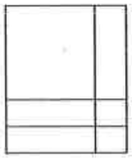
Factoring: $ax^2 + bx + c$ (where a=1) with tiles.

Hint: 3 terms, no common factor, leading coefficient is 1.

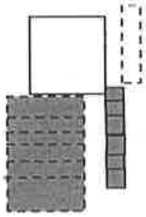
Eg.1. Consider $x^2 + 3x + 2$. The trinomial can be represented by the rectangle below.

Recall, the side lengths will give us the "factors".

$\therefore\ x^2 + 3x + 2 = (x + 1)(x + 2)$



Eg.2. Factor $x^2 - 5x - 6$

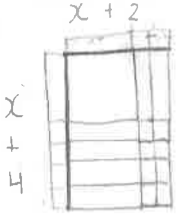


Start by placing the "x^2 tile" and the six "-1 tiles" at the corner. Then you can fill in the "x tiles". You'll need one x tile and six –x tiles.

$\therefore\ x^2 - 5x - 6 = (x + 1)(x - 6)$

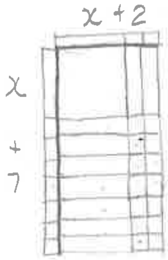
Factor the following trinomials using algebra tiles.

189. $x^2 + 6x + 8$



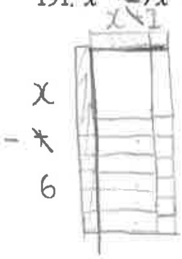
(x+4)(x+2)

190. $x^2 + 9x + 14$



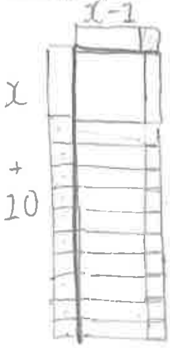
(x+7) (x+2)

191. $x^2 - 7x + 6$



(x-6)(x-1)

192. $x^2 + 9x - 10$



(x+10) (x-1)

✓

Factoring: $ax^2 + bx + c$ (where a=1) without tiles.

Did you see the pattern with the tiles?

If a trinomial in the form $x^2 + bx + c$ can be factored, it will end up as $(x + \_\_)(x + \_\_)$.

The trick is to find the numbers that fill the spaces in the brackets.

The Method...

If the trinomial is in the form: $x^2 + bx + c$, look for two numbers that multiply to c, and add to b.

Eg.1.

Factor. $x^2 + 6x + 8$ $(x^2+4x)+(2x+8)$

$(x + \_\_)(x + \_\_)$ What two numbers multiply to +8 but add to +6? 2 and 4

$= (x + 2)(x + 4)$ The numbers 2 and 4 fill the spaces inside the brackets.

Eg.2. Factor. $x^2 - 11x + 18$

$(x + \_\_)(x + \_\_)$ What two numbers multiply to +18 but add to -11? -2 and -9

$= (x - 2)(x - 9)$ The numbers -2 and -9 fill the spaces inside the brackets.

Eg.3. Factor. $x^2 - 7xy - 60y^2$ The y's can be ignored temporarily to find the two numbers. Just write them in at the end of each bracket.

$(x + \_\_y)(x + \_\_y)$ What two numbers multiply to -60 but add to -7? -12 and +5

$= (x - 12y)(x + 5y)$ The numbers -12 and +5 fill the spaces in front of the y's.

Factor the trinomials if possible.

193. $a^2 + 6a + 5$	194. $n^2 + 7n + 10$	195. $x^2 - x - 30$
$(a+1)(a+5)$	$(n+2)(n+5)$	$(x+5)(x-6)$

✓

Factor the trinomials if possible.

196. $q^2 + 2q - 15$

$(q+5)(q-3)$

197. $k^2 + k - 56$

$(K-7)(K+8)$

198. $t^2 + 11t + 24$

$(t+3)(t+8)$

199. $y^2 - 7y - 30$

$(y-10)(y+3)$

200. $g^2 - 11g + 10$

$(g-10)(g-1)$

201. $s^2 - 2s - 80$

$(s-10)(s+8)$

202. $m^2 - 12m + 27$

$(m-9)(m-3)$

203. $x^2 - 27 - 6x$

$x^2-6x-27$

$(x-9)(x+3)$

204. $p^2 + 3p - 54$

$(p-6)(p+9)$

205. $2a^2 - 16a + 32$

$2(a^2-8a+16)$

$2(a-4)(a-4)$

$2(a-4)^2$

206. $a^2 - 14a + 45$

$(a-9)(a-5)$

207. $6x + 2x^2 - 20$

$2x^2+6x-20$

$2(x^2+3x-10)$

$2(x+5)(x-2)$

✓

☆ Keep in order? ☆

Factor the trinomials if possible.

208. $x^4 - 3x^2 - 10$

$(x^2 - 5\cancel{x})(x^2 + 2\cancel{x})$

209. $w^6 + 7w^3 + 12$

$(w^3 + 3)(w^3 + 4)$

210. $p^8 - 4p^4 - 21$

$(p^4 - 7\cancel{p^2})(p^4 + 3\cancel{p^2})$

211. $56x + x^2 - x^3$

$-x^3 + x^2 + 56x$

$-x(x^2 - x + 56)$

$-x(x+7)(x+8)$

☆ order?)

212. $x^4 + 11x^4 - 80$

$(x^2 + 16)(x^2 - 5)$

213. $x^2 - 3x + 7$

* not factorable *

214. $x^2 - 6xy + 5y^2$

$x^2 + 5y^2 - 6xy$

$(x - \cancel{1}y)(x - 5y)$

☆ $(x - y)(x - 5y)$ ☆

215. $x^2 + 5xy - 36y^2$

$(\underset{y}{x + 9})(x - \underset{-y}{4}) - y$

☆ $(x + 9y)(x - 4y)$ ☆

216. $a^2b^2 - 5ab + 6$

ab -2

ab -3

$-3ab + 2ab = -5ab$

$(ab - 2)(ab - 3)$

Challenge Question

Factor $2x^2 + 7x + 6$.

$2x^2 + 4x + 3x + 6$

$2x(x+2) + 3(x+2)$

$\boxed{(2x+3)(x+2)}$

✓

Factoring $ax^2 + bx + c$ where $a \neq 1$

When the trinomial has an x^2 term with a coefficient other than 1 on the x^2term, you cannot use the same method as you did when the coefficient is 1.

We will discuss 3 other methods:
1. Trial & Error 2. Decomposition 3. Algebra Tiles

Trial & Error:

Eg.1. Factor $2x^2 + 5x + 3$.

$2x^2 + 5x + 3 = (\quad)(\quad)$ We know the first terms in the brackets have product of $2x^2$

$2x^2 + 5x + 3 = (2x \quad)(x \quad)$ $2x$ and x have a product of $2x^2$, place them at front of brackets.

The product of the second terms is 3. (1, 3 or -1, -3).
These will fill in the second part of the binomials.

List the possible combinations of factors.

$(2x + 1)(x + 3)$
$(2x + 3)(x + 1)$
$(2x - 1)(x - 3)$
$(2x - 3)(x - 1)$

IF $2x^2 + 5x + 3$ is factorable, one of these must be the solution.

Expand each until you find the right one.

$(2x + 3)(x + 1) = 2x^2 + 2x + 3x + 3 = 2x^2 + 5x + 3.$ This is the factored form.

Decomposition:
Using this method, you will break apart the middle term in the trinomial, then factor by grouping.

To factor $\boldsymbol{ax^2 + bx + c}$, look for two numbers with a product of $\boldsymbol{ac}$ and a sum of $\boldsymbol{b}$.

Eg.1. Factor. $3x^2 - 10x + 8$

1. We see that $ac = 3 \times 8 = 24; and\ b = -10$
   We need two numbers with a product of 24, but add to -10...
   -6 and -4.

$3x^2 - 6x - 4x + 8$ 2. Break apart the middle term.
$3x(x - 2) - 4(x - 2)$ 3. Factor by grouping.

$= (x - 2)(3x - 4)$

Eg.2. Factor. $3a^2 - 22a + 7$

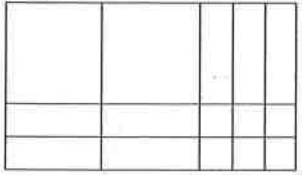
We need numbers that multiply to 21, but add to -22...
-21 and **-1**

$3a^2 - \mathbf{21}a - \mathbf{1}a + 7$ Decompose middle term.

$3a(a - 7) - 1(a - 7)$ Factor by grouping.

$= \mathbf{(a - 7)(3a - 1)}$

Eg.3. Factor $2x^2 + 7x + 6$ using algebra tiles.



Arrange the tiles into a rectangle (notice the "ones" are again grouped together at the corner of the x² tiles)

Side lengths are $(2x + 3)$ and $(x + 2)$ $\therefore 2x^2 + 7x + 6 = (2x + 3)(x + 2)$

Your notes here...

Factor the following if possible.

217. $2a^2 + 11a + 12$

$a^2+11a+24$

$(\underbrace{a+8}_{2})(\underbrace{a+3}_{2})$

$\boxed{(a+4)(2a+3)}$

218. $5a^2 - 7a + 2$

$a^2-7a+10$

X

★not factorable★

$a^2-7a+10$

$(\underbrace{a-5}_{5})(\underbrace{a-2}_{5})$

$\boxed{(a-1)(5a-2)}$

219. $3x^2 - 11x + 6$

$x^2-11x+18$

$(\underbrace{x-9}_{3})(\underbrace{x-2}_{3})$

$\boxed{(x-3)(3x-2)}$

✓

Factor the following if possible.

220. $2y^2 + 9y + 9$

$y^2+9y+18$

$\left(\frac{y+3}{2}\right)\left(\frac{y+6}{2}\right)$

$\boxed{(2y+3)(y+3)}$

221. $5y^2 - 14y - 3$

$y^2-14y-15$

$\left(\frac{y-15}{5}\right)\left(\frac{y+1}{5}\right)$

$\boxed{(y-3)(5y+1)}$

222. $10x^2 - 17x + 3$

$x^2-17x+30$

$\left(\frac{x-\overset{3}{\cancel{15}}}{\cancel{10}\;2}\right)\left(\frac{x-\overset{1}{\cancel{2}}}{\cancel{10}\;5}\right)$

$\boxed{(2x-3)(5x-1)}$

223. $2x^2 + 3x + 1$

x^2+3x+2

$\left(\frac{x+2}{2}\right)\left(\frac{x+1}{2}\right)$

$\boxed{(x+1)(2x+1)}$

224. $6k^2 - 5k - 4$

$K^2-5K-24$

$\left(\frac{K-\overset{4}{\cancel{8}}}{\cancel{6}\;3}\right)\left(\frac{K+\overset{1}{\cancel{3}}}{\cancel{6}\;2}\right)$

$\boxed{(3K-4)(2K+1)}$

225. $6y^2 + 11y + 3$

$y^2+11y+18$

$\left(\frac{y+\overset{3}{\cancel{9}}}{\cancel{6}\;2}\right)\left(\frac{y+\overset{1}{\cancel{2}}}{\cancel{6}\;3}\right)$

$\boxed{(2y+3)(3y+1)}$

226. $3x^2 - 16x - 12$

$x^2-16x-36$

$\left(\frac{x+2}{3}\right)\left(\frac{x-\overset{6}{\cancel{18}}}{\cancel{3}\;1}\right)$

$\boxed{(3x+2)(x-6)}$

227. $3x^3 - 5x^2 - 2x$

x^3-5x^2-6x

$x\left(\frac{x-\overset{2}{\cancel{6}}}{\cancel{3}\;1}\right)\left(\frac{x+1}{3}\right)$

$\boxed{x(x-2)(3x+1)}$

228. $9x^2 + 15x + 4$

$x^2+15x+36$

$\left(\frac{x+\overset{4}{\cancel{12}}}{\cancel{9}\;3}\right)\left(\frac{x+\overset{1}{\cancel{3}}}{\cancel{9}\;3}\right)$

$\boxed{(3x+4)(3x+1)}$

✓

Factor the following if possible.

229. $21x^2 + 37x + 12$

28×9

$x^2 + 37x + 252$

$(x + \overset{4}{\cancel{28}})(x + \overset{3}{\cancel{9}})$

$\cancel{21}\ 3 \quad \cancel{21}\ 7$

$\boxed{(3x+4)(7x+3)}$

230. $6x^3 - 15x - x^2$

$x^3 - x^2 - 15x$

$x(x^2 - x - 90)$

$x(x + \overset{5}{\cancel{10}})(x + \overset{3}{\cancel{9}})$

$\cancel{6}\ 3 \quad \cancel{6}\ 2$

$\boxed{x(3x+5)(2x+3)}$

231. $4t + 10t^2 - 6$

$\frac{10t^2 + 4t - 6}{2}$

$2(5t^2 + 2t - 3)$

5 -3

1 1

$t+5-3=2$

$2(5t-3)(t+1)$

232. $3x^2 - 22xy + 7y^2$

~~$3x^2 + 7y^2 - 22xy$~~

~~$x^2 + 7y^2 - 66xy$~~

don't change order!

$x^2 - 22xy + 21y^2$

$(x - 21y)(x - 1y)$

3 3

$\boxed{(x-7y)(3x-1y)}$

233. $4c^2 - 4cd + d^2$

$c^2 - 4cd + 4d^2$

$(c - \overset{1}{\cancel{2}}d)(c - \overset{2}{\cancel{2}}d)$

$\cancel{4}\ 2 \quad \cancel{4}\ 2$

$\boxed{(2c-d)(2c-d)}$

or

$\boxed{(2c-d)^2}$

234. $2x^4 + 7x^2 + 6$

$x^4 + 7x^2 + 12$

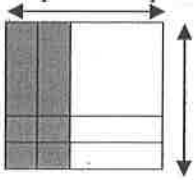
$(x^2+3)(x^2+\overset{2}{\cancel{4}})$

2 $\cancel{4}\ 2$

$\boxed{(2x^2+3)(x^2+2)}$

Challenge Question

Write a simplified expression for the following diagram of algebra tiles.



$(x-2)(x+2)$

(x^2-4)

(x^2-4)

$(a+b)(a-b)$

(x^2-9)

$(x-3)(x+3)$

What two binomials are being multiplied in the diagram above?

$(x-2)(x+2)$

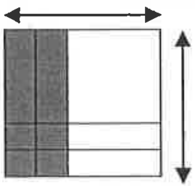
Write an equation using the binomials above and the simplified product.

$(x-2)(x+2) = x^2$

✓

A Difference of Squares

235. Write a simplified expression for the following diagram.



Solution: $x^2 - 2x + 2x - 4$

What two binomials are being multiplied in the diagram above?

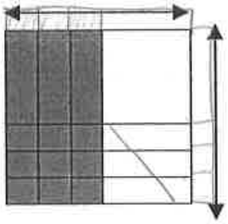
$(x-2)(x+2)$

Write an equation using the binomials above and the simplified product.

$x^2 - 4 = (x-2)(x+2)$

↗ Factored Form

236. Write a simplified expression for the following diagram.



$\boxed{x^2 - 3x + 3x + 9}$

☆ $\boxed{x^2 - 9}$ ☆

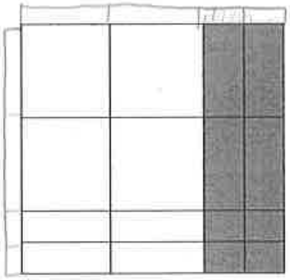
What two binomials are being multiplied above?

$\boxed{(x+3)(x-3)}$

Write an equation using the binomials above and the simplified product.

$\boxed{x^2 - 9 = (x+3)(x-3)}$

237. Write a simplified expression for the following diagram.



$4x^2 - 4x + 4x - 4$

☆ $\boxed{4x^2 - 4}$ ☆

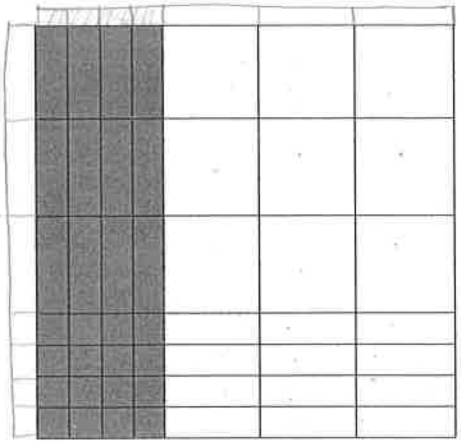
What two binomials are being multiplied above?

$\boxed{(2x+2)(2x-2)}$

☆ Factor the polynomial represented above by writing the binomials as a product (multiplication).

$\boxed{4x^2 - 4 = (2x+2)(2x-2)}$

238. Write a simplified expression for the following diagram.



$9x^2 - 12x + 12x - 16$

☆ $\boxed{9x^2 - 16}$ ☆

What two binomials are being multiplied above?

$(3x+4)(3x-4)$

Factor the polynomial represented above by writing the binomials as a product (multiplication).

$\boxed{9x^2 - 16 = (3x+4)(3x-4)}$

✓