Quadratic Functions and Parabolic Motion

This booklet belongs to:____

LESSON #	DATE	QUESTIONS FROM NOTES	Questions that I find difficult
1.	Dec 9	Pg. 1-9	
2.		Pg.	
3.		Pg.	
4.		Pg.	
5.		Pg.	
6.		Pg.	
7.		Pg.	
8.		Pg.	
9.		Pg.	
10.		Pg.	
11.		Pg.	
12.		Pg.	
13.		REVIEW	
14.		TEST	

Your teacher has important instructions for you to write down below.

The answers to challenges are usually on the next page.

Definitions

		Examples
Axis of symmetry	A vertical line that bisects a parabola.	Given $y=-3(x-4)^2+5$ the axis of symmetry is x=4.
Completing the square	This technique is used to turn a quadratic function in standard form, $y=ax^2+bx+c$, into vertex from $y=a(x-p)^2+q$	
Find the maximum value.	When a quadratic is in the vertex form, $y=a(x-p)^2+q$, the maximum value is the q value.	Given $y=-3(x-4)^2+5$ the maximum value is 5.
Find the value that creates a maximum value.	When a quadratic is in the vertex form, $y=a(x-p)^2+q$, the value that creates the maximum value is the p value.	Given $y=-3(x-4)^2+5$ there is a maximum when x=4.
How high was the object when the object was released?	This is word problem question. It is asking for the y-intercept. This can be found by setting x=0 and solving for y.	
How long until an object hits the ground.	This is word problem question. It is asking for the x-intercept. This can be found by setting y=0 and solving for x.	
Maximum value	A parabola has a maximum value when it opens down. The q value from the vertex (p,q) is the maximum value.	
Minimum value	A parabola has a minimum value when it opens up. The q value from the vertex (p,q) is the minimum value.	
Parabola	The symmetrical graph of a quadratic function.	
Quadratic Function	A polynomial function of degree 2.	
Standard form	$y=ax^2+bx+c$	
Vertex form	$y=a(x-p)^2+q$	
Vertex of a parabola	The highest or lowest point.	Given $y=a(x-p)^2+q$, the vertex is (p,q).
Y-intercept	The coordinate where the curve crosses the y-axis. This can be found algebraically by setting x=0 and solving for y.	
X-intercept	The coordinate where the curve crosses the x-axis. This can be found algebraically by setting y=0 and solving for x.	

Quadratic Functions $y=a(x-p)^2+q$

The Blob Jump World Record:

What is a Blob Jump?

A blob is a large inflatable pillow that sits in the water. One person sits on one end and another person or three jump from a raised platform and land on the other end of the blob. The resulting air displacement within the blob sends the reclining person into the air.

What is the world record?

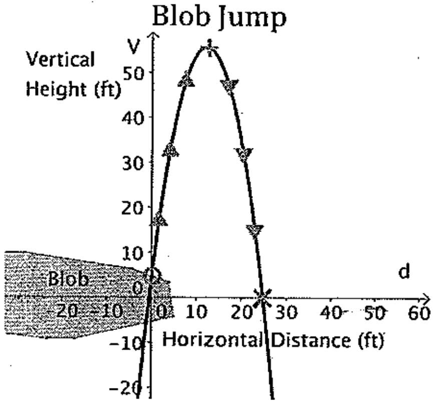
The picture to the right shows the world record height of 55feet. This record was achieved on July 21st 2011 in Cham Switzerland.

07/21/2011 World Record

See the video at→ http://www.todaysbigthing.com/2011/07/21

The above world record is an example of parabolic motion. This blob jump can be graphed in the form of a **parabola** and written in the form of a **quadratic function.**

Blob Jump



1. Which shape represents the y-intercept?
   0 ✓ + ×
2. Which shape represents the vertex?
   0 + ✓ ×
3. Which shape represents the x-intercept?
   0 + × ✓

Challenge #1: Use the graph to the left to help answer questions about the equations below. A parabola can be written in following quadratic forms"

Standard form: $V=-0.35d^2+8.5d+4$

4. At time zero how high was the person sitting on the blob?
   4 feet

Vertex form: $V=-0.35(d-12)^2+55$

5. What is the person's maximum height?
   55 feet
6. What was the horizontal distance when the person reached the maximum height?
   12 feet

Quadratic Functions: Terminology

Quadratic Function: A polynomial function of degree 2. i.e. $y=x^2$ or $y=-x^2=5x-7$.

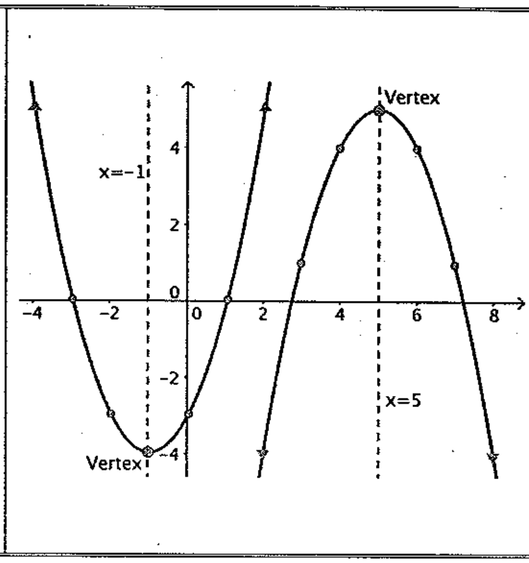
Parabola: The symmetrical graph of a quadratic function.

Vertex: Highest or lowest point of parabola.

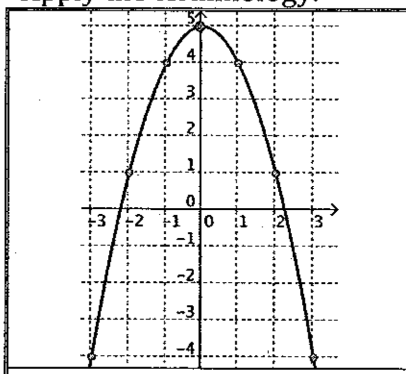
Maximum: Describes a vertex when it is biggest. A parabola that opens down has a maximum value.

Minimum: Describes a vertex when it is smallest. A parabola that opens up has a minimum value.

Axis of Symmetry: The vertical line that bisects a parabola



Apply the terminology.



7. Vertex:

(0,5)

8. Maximum or Minimum:

Maximum

9. Direction of opening:

Down

10. Axis of symmetry:

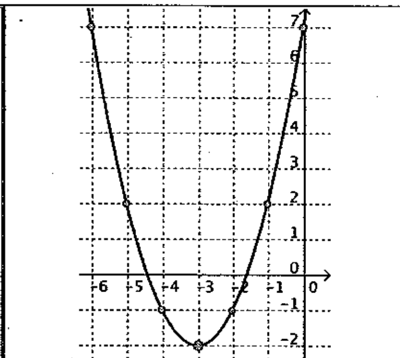
X=0

11. Domain:

All real numbers.

12. Range:

$y \le 5$



13. Vertex:

(−3,−2)

14. Maximum or Minimum:

Minimum

15. Direction of opening:

Up

16. Axis of symmetry:

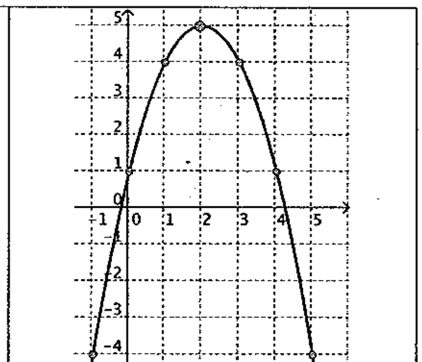
x=−3

17. Domain:

All real numbers

18. Range:

$y \geq -2$



19. Vertex:

(2,5)

20. Maximum or Minimum:

Maximum

21. Direction of opening:

Down

22. Axis of symmetry:

x=2

23. Domain:

All real numbers

24. Range:

$y \le 5$

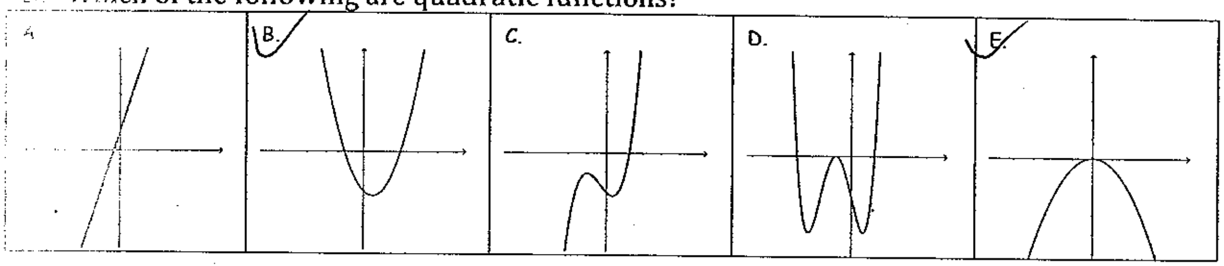
25. How is the vertex of a parabola related to its axis of symmetry?

The x-coordinate of the vertex is the axis of symmetry.

26. How is the range of a parabola related to its vertex?

The y-coordinate is either the minimum value or the maximum value.

27. Which of the following are quadratic functions?



28. Which of the following are quadratic functions?

A. $y = -x^2 + x$	B. $y = x(2x+3)$	C. $y = (x+3)^2$	D. $y = 0$	E. $y = 6(x-1)$
F. $y = (x-3)(x+3)$	G. $y = 3x^2 - 3$	H. $y = \sqrt{x} - x$	I. $y = 5x^3 + x^2 + 1$	J. $y = -x^2 + x^4$

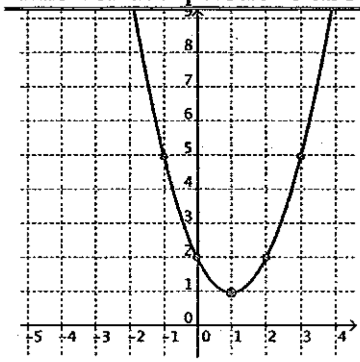
29. What are the characteristics of the graph of a quadratic function?

Symmetric about an axis of symmetry. Can open up and down.
It is approximately "U" shaped.

30. What are the characteristics of the equation of a quadratic function?

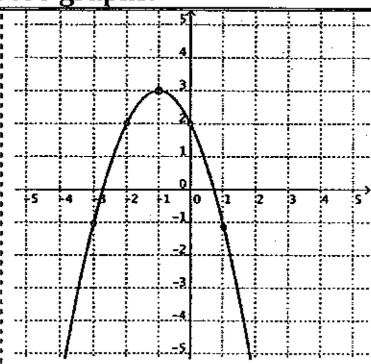
No term has a degree higher than 2

Answer the questions about these graphs.



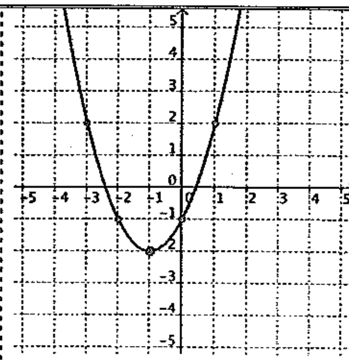
31. Vertex: (1,1)

32. Axis of Symmetry: $x=1$



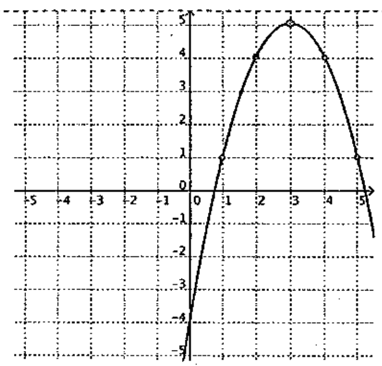
33. Max or Min: Maximum

34. Y-intercept: (0,2)



35. Domain: All real numbers

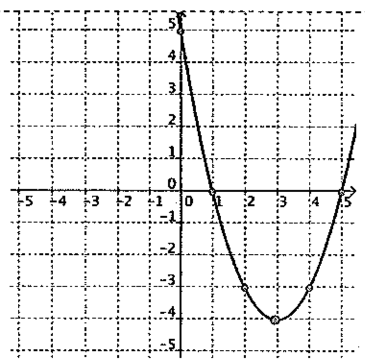
36. Range: $y \geq -2$



37. Vertex: (3,5)

38. Range: $y \leq 5$

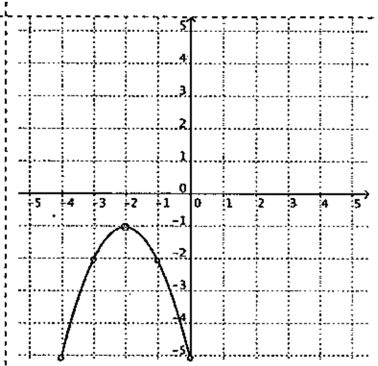
39. Direction of opening: Down



40. Vertex: (3,-4)

41. X-intercepts: (1,0) , (5,0)

42. Range: $y \geq -4$



43. Vertex: (-2,-1)

44. Axis of symmetry: $x=-2$

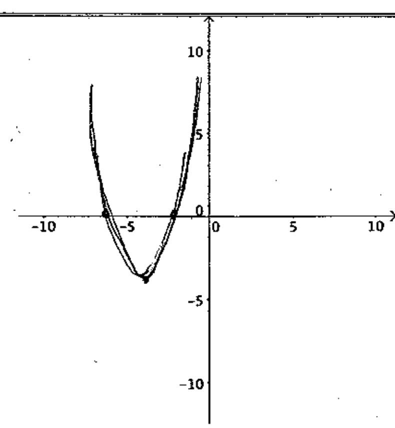
45. Domain: All real numbers

Challenge #:

46. Graph $y = x^2 + 8x + 12$, using the x and y-intercept to approximate the function.

x-intercept: -2, -6

y-intercept: 12



Determine the x and y intercepts of each quadratic function and sketch the graph.

47. $y = x^2 + 8x + 12$

X=0 at the y-intercept.
Substitute x=0

$y = 0^2 + 8(0) + 12$

$y = 12$

The y-intercept is 12 or (0,12)

y=0 at the x-intercept.
Substitute y=0

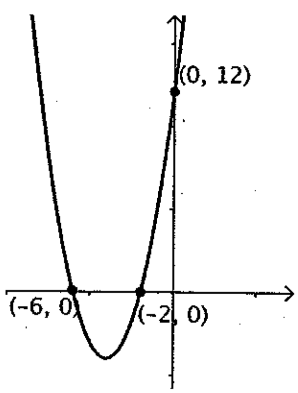
$0 = x^2 + 8x + 12$

Factor

$0 = (x+2)(x+6)$

y=0 when x=-2 or x=-6.

The x-intercepts are -2 and -6.



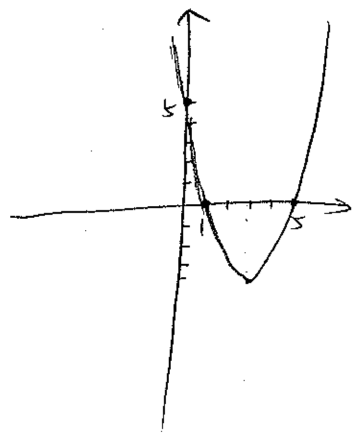
* The vertex was approximated.

48. $y = x^2 - 6x + 5$

$y = (x-1)(x-5)$

x-intercepts : 1, 5

y-intercept : 5

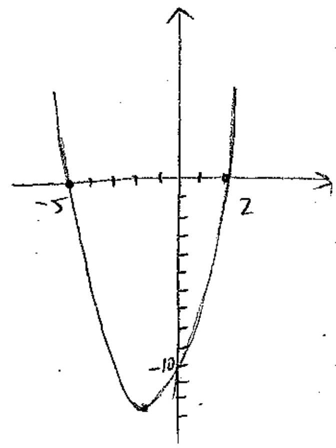


49. $y = x^2 + 3x - 10$

$y = (x-2)(x+5)$

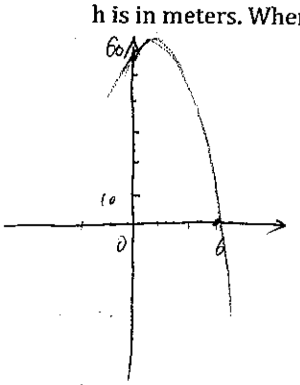
x-intercepts : 2, −5

y-intercept : −10



Challenge #:

50. A model rocket is launched at 19.6 meters per second (m/s) from a 58.8-meter tall platform. The equation for the object's height h at time t seconds after launch is $h = -4.9t^2 + 19.6t + 58.8$, where h is in meters. When does the object strike the ground?



$h = -4.9(t^2 - 4t - 12)$

$= -4.9(t-6)(t+2)$

$\therefore t = 6 ; t = -2$

$\because$ time can't be negative

$\therefore t = 6$ seconds.

Write down the steps to solve the challenge to the left.

Draw a picture

We are looking for the x-intercepts

Factor out the GCF.

Factor the trinomial

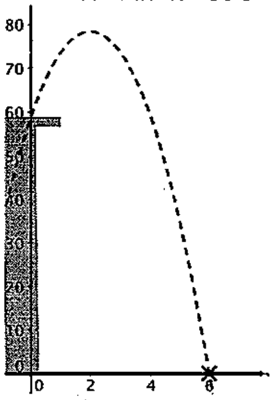
Interpret the x-intercepts.

State the solution.

51. A model rocket is launched at 19.6 meters per second (m/s) from a 58.8-meter tall platform. The equation for the object's height s at time t seconds after launch is $h=-4.9t^2+19.6t+58.8$, where h is in meters. When does the object strike the ground?

Solution:
Draw a picture.

$h=-4.9t^2+19.6t+58.8$



H=0 when the rocket hits the ground.

$0=-4.9t^2+19.6t+58.8$

Factor out the GCF

$0=-4.9(t^2-4t-12)$

Factor the trinomial

$0=-4.9(t-6)(t+2)$

Solve for t.
t=6 or t=-2
Reject t=-2. (negative time does not make sense.)
The solution is positive 6 seconds not -2.

52. A pebble is launched directly upward at 64 feet per second (ft/s) from a cliff that is 80 feet high. The equation for the pebble's height h at time t seconds after launch is $h=-16t^2+64t+80$. How long will it take the pebble to hit the ground?

When the pebble hits the ground, h=0

$0=-16t^2+64t+80$

$0=-16(t^2-4t-5)$

$0=-16(t-5)(t+1)$

$t=5,\ t=-1$

time can't be negative

$\therefore t=5$ seconds.

53. How high will the pebble be after 1 second, 2 seconds and 3 seconds? What do you think the maximum height is?

$t=1,\ h=-16\times1+64+80=128$ feet

$t=2,\ h=-16\times4+64\times2+80=144$ feet

$t=3,\ h=-16\times9+64\times3+80=128$ feet

The maximum height is 144 feet.

54. Jason wants to see how high his slingshot can send an object into the air. He attaches his digital altimeter to a potato and then shoots the pair into the air. The potato's height in relation to time follows the pathway $h=-7t^2+21t+5$, where h is height in feet and t is time in seconds. At what height was the potato at the exact moment it was released?

5 feet.

55. How high will the potato and the altimeter be after 1 second? After 2 seconds? How long do you think it will take for the potato to reach a maximum height?

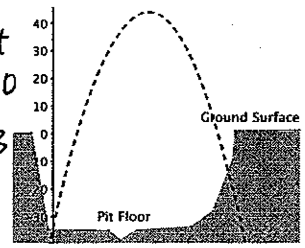
$t=1,\ h=-7\times1+21+5=19$ feet

$t=2,\ h=-7\times4+21\times2+5=19$ feet

After 1.5 seconds.

56. An object is launched from the bottom of pit 34.3 meters below the ground surface, directly upward at 39.2 m/s. The equation for the object's height h at time t seconds after launch is $h=-4.9t^2+39.2t-34.3$, where h is in meters. For how long is the object at or above the ground surface?

When the object is at the ground surface, h=0



$0=-4.9t^2+39.2t-34.3$

$=-4.9(t^2-8t+7)$

$=-4.9(t-1)(t-7)$

$t=1,\ t=7$

$7-1=6$ seconds

$\therefore$ the object is at or above the ground surface for 6 seconds.

57. True or False: The axis of symmetry always goes through the vertex. Explain.

True. The vertex is in the middle and the axis of symmetry always goes through the middle.

58. True or False: If a parabola has a maximum value then it opens up. Explain.

False A maximum is at the top of the parabola that opens down.

59. True or False: The domain for $y=x^2$ is always all real numbers. Explain.

True. There are no restrictions on the domain

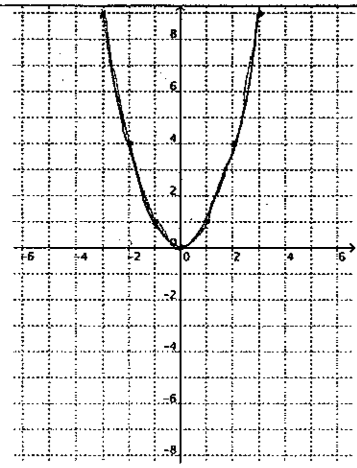
Quadratic Functions: The A-Value: $f(x)=x^2$ & $f(x)=ax^2$

- You will learn to graph quadratic functions in this section.

60. Graph $y=x^2$

62. Confirm the table of values.

$y=x^2$	
X	Y
3	9
2	4
1	1
0	0
-1	1
-2	4
-3	9



64. Vertex: (0,0)

65. Max or Min? Minimum

66. Axis of Symmetry: x=0

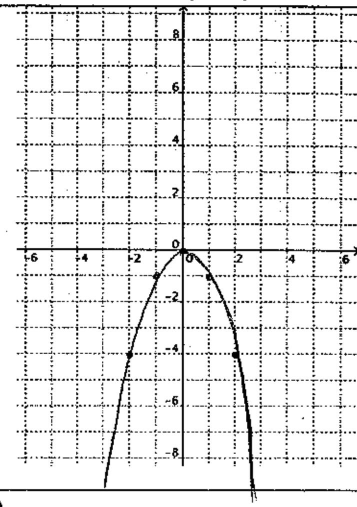
67. Domain: All real numbers

68. Range: $y \geq 0$

61. Graph $y=-x^2$, notice $(a<0)$

63. Fill out the table of values:

$y=-x^2$	
X	Y
3	-9
2	-4
1	-1
0	0
-1	-1
-2	-4
-3	-9



69. Vertex: (0,0)

70. Max or Min? Maximum

71. Axis of Symmetry: x=0

72. Domain: real

73. Range: $y \leq 0$

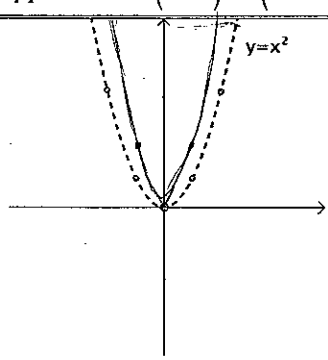
74. What happens to the graph of $y=x^2$ when it is transformed to $y=-x^2$, $(a<0)$?

Reflected over the x-axis.

Challenge #: What happens when $(a>1)$ or $(0<a<1)$?

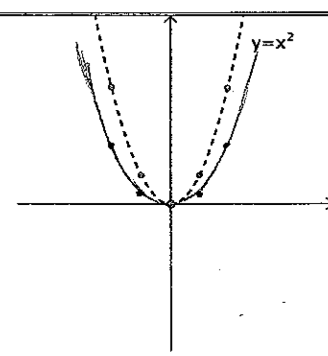
75. **Take a guess☺**

If the graph to the right is $y=x^2$, sketch what you think $y=2x^2$ might look like.



76. **Take a guess☺**

If the graph to the right is $y=x^2$, sketch what you think $y=\frac{1}{2}x^2$ might look like.



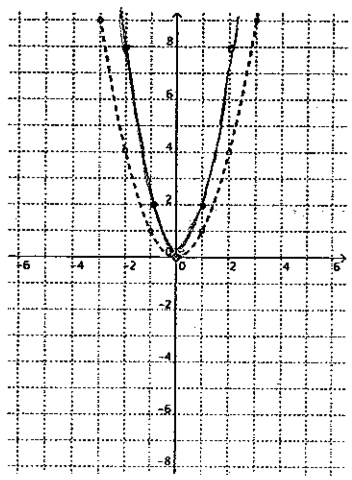
Use a table of values to determine what the following graphs look like.

77. Graph $y=2x^2$, notice $(a>1)$

78. Graph $y=\frac{1}{2}x^2$, notice $(0<a<1)$

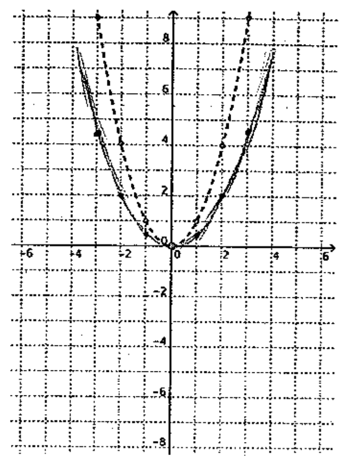
79. Complete the table of values.

$y=2x^2$	
X	Y
3	18
2	8
1	2
0	0
-1	2
-2	8
-3	18



80. Complete the table of values.

$y=\frac{1}{2}x^2$	
X	Y
3	4.5
2	2
1	0.5
0	0
-1	0.5
-2	2
-3	4.5



81. Vertex: (0, 0)

82. Max or Min? Minimum

83. Vertex: (0, 0)

84. Max or Min? Minimum

The A-value:

How do changes to the a-value change the graph of $y=ax^2$?*

85. If $a>1$ then the graph of $y=ax^2$:

A. Reflects over the x-axis,
B. Vertically expands, ✓
C. Vertically compresses

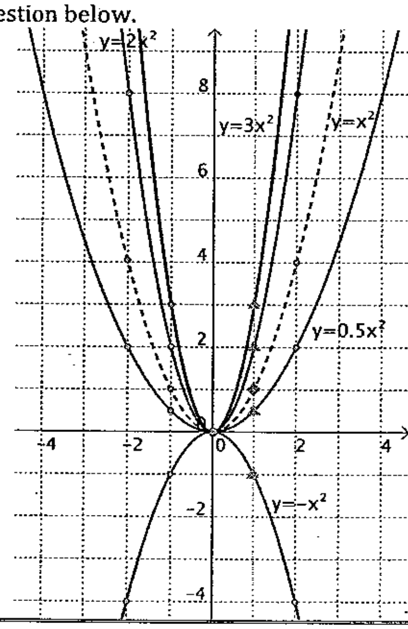
86. If $0<a<1$ then the graph of $y=ax^2$:

A. Reflects over the x-axis,
B. Vertically expands,
C. Vertically compresses ✓

87. If $a<0$ then the graph of $y=ax^2$:

A. Reflects over the x-axis, ✓
B. Vertically expands,
C. Vertically compresses

Use the graph to answer the questions to the left and the question below.



* If you are unsure, use the answer key right away to ensure your understanding.

What does the a-value do to all y-values of the original graph?

Fill out the table of values to help you answer the question above.

Original Graph

$y=x^2$	
x	**y**
1	**1**
2	**4**

88.

$y=2x^2$	
x	y
1	2
2	8

89.

$y=3x^2$	
x	y
1	3
2	12

90.

$y=\frac{1}{2}x^2$	
x	y
1	0.5
2	2

91.

$y=-x^2$	
x	y
1	-1
2	-4

92. What does the a-value do to all y-values of the original graph?

The a-value multiplies the y-values

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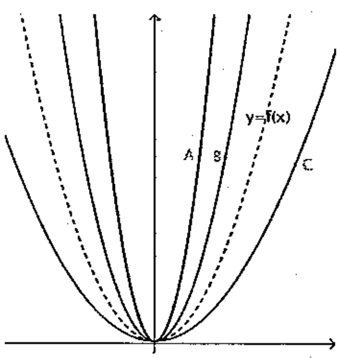
93. If $f(x)=x^2$ match the letters with the corresponding equations.

A $y=4x^2$

B $y=\frac{7}{2}x^2$

C $y=0.7x^2$

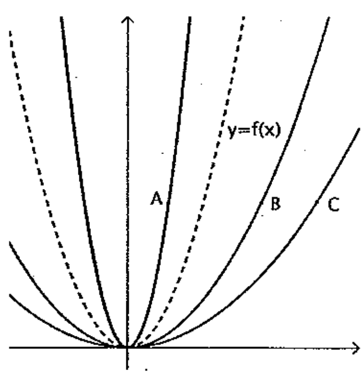


94. If $f(x)=0.9x^2$ match the letters with the corresponding equations.

C $y=\frac{1}{4}x^2$

A $y=x^2$

B $y=\frac{2}{3}x^2$

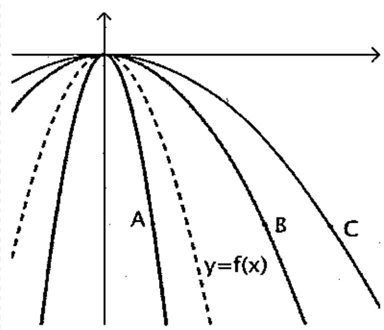


95. If $f(x)=-5x^2$ match the letters with the corresponding equations.

C $y=-\frac{4}{5}x^2$

B $y=-2x^2$

A $y=-\frac{11}{2}x^2$



How will each equation change after the following transformations?

96. How will the equation $y=3x^2$ change, if it reflects over the x-axis?

$y=-3x^2$

97. How will the equation $y=x^2$ change, after a vertical expansion by a factor of 7?

$y=7x^2$

98. How will the equation $y=x^2$ change after a vertical compression by a factor of $\frac{2}{3}$?

$y=\frac{2}{3}x^2$

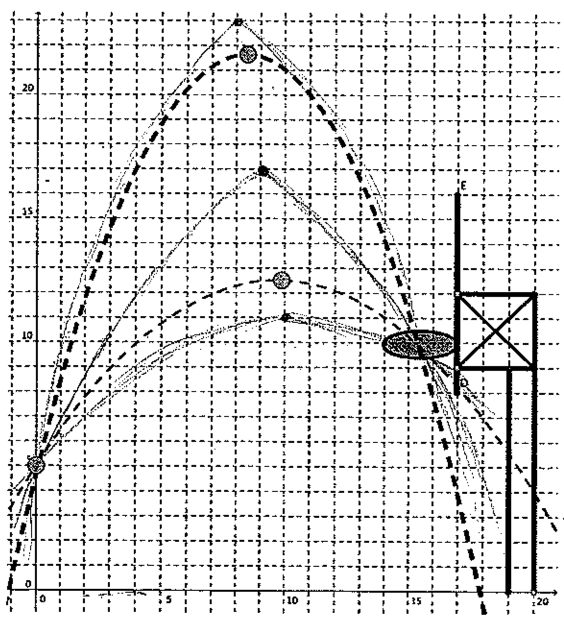
Michael Jordan,(6 NBA Titles) and Parabolic Motion

Putting a basketball through a ten-foot hoop would be extremely difficult without parabolic motion. And with parabolic motion comes quadratic equations. Let's be honest, NBA stars do not need to calculate quadratic equations to score 3-point shots. That being said, it is interesting to note that while the average NBA player may have no concept of the equation $y=a(x-p)^2+q$, the ball follows this pathway EVERY single time. Let's take a closer look.

Take a look at this video clip where we see basketball legend, Michael Jordan,
as he demonstrates his mastery of basketball and parabolic motion.
http://www.youtube.com/watch?v=alijpYhHtyk

Samual takes two shots and scores both of them. It is clear to see that the ball follows a parabolic pathway.

99. Sketch 3 more pathways that will lead Sam to scoring the basketball.
    - Draw a pathway that is higher than Sam's highest shot.
    - Draw a pathway that is in between Sam's two shots
    - Draw a pathway that is below Sam's lowest shot.



100. Can the vertices of scoring pathways have the same height but different horizontal values? Explain.

Yes, ~~bect~~ but the spread is very small.

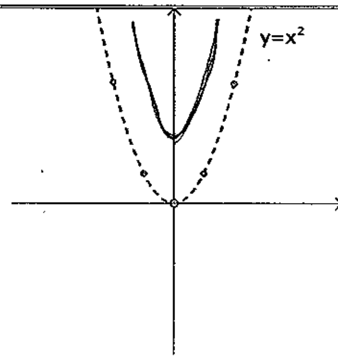
The P & Q Values: $f(x)=x^2+q$ & $f(x)=(x+p)^2$

- What we know so far: Terminology and what the a-value does in $f(x)=ax^2$
- Today's Lesson: How do the "p" and "q" values in $f(x)=x^2+q$ & $f(x)=(x+p)^2$ impact $f(x)=x^2$

Challenge #:

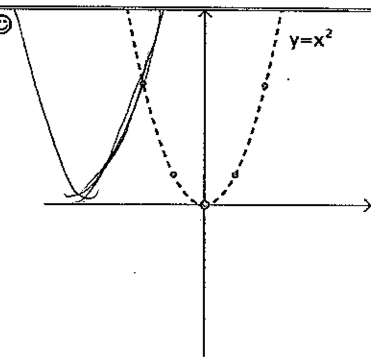
101. **Take a guess☺**

If the graph to the right is $y=x^2$, sketch what you think $y=x^2+2$ might look like.



102. **Take a guess☺**

If the graph to the right is $y=x^2$, sketch what you think $y=(x+4)^2$ might look like.

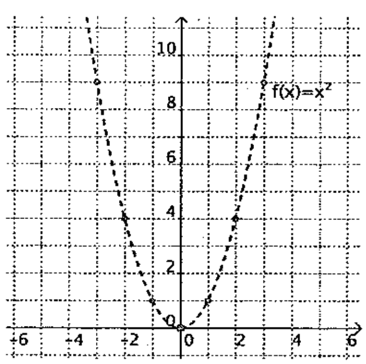


Use a table of values to determine what the following graphs look like.

103. Graph $y=x^2+2$.

105. Complete the table of values.

$y=x^2+2$	
X	Y
3	11
2	6
1	3
0	2
-1	3
-2	6
-3	11



107. Vertex: (0,2)

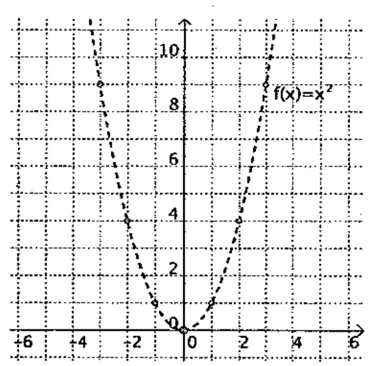
108. Axis of symmetry X=0

The graph of $y=x^2+2$ is two units higher than $y=x^2$.

104. Graph $y=(x+4)^2$.

106. Complete the table of values.

$y=(x+4)^2$	
X	Y
-1	9
-2	4
-3	1
-4	0
-5	1
-6	4
-7	9



109. Vertex: (-4,0)

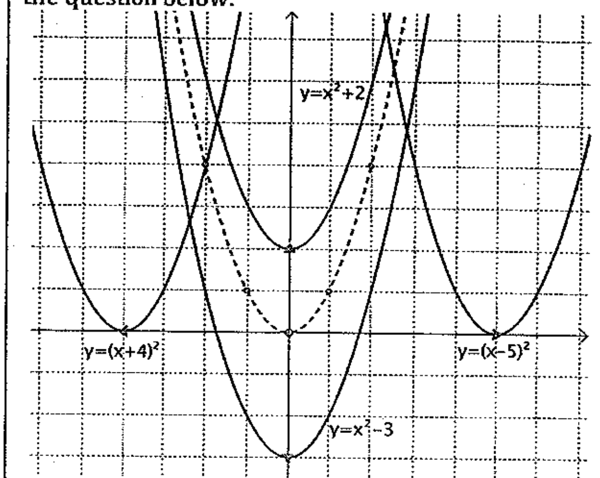
110. Axis of symmetry X=-4

The graph of $y=(x+4)^2$ is 4 units to the left of $y=x^2$.

The "p"-value and the "q"-value:

How do changes to the p & q values change the graph of $y=(x-p)^2+q$?

Use the graph to answer the questions to the left and the question below.



111. If $q>0$ then the graph of $y=x^2+q$ translates:

Up or Down

112. If $q<0$ then the graph of $y=x^2+q$:

Up or Down

113. If $p>0$ then the graph of $y=(x-p)^2$ translates:

Left or Right

114. If $p<0$ then the graph of $y=(x-p)^2$ translates:

Left or Right

*Use the answer key if you are unsure of the definitions.

115. True/False $y=x^2$ and $y=(x-4)^2+7$ are congruent. Explain.

True. The a-value is same in both equations.

116. True/False $y=x^2$ and $y=(x+4)^2$ have the same domain.

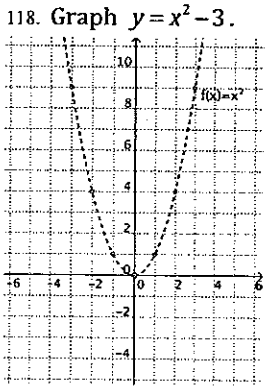
True. All parabolas of the form $y=a(x-p)^2+q$ have a domain of real values.

117. True/False $y=x^2$ and $y=x^2+6$ have the same axis of symmetry.

True. There is no horizontal translation so the axis of symmetry is $x=0$ for both parabolas.

Challenge #:

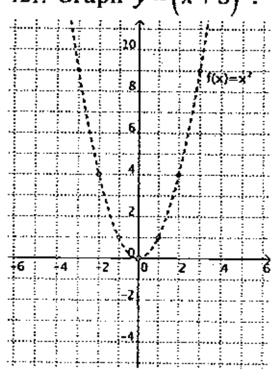
118. Graph $y=x^2-3$.



119. Vertex: $(0,-3)$

120. Range: $y \geq -3$

121. Graph $y=(x+3)^2$.

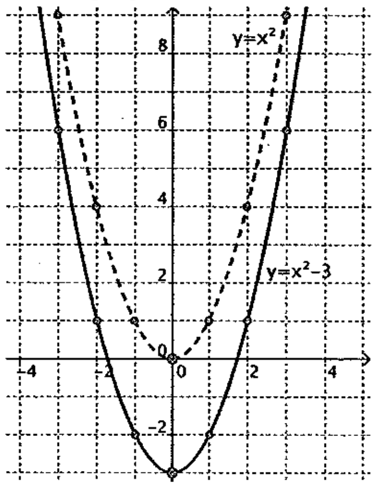


122. Vertex: $(-3,0)$

123. Axis of symmetry: $x=-3$

Graph the following parabolas with or without the use of a table of values.

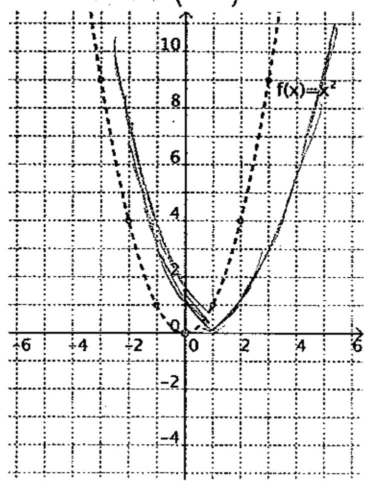
124. Graph $y=x^2-3$.



125. Vertex: **(0,-3)**

126. Range: $y\geq -3$

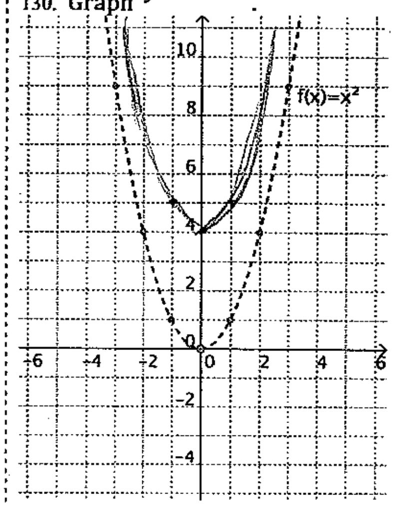
127. Graph $y=(x-1)^2$.



128. Vertex: (1,0)

129. Axis Symmetry: $x=1$

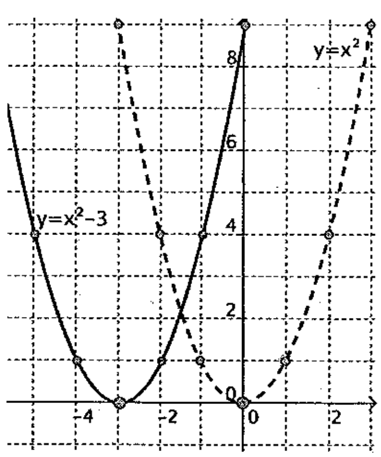
130. Graph $y=x^2+4$.



131. Vertex: (0,4)

132. Range: $y\geq 4$

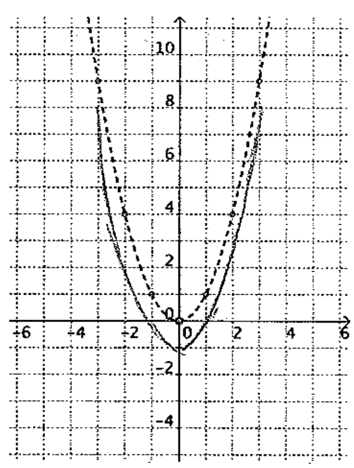
133. Graph $y=(x+3)^2$.



134. Vertex: **(-3,0)**

135. Axis of symmetry: **x=-4**

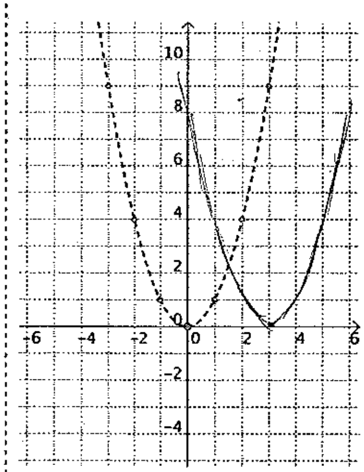
136. Graph $y=x^2-1$.



137. Vertex: (0,-1)

138. Axis of symmetry: $x=0$

139. Graph $y=(x-3)^2$.



140. Vertex: (3,0)

141. Range: $y\geq 0$

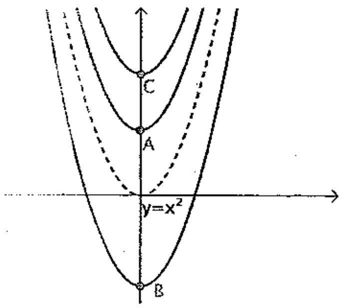
Match the parabola to the quadratic function in the form $y=(x-p)^2+q$

142. If $y=x^2$, match the letters with the corresponding equations.

A $y=x^2+2$

B $y=x^2-3$

C $y=x^2+4$

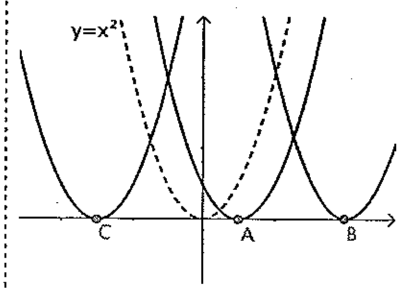


143. If $y=x^2$, match the letters with the corresponding equations.

C $y=(x+3)^2$

B $y=(x-4)^2$

A $y=(x-1)^2$

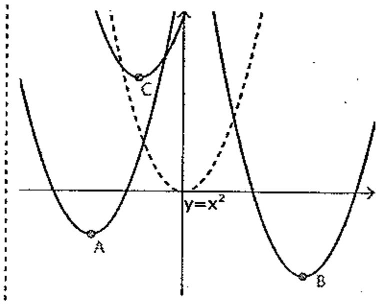


144. If $y=x^2$, match the letters with the corresponding equations.

A $y=(x+4)^2-2$

B $y=(x-5)^2-3$

C $y=(x+2)^2+4$



Write the equation with the following transformations.

145. Write the equation, $f(x)=x^2$, after a translation 2 units right and 5 units down.

$f(x)=(x-2)^2-5$

146. Write the equation, $f(x)=x^2$, after a translation 6 units left and 4 units down.

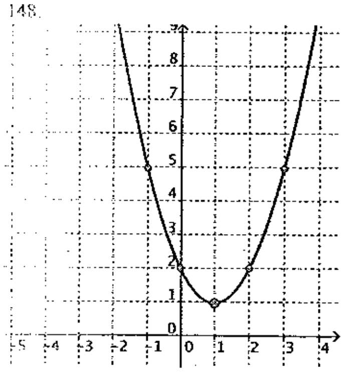
$f(x)=(x+6)^2-4$

147. Write the equation, $f(x)=x^2$, after a translation 3 units right and 1 units up.

$f(x)=(x-3)^2+1$

Write the equation of each parabola in the form $y=(x-p)^2+q$.

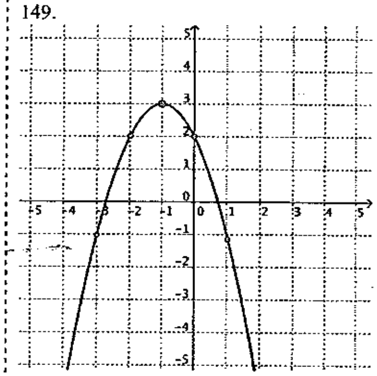
148.



Equation: $f(x)=(x-1)^2+1$

P-value : 1 Q-value: 1

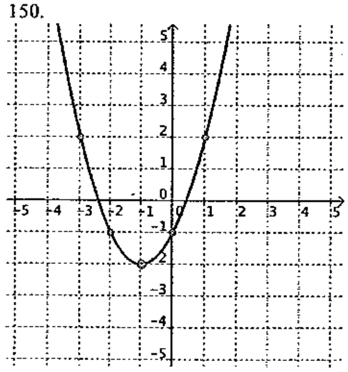
149.



Equation: $f(x)=-(x+1)^2+3$

P-value : -1 Q-value: 3

150.

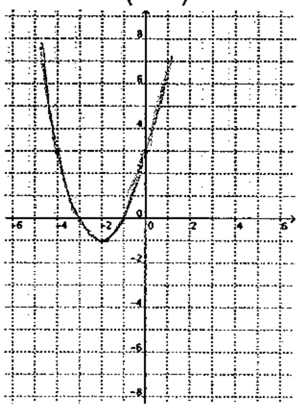


Equation: $f(x)=(x+1)^2-2$

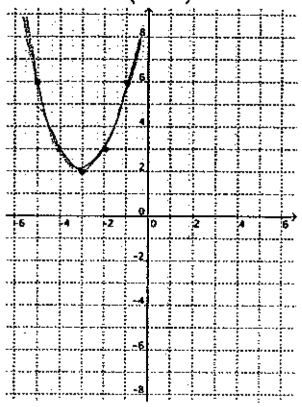
P-value : -1 Q-value: -2

Graph each parabola. Plot 5 points.

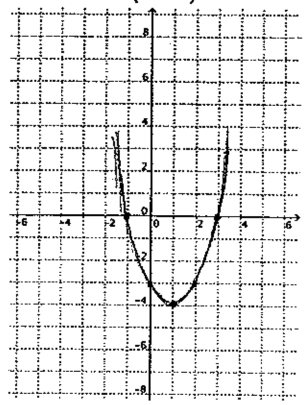
151. Graph $y=(x+2)^2-1$.



152. Graph $y=(x+3)^2+2$.

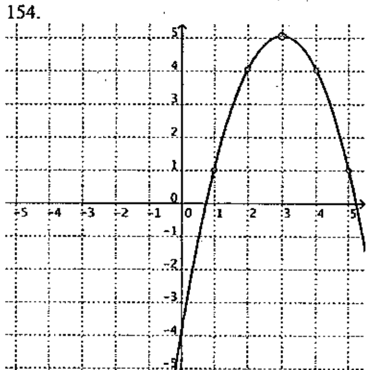


153. Graph $y=(x-1)^2-4$.



Write the equation of each parabola.

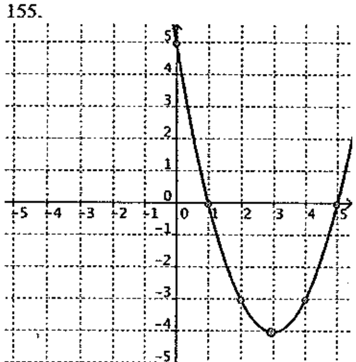
154.



Equation: $f(x)=-(x-3)^2+5$

Axis of Symmetry: $x=3$

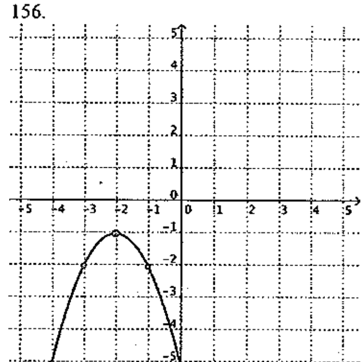
155.



Equation: $f(x)=(x-3)^2-4$

Range: $y\geq -4$

156.

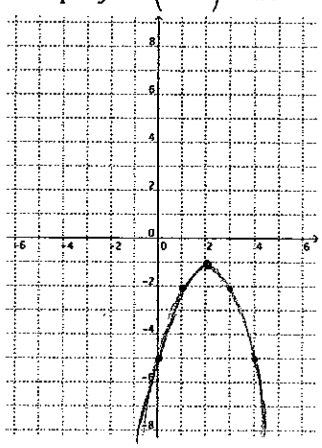


Equation: $f(x)=-(x+2)^2-1$

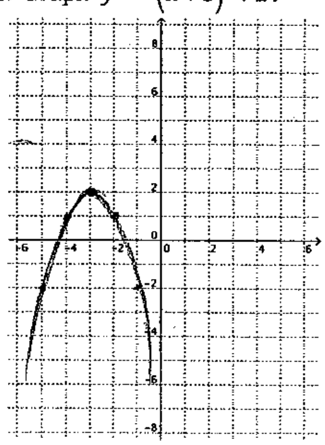
Max or Min: Maximum

Graph each parabola and plot 5 points.

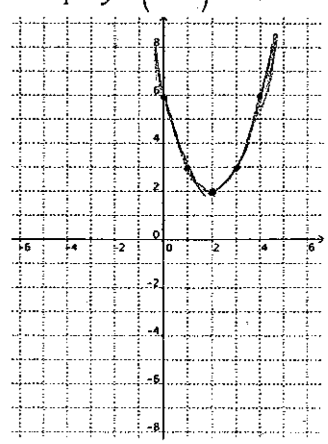
157. Graph $y=-(x-2)^2-1$.



158. Graph $y=-(x+3)^2+2$.



159. Graph $y=(x-2)^2+2$.



Determine the y and x-intercepts. Round your answer to the nearest tenth where appropriate.

160. $y=2(x+3)^2-8$

Solution:
X=0 at the y-intercept.
Substitute x=0

$y=2([0]+3)^2-8$

$y=2(9)-8$

$y=10$

Solution:
Option #1. Expand, simplify and factor.

OR Option #2.
Isolate the quadratic.

$y=2(x+3)^2-8$

$0=2(x+3)^2-8$

$8=2(x+3)^2$

$4=(x+3)^2$

Square root both sides and solve.

$2=\pm(x+3)$

$2=x+3$, $x=-1$

$2=-x-3$, $5=-x$, $x=-5$

161. $y=(x-1)^2-4$

if $x=0$

$y=(0-1)^2-4=1-4=-3$

if $y=0$

$0=(x-1)^2-4$

$(x-1)^2=4$

$x-1=\pm 2$

$x=3, -1$

162. $y=4(x+1)^2-20$

if $x=0$

$y=4(0+1)^2-20$

$=-16$

if $y=0$

$0=4(x+1)^2-20$

$4(x+1)^2=20$

$(x+1)^2=5$

$x+1=\pm\sqrt{5}$

$x=\sqrt{5}-1=1.2$

$x=-\sqrt{5}-1=-3.2$

163. $y=(x-1)^2-7$

if $x=0$

$y=(0-1)^2-7=-6$

if $y=0$

$0=(x-1)^2-7$

$(x-1)^2=7$

$x-1=\pm\sqrt{7}$

$x=\sqrt{7}+1=3.6$

$x=-\sqrt{7}+1=-1.6$

164. $y=(x-5)^2$

if $x=0$

$y=(0-5)^2=25$

if $y=0$

$0=(x-5)^2$

$x-5=0$

$x=5$

165. $y=-3(x+2)^2+21$

if $x=0$

$y=-3(0+2)^2+21=9$

if $y=0$

$0=-3(x+2)^2+21$

$-3(x+2)^2=-21$

$(x+2)^2=7$

$x+2=\pm\sqrt{7}$

$x=\sqrt{7}-2=0.6$

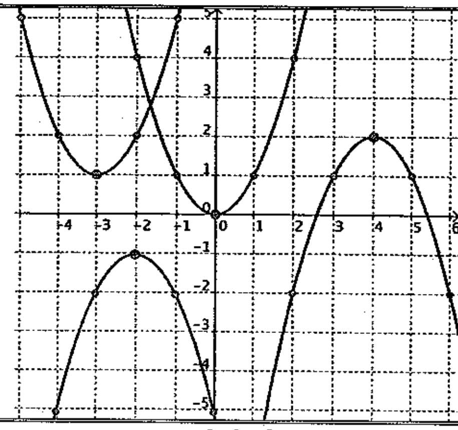
$x=-\sqrt{7}-2=-4.6$

Congruence:

Two parabolas are **congruent:**
- If they have the same shape.
- If they have the same $|a|$ value.

The following parabolas are congruent:
- $y=2x^2$, $y=2(x+7)^2-5$, $y=-2x^2+8$

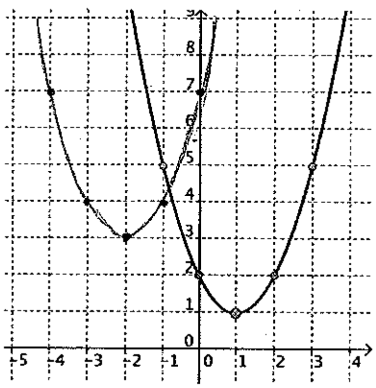
The parabolas to the right are all congruent.



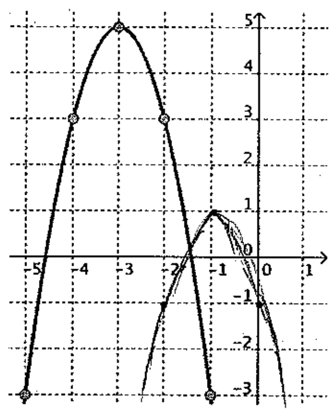
166. True or False. A parabola that opens down can never be congruent to a parabola that opens up.

False. It is the absolute value of the a-value that makes the parabola congruent not the sign of the a-value.

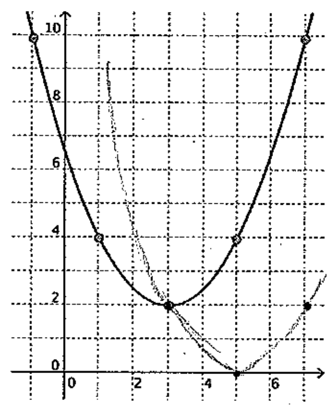
167. Draw a parabola that opens up, has vertex is $(-2,3)$ and is conguent to the parabola below.



168. Draw a parabola that opens down, has vertex is $(-1,1)$ and is conguent to the parabola below.



169. Draw a parabola that opens up, has vertex is $(5,0)$ and is conguent to the parabola below.



In each group which quadratic functions are congruent to the given function.

170. Given: $y=-x^2$

A. ✓ $y=x^2+5x-7$

B. $y=-x+5$

C. ✓ $y=(x+3)^2+2$

171. Given: $y=-5x^2+2$

A. ✓ $y=5x^2+17$

B. $y=-2x^2+5$

C. $y=-5^2+5$

172. Given: $y=ax^2+bx+c$

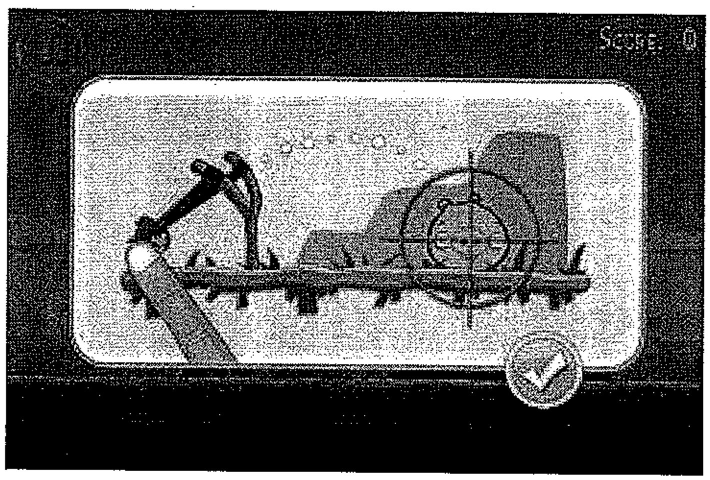
A. ✓ $y=ax^2-cx+b$

B. ✓ $y=-a(x-p)^2+q$

C. ✓ $y=-a(x^2-2p+p^2)+q$

Quadratic Transformations and Angry Birds

Angry Birds is one of the most popular cell phone games of 2011. It has a simple premise, some pigs stole some eggs and the birds are angry. The bird's revenge is simple; Slingshot themselves at the pigs. The pigs are ultimately destroyed by two powerful forces; Angry Birds and parabolic motion.



Check out quadratic functions at:
http://chrome.angrybirds.com/

You do not need to understand quadratic functions to be an Angry Bird or to play the game Angry Birds. However, the developers of this game have deep understanding of parabolic motion and the quadratic functions that create the bird's parabolic pathways. They have mastered the meanings of the a, p & q values in the equation $y=a(x-p)^2+q$.

Quadratic transformations $y=a(x-p)^2+q$

173. Write the equation of a parabola with a vertex (3,4) and that is congruent to $y=-3(x-5)^2+7$ and opens up.

$y=3(x-3)^2+4$

174. Write the equation of a parabola with an axis of symmetry of x=7, a minimum value of 1 and that is vertically expanded by a factor of 2.

$y=2(x-7)^2+1$

Write the equation of the parabola.

175.
Write the equation of a parabola with a vertex (3,4) and that is congruent to $y=-3(x-5)^2+7$ and opens up.
Solution:

$y=a(x-p)^2+q$

Substitute the p & q value.

$y=a(x-3)^2+4$

The a-value could be ±3 since the a-value of $y=-3(x-5)^2+7$ is -3.

$y=\pm3(x-3)^2+4$

The a-value is positive since the parabola opens up.

$y=+3(x-3)^2+4$

176. Write the equation of a parabola with an axis of symmetry of $x=2$, a maximum value of 3 and vertically compressed by a factor of $\frac{1}{8}$.

$f(x)=-\frac{1}{8}(x-2)^2+3$

177. Write the equation of a parabola with a vertex (-2,5) and that is congruent to $y=\frac{1}{2}(x-5)^2+1$ and a vertex that is a maximum.

$y=-\frac{1}{2}(x+2)^2+5$

178.
Write the equation of a parabola with an axis of symmetry of x=7, a minimum value of 1 and that is vertically expanded by a factor of 2.
Solution:

$y=a(x-p)^2+q$

p=7 since the axis of symmetry is x=7 and q=1 since the minimum value is 1.

$y=a(x-7)^2+1$

The a-value is +2 since it is expanded by a factor of 2 and the vertex is at a minimum.

$y=2(x-7)^2+1$

179. Write the equation of a parabola with a vertex (-2,3) and that is congruent to $y=-3x^2$ and opens down.

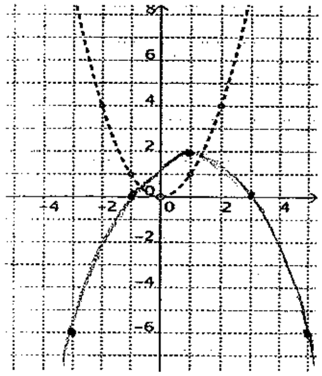
$y=-3(x+2)^2+3$

180. Write the equation of a parabola with an axis of symmetry of x=8, a maximum value of -2 and vertically compressed by a factor of $\frac{2}{3}$

$y=-\frac{2}{3}(x-8)^2-2$

Challenge #:

181. Graph. $y=-\frac{1}{2}(x-1)^2+2$

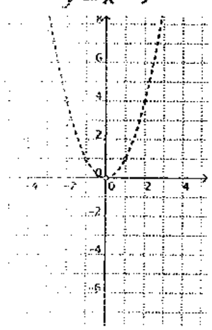


Write down the steps to solve the challenge to the left.

1. Translate the axis one unit right and 2 units up.
2. Reflect and compress in relation to the dotted lines

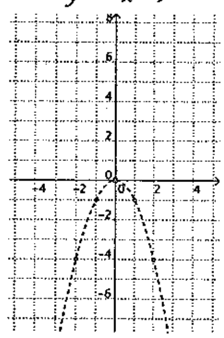
Solution #1: Graph $y=-\frac{1}{2}(x-1)^2+2$.

$y=x^2 \rightarrow$



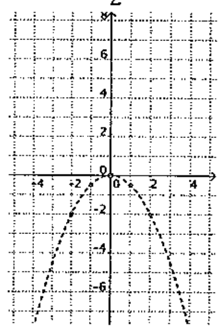
Reflect over the x-axis.

$y=-x^2 \rightarrow$



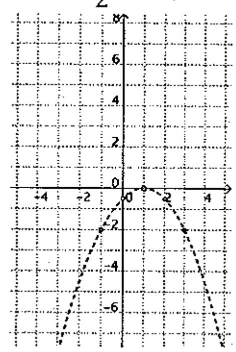
Vertically compress by a half

$y=-\frac{1}{2}x^2 \rightarrow$



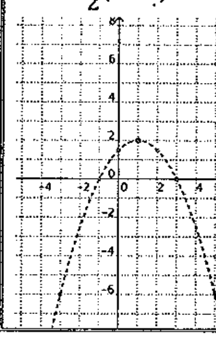
Horizontally translate 1 unit right

$y=-\frac{1}{2}(x-1)^2 \rightarrow$



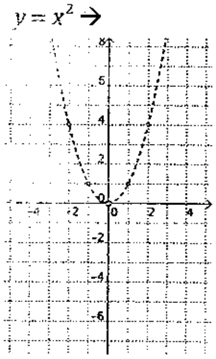
Vertically translate 2 units up.

$y=-\frac{1}{2}(x-1)^2+2$



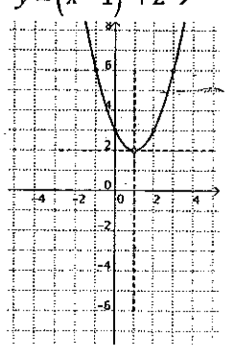
Solution #2: $y=-\frac{1}{2}(x-1)^2+2$.

$y=x^2 \rightarrow$



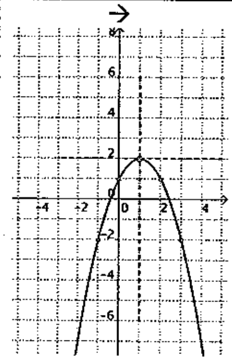
$\rightarrow$

$y=(x-1)^2+2 \rightarrow$

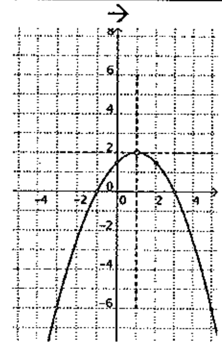


Translate the axis one unit right and 2 units up. $\rightarrow$

$\rightarrow$

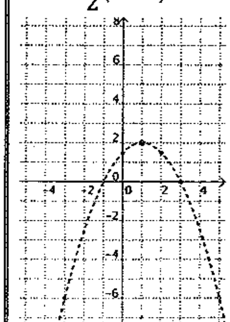


$\rightarrow$



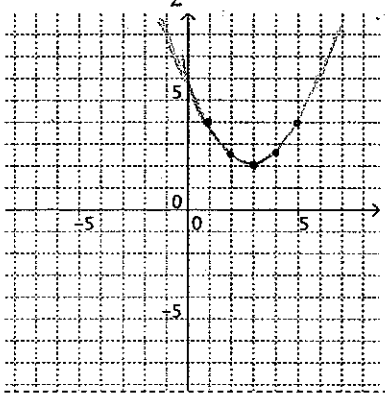
Reflect and compress in relation to the dotted lines; x=1 and y=2. $\rightarrow$

$y=-\frac{1}{2}(x-1)^2+2$

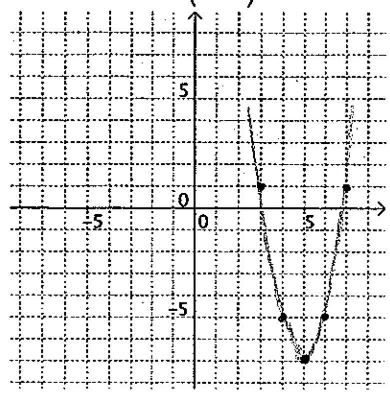


Graph each of the quadratic functions and plot at least 5 points.

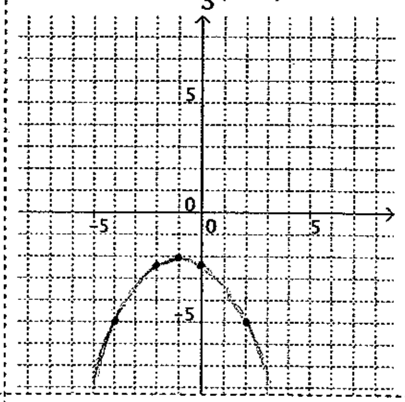
182. Graph $y=\frac{1}{2}(x-3)^2+2$.



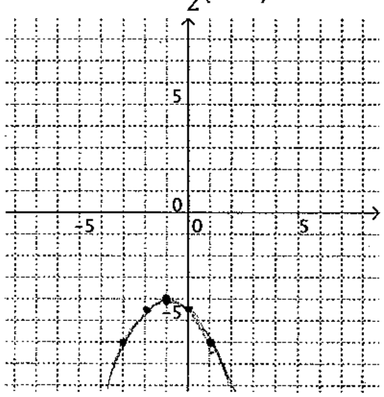
183. Graph $y=2(x-5)^2-7$.



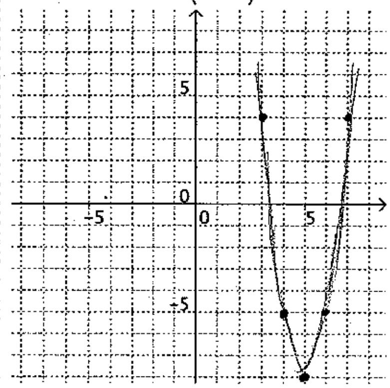
184. Graph $y=-\frac{1}{3}(x+1)^2-2$.



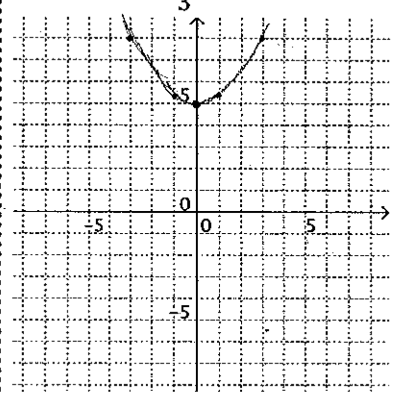
185. Graph $y=-\frac{1}{2}(x+1)^2-4$.



186. Graph $y=3(x-5)^2-8$.

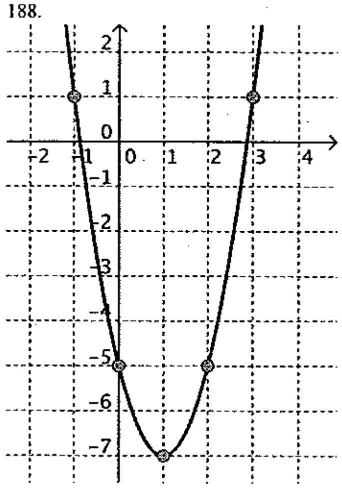


187. Graph $y=\frac{1}{3}x^2+5$.



Write the equation of each of the following parabolas.

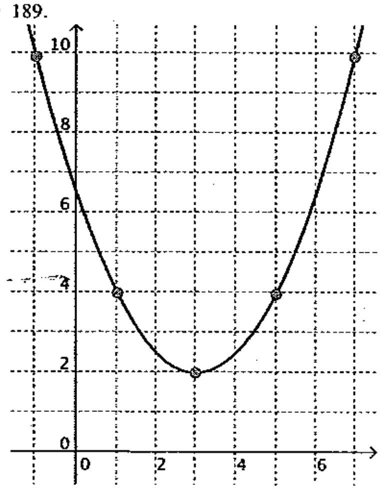
188.



Equation:

$y=2(x-1)^2-7$

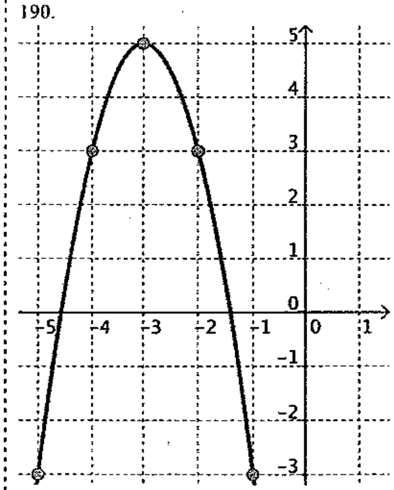
189.



Equation:

$y=\frac{1}{2}(x-3)^2+2$

190.

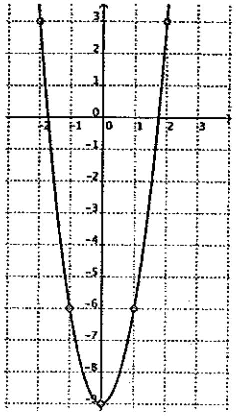


Equation:

$y=-2(x+3)^2+5$

Write the equation of each of the following parabolas.

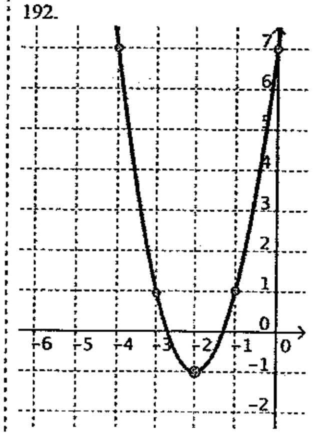
191.



Equation:

$y=3x^2-9$

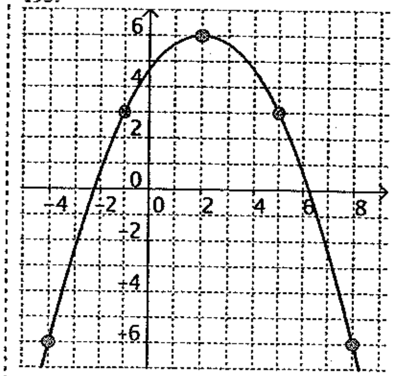
192.



Equation:

$y=2(x+2)^2-1$

193.



Equation:

$y=-\frac{1}{3}(x-2)^2+6$

Determine the following without graphing:

194. $y=\frac{1}{2}(x-10)^2+11$	195. $y=-\frac{1}{2}(x+7)^2-11$	196. $y=3(x+21)^2-1$	197. $y=-2(x-7)^2+9$
Vertex: $(10,11)$	Vertex: $(-7,-11)$	Vertex: $(-21,-1)$	Vertex: $(7,9)$
Axis of Symmetry: $x=10$	Axis of Symmetry: $x=-7$	Axis of Symmetry: $x=-21$	Axis of Symmetry: $x=7$
Max or min? What is it? Min (11)	Max or min? What is it? Max (-11)	Max or min? What is it? Min (-1)	Max or min? What is it? Max (9)
Domain: real	Domain: real	Domain: real	Domain: real
Range: $y\geq 11$	Range: $y\leq -11$	Range: $y\geq -1$	Range: $y\leq 9$
Does it have X int? No YES or NO	Does it have X int? No YES or NO	Does it have X int? Yes YES or NO	Does it have X int? Yes YES or NO

Challenge #:

198. Use the a and q values to determine whether $y=2(x-5)^2-7$ has zero x-intercepts, 1 x-intercept or 2 x-intercepts.

2 x-intercepts.

199. Write the equation of a parabola with a vertex (4,5) and has a y-intercept of -3.

$y=-\frac{1}{2}(x-4)^2+5$

Use the a and q value to determine whether each quadratic function, $y=a(x-p)^2-q$, has zero x-intercepts, one x-intercept or two x-intercepts.

200. $y=2(x-5)^2-7$

Solution:
2 x-intercepts.
Since the vertex is below the x-axis and the parabola opens up, there will be two x-intercepts.

201. $y=-2(x+10)^2+12$

two x-intercepts.

202. $y=-2(x-4)^2-1$

zero x-intercepts

203. How many x-intercepts will $y=a(x-p)^2+q$ have if a>0 and q=0?

one x-intercept

204. How many x-intercepts will $y=a(x-p)^2+q$ have if have if a<0 and q<0?

zero x-intercepts

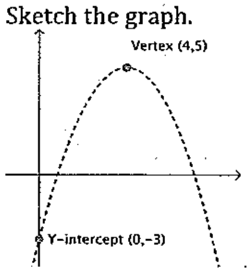
205. How many x-intercepts will $y=a(x-p)^2+q$ have if a>0 and q<0?

two x-intercepts.

Write the equation of a parabola passing through a specific point.

206. Write the equation of a parabola with a vertex (4,5) and has a y-intercept of -3.

Solution:
Sketch the graph.



$y=a(x-p)^2+q$

Substitute the (p,q) with (4,5).

$y=a(x-4)^2+5$

Find the a-value that makes the parabola go through the point $(0,-3)$. Substitute $(0,-3)$ for (x,y).

$-3=a(0-4)^2+5$

Simplify and solve for a.

$-3-5=a(16) \rightarrow a=\frac{-8}{16}=-\frac{1}{2}$

$y=-\frac{1}{2}(x-4)^2+5$

207. Write the equation of a parabola with a vertex (3,-2) and passes through the point (5,7).

$y=a(x-p)^2+q$

$y=a(x-3)^2-2$

$7=a(5-3)^2-2$

$a=\frac{9}{4}$

$y=\frac{9}{4}(x-3)^2-2$

208. Write the equation of a parabola with a vertex (-4,3) and passes through the point (2,5)

$y=a(x-p)^2+q$

$y=a(x+4)^2+3$

$5=a(2+4)^2+3$

$a=\frac{1}{18}$

$y=\frac{1}{18}(x+4)^2+3$

How many x-intercepts will each quadratic function have?

209. $y=4(x-8)^2+2$ 0	210. $y=-x^2+9$ 2	211. $y=2(x-9)^2$ 1
212. How many x-intercepts will $y=a(x-p)^2+q$ have if a<0 and q>0? 2	213. How many x-intercepts will $y=a(x-p)^2+q$ have if have if a<0 and q=0? 1	214. How many x-intercepts will $y=a(x-p)^2+q$ have if a>0 and q>0? 0

Describe how $y=x^2$ was transformed to the following quadratic functions. Use words like reflection, expansion, compression and translation

215. $y=\frac{1}{2}(x+2)^2-3$ Vertical compression by $\frac{1}{2}$, horizontal translation 2 left, vertical translation 3 down	216. $y=-3(x-1)^2+2$ Reflection over the x-axis, vertical expansion by 3, horizontal translation 1 right, vertical translation 2 up	217. $y=\frac{1}{4}(x+1)^2+1$ Vertical compression by $\frac{1}{4}$, horizontal translation 1 left, vertical translation 1 up.

218. Write the equation of a parabola with a vertex (10,5) and has a y-intercept of -2. $y=a(x-p)^2+q$ $y=a(x-10)^2+5$ $-2=a(-10)^2+5$ $-2=100a+5$ $a=-\frac{7}{100}$ $y=-\frac{7}{100}(x-10)^2+5$	219. Write the equation of a parabola with a vertex $(2,-1)$ and has x-intercepts of1 and 3. $y=a(x-p)^2+q$ $y=a(x-2)^2-1$ $0=a(1-2)^2-1$ $0=a(3-2)^2-1$ $a=1$ $y=(x-2)^2-1$	220. Write the equation of a parabola with a vertex (5,2) and passes through the point (4,1). $y=a(x-p)^2+q$ $y=a(x-5)^2+2$ $1=a(4-5)^2+2$ $a=-1$ $y=-(x-5)^2+2$

Quadratic functions in Standard form: $y=ax^2+bx+c$.

Challenge #:

Answer the questions about $y=2x^2+16x-7$ by observation or select "Can not answer by observation."

221. Y-intercept:

(0, -7)

"Can not answer by observation."

222. X=intercept(s):

"Can not answer by observation."

223. Opens up or down:

up

"Can not answer by observation."

224. Vertex is a maximum or a minimum:

minimum

"Can not answer by observation."

225. Vertex:

"Can not answer by observation."

226. Expansion or compression:

Expansion

"Can not answer by observation."

227. Domain:

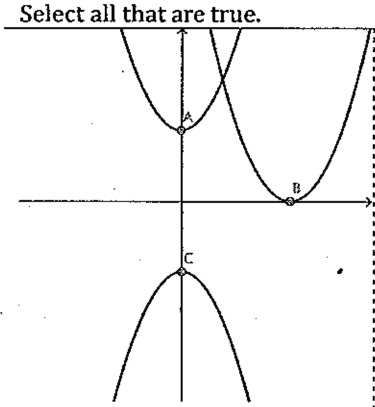
real

"Can not answer by observation."

228. Range:

"Can not answer by observation."

Select all that are true.



229. Which of the following could represent graph A to the left?

a. $y=x^2$
b. $y=x^2+4$
c. $y=(x+4)^2$
d. $y=x^2-4$
e. $y=x^2+c$, $c>0$

230. Which of the following could represent graph B to the left?

a. $y=(x+4)^2$
b. $y=x^2+8x+16$
c. $y=(x-4)^2$
d. $y=x^2-8x+16$
e. $y=x^2+bx+25$, $b<0$

231. Which of the following could represent graph C to the left?

a. $y=-x^2+4$
b. $y=-x^2$
c. $y=-x^2+c$, $c>0$
d. $y=-x^2+c$, $c<0$
e. $y=-(x-4)^2$

Select all that are true.

232. Which of the following have a vertex to the right of the y-axis?

A. $y=(x+4)^2\pm c$
B. $y=x^2+8x+60$
C. $y=(x-4)^2\pm c$
D. $y=x^2-8x+60$

233. Which of the following have a negative y-intercept

A. $y=x^2-7$
B. $y=x^2+8x-60$
C. $y=x^2+8x+c$, $c<0$
D. $y=x^2-8x+c$, $c>0$

234. Which of the following open down?

A. $y=(x+4)^2\pm c$
B. $y=-(x+4)^2\pm c$
C. $y=-x^2+8x+6$
D. $y=ax^2-8x+6, a<0$

235. Order the parabolas from widest to narrowest.

A. $y=-\frac{4}{5}(x+4)^2\pm c$
B. $y=-3x^2+60$
C. $y=\frac{1}{6}x^2+8x+c$
D. $y=x^2+8x+6$

CADB

Commerce, Business, Completing the square

$$\mathrm{Re}venue = \mathrm{Pr}ice \times Quantity\ Sold$$

David is the CEO of a solar battery company. He owns 150 battery kiosks across Canada and Western USA.

Price per battery: $10 each
Number sold per day: 100 batteries per kiosk.
Revenue per day per kiosk: $\$10 \times 100 = \$1000$

He paid an independent company $50000 dollars to do some market research. They determined that he will sell more batteries if he reduces the price. This was obvious but they were more specific.

- For every $2.50 price increase he will sell 10 fewer batteries per day per kiosk.
- For every $2.50 price decrease he will sell 10 more batteries per day per kiosk.

Here is what David Knows:

- His current daily revenue in all his Kiosks is $\$10 \times 100 \times 150 = \$150000$ /Day.

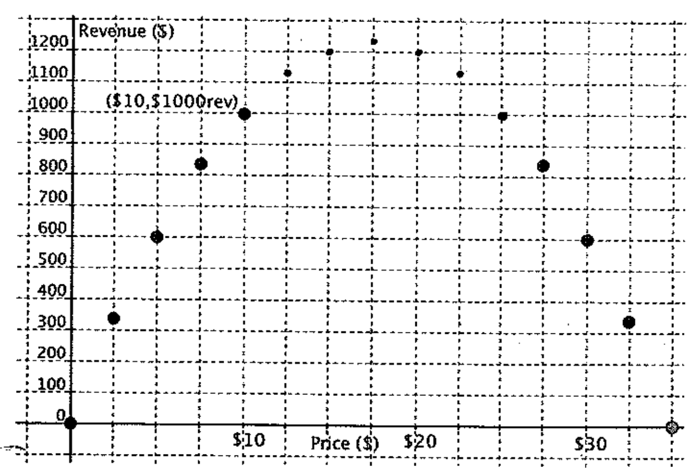
Here is what he wants to know:

- Is $10 the best price for his batteries?
- If not, WHAT IS the best price?

236. Fill out the table and plot the points to help David determine the best price.

Price	Number sold	Revenue
$0	140	0
$2.50	130	$325
$5	120	$600
$7.50	110	$825
$10	*100*	*$1000*
$12.50	90	$1125
$15	80	$1200
$17.50	70	$1225
$20	60	$1200
$22.50	50	$1125
$25	40	$1000
$27.50	30	$825
$30	20	$600
$32.50	10	$325
$35	0	0

Daily Revenue of single Battery Kiosk



237. **What is the best price?** $17.50
238. **How many will be sold per day?** 70
239. **What is the new revenue per store?** $1225

240. How much more revenue will David's company earn per day if the market research is accurate?

New model $\$NewPrice \times NewQuatitySold \times 150 =$

Current model: $\$10 \times 100 \times 150 = \$150000$

Quadratic functions: Completing the Square

We know how to graph $y=(x+3)^2+5$, but how would we graph $y=x^2+6x+14$? In this section we will learn how turn a quadratic function in standard form, $y=x^2+6x+14$, into vertex or graphing form, $y=(x+3)^2+5$.

Review: Refresh your memory by reading each of the following expansions.

$y=(x+5)^2$ 241. $y=(x+5)(x+5)$ $y=x^2+10x+25$ ✓	$y=(x-6)^2$ 242. $y=(x-6)(x-6)$ $y=x^2-12x-36$ ✓	$y=(x+10)^2$ 243. $y=(x+10)(x+10)$ $y=x^2+20x+100$ ✓

Determine the vertex of each quadratic function.

244. $y=x^2+2x+1$ $y=(x+1)^2$ $(-1,0)$	245. $y=x^2-8x+16$ $y=(x-4)^2$ $(4,0)$	246. $y=x^2+18x+81$ $y=(x+9)^2$ $(-9,0)$

Complete the square: Make each binomial a perfect square.

- Remember, what ever is added in the brackets, needs to be subtracted outside of the brackets.

247. $y=(x^2+8x+\underline{16})-\underline{16}$ $y=(x\underline{+4})^2-\underline{\quad}$	248. $y=(x^2+14x+\underline{49})-\underline{49}$ $y=(x\underline{+7})^2-\underline{49}$	249. $y=(x^2-6x+\underline{9})-\underline{9}$ $y=(x\underline{-3})^2-\underline{9}$

Challenge #:

250. Complete the square and state the vertex $y=x^2+6x+11$.

Write down the steps to solve this problem.

$y=(x^2+6x+[0])+11$ — Exclude the constant

$y=(x^2+6x+[9-9])+11$ — Complete the square.

$y=(x^2+6x+9)-9+11$ — Add half of 6 and square it.

$y=(x+3)^2+2$ — -3,2 — Rearrange and factor the perfect square.

Vertex: $(-3,2)$

Complete the square and determine the vertex.

251. $y=x^2+6x+11$

Solution:

$y=x^2+6x+11$

Exclude the constant.

$y=\left(x^2+6x+[0]\right)+11$

Complete the square. Add half of 6 and square it

$y=\left(x^2+6x+[9-9]\right)+11$

Rearrange and factor the perfect square.

$y=\left(x^2+6x+[9]\right)-[9]+11$

$y=(x+3)^2+2$

Vertex (-3,2)

252. $y=x^2+8x+7$

$y=(x^2+8x+16)-16+7$

$y=(x+4)^2-9$

$(-4,-9)$

253. $y=x^2+10x+10$

$y=(x^2+10x+25)-25+10$

$y=(x+5)^2-15$

$(-5,-15)$

254. $y=x^2-4x+1$

$y=(x^2-4x+4)-4+1$

$y=(x-2)^2-3$

$(2,-3)$

255. $y=x^2+20x-6$

$y=(x^2+20x+100)-100-6$

$y=(x+10)^2-106$

$(-10,-106)$

256. $y=x^2+12x-9$

$y=(x^2+12x+36)-36-9$

$y=(x+6)^2-45$

$(-6,-45)$

257. $y=x^2+3x$

$y=(x^2+3x+2.25)-2.25$

$y=(x+1.5)^2-2.25$

$(-1.5,-2.25)$

258. $y=x^2-9x$

$y=(x^2-9x+20.25)-20.25$

$y=(x-4.5)^2-20.25$

$(4.5,-20.25)$

259. $y=x^2+14x-10$

$y=(x^2+14x+49)-49-10$

$y=(x+7)^2-59$

$(-7,-59)$

260. State the minimum value of $y=x^2-16x+2$

$y=(x^2-16x+64)-64+2$

$y=(x-8)^2-62$

-62

261. Determine the minimum value of $y=x^2+24x+100$

$y=(x^2+24x+144)+100-144$

$y=(x+12)^2-44$

-44

262. State the vertex of $y=x^2-5x+14$

$y=(x^2-5x+6.25)-6.25+14$

$y=(x-2.5)^2+7.75$

$(2.5, 7.75)$

263. State the vertex $y=x^2+16x-7$

$y=(x^2+16x+64)-64-7$

$y=(x+8)^2-71$

$(-8,-71)$

264. Determine the range of $y=x^2-9x+20$

$y=(x^2-9x+20.25)-20.25+20$

$y=(x-4.5)^2-0.25$

$y\geq-0.25$

265. Determine the range of $y=x^2+24x-100$

$y=(x^2+24x+144)-144-100$

$y=(x+12)^2-244$

$y\geq-244$

Challenge #:

266. State the vertex of $y=2x^2+12x+7$

$y=2(x^2+6x+9)-18+7$

$y=2(x+3)^2-11$

$(-3,-11)$

Write down the steps to solve this problem.

Completing the Square

Write $y=2x^2+12x+7$ in standard form.	Rules	Write $y=-2x^2-10x+1$ in standard form.
$y=(2x^2+12x)+7$	Group the x terms	$y=(-2x^2-10x)+1$
$y=2(x^2+6x+[0])+7$ Square half of 6 & add and subtract it. $y=2(x^2+6x+[3^2-3^2])+7$	Factor out the x^2 coefficient. & Complete the square. Add and subtract the square of half the x coefficient	$y=-2(x^2+5x+[0])+1$ Square half of 5. & add and subtract it. $y=-2\left(x^2+5x+(-2.5)^2-(-2.5)^2\right)+1$
$y=2((x^2+6x+9)-9)+7$ $y=2\left((x+3)^2-9\right)+7$ Expand by 2 $y=2(x+3)^2+2(-9)+7$	Group and factor the perfect square trinomial Expand	$y=-2([x^2+5x+6.25]-6.25)+1$ $y=-2\left([x+2.5]^2-6.25\right)+1$ Expand by -2 $y=-2[x+2.5]^2-2(-6.25)+1$
$y=2(x+3)^2-18+7$	Simplify	$y=-2(x+2.5)^2+12.5+1$
$y=2(x+3)^2-11$		$y=-2(x+2.5)^2+13.5$

Complete the square and state the vertex.

267. $y=4x^2+8x-7$

$y=4(x^2+2x+1)-4-7$

$y=4(x+1)^2-11$

$(-1,-11)$

268. $y=5x^2-30x+44$

$y=5(x^2-6x+9)-45+44$

$y=5(x-3)^2-1$

$(3,-1)$

269. $y=3x^2-6x+5$

$y=3(x^2-2x+1)-3+5$

$y=3(x-1)^2+2$

$(1,2)$

270. $y=-10x^2+80x-7$

$y=-10(x^2-8x+16)+160-7$

$y=-10(x-4)^2+153$

$(4,153)$

271. $y=-5x^2-60x+2$

$y=-5(x^2+12x+36)+180+2$

$y=-5(x+6)^2+182$

$(-6,182)$

272. $y=7x^2-42x-3$

$y=7(x^2-6x+9)-63-3$

$y=7(x-3)^2-66$

$(3,-66)$

State the vertex of each quadratic function.

273. $y=9x^2-18x-10$

$y=9(x^2-2x+1)-9-10$

$y=9(x-1)^2-19$

$(1,-19)$

274. $y=-20x^2+400x+20$

$y=-20(x^2-20x+100)+2000+20$

$y=-20(x-10)^2+2020$

$(10,2020)$

275. $y=\frac{1}{2}x^2-x+2$

$y=\frac{1}{2}(x^2-2x+1)-\frac{1}{2}+2$

$y=\frac{1}{2}(x-1)^2+\frac{3}{2}$

$(1,\frac{3}{2})$

276. $y=\frac{1}{3}x^2-2x+1$

$y=\frac{1}{3}(x^2-6x+9)-3+1$

$y=\frac{1}{3}(x-3)^2-2$

$(3,-2)$

277. $y=-1.5x^2-15x+1$

$y=-1.5(x^2+10x+25)+37.5+1$

$y=-1.5(x+5)^2+38.5$

$(-5,38.5)$

278. $y=\frac{2}{3}x^2-6x+1$

$y=\frac{2}{3}(x^2-9x+20.25)-13.5+1$

$y=\frac{2}{3}(x-4.5)^2-12.5$

$(4.5,-12.5)$

279. $y=-\frac{1}{4}x^2-\frac{1}{2}x+1$

$y=-\frac{1}{4}(x^2+2x+1)+\frac{1}{4}+1$

$y=-\frac{1}{4}(x+1)^2+\frac{5}{4}$

$(-1,\frac{5}{4})$

280. $y=\frac{1}{9}x^2+\frac{2}{3}x+100$

$y=\frac{1}{9}(x^2+6x+9)-1+100$

$y=\frac{1}{9}(x+3)^2+99$

$(-3,99)$

281. $y=0.6x^2+3x+7$

$y=0.6(x^2+5x+6.25)-3.75+7$

$y=0.6(x+2.5)^2+3.25$

$(-2.5,3.25)$

282. State the range of $y=-2x^2-4x+1$.

$y=-2(x^2+2x+1)+2+1$

$y=-2(x+1)^2+3$

$(-1,3)$

$y\le 3$

283. Determine the axis of symmetry of $y=\frac{1}{2}x^2-4x+7$.

$y=\frac{1}{2}(x^2-8x+16)-8+7$

$y=\frac{1}{2}(x-4)^2-1$

$(4,-1)$

$y\ge -1$

$x=4$

284. State the minimum value of $y=x^2+24x-100$.

$y=(x^2+24x+144)-144-100$

$y=(x+12)^2-244$

-244

Which line did the error first occur in? Fix it.

285. $y=2x^2+4x-5$

i. $y=2(x^2+2x)-5$

ii. $y=2(x^2+2x+1)-5$

$y=2(x^2+2x+1)-2-5$

iii. $y=2(x+1)^2-5$

$y=2(x+1)^2-7$

286. $y=-x^2-2x+4$

i. $y=-(x^2+2x)+4$

ii. $y=-(x^2+2x+1-1)+4$

iii. $y=-(x+1)^2+1+4$

$y=-(x+1)^2+5$

iv. $y=-(x+1)^2+3$

287. $y=-\frac{1}{3}x^2+6x-1$

i. $y=-\frac{1}{3}(x^2+18x)-1$

$y=-\frac{1}{3}(x^2-18x)-1$

ii. $y=-\frac{1}{3}(x^2+18x+81-81)-1$

$y=-\frac{1}{3}(x^2-18x+81)+27-1$

iii. $y=-\frac{1}{3}(x+9)^2-\frac{1}{3}(-81)-1$

iv. $y=-\frac{1}{3}(x+9)^2+27-1$

v. $y=-\frac{1}{3}(x+9)^2+26$

$y=-\frac{1}{3}(x-9)^2+26$

288. **Determine the range of** $y=-x^2+4x+18$.

$y=-(x^2-4x+4)+4+18$

$y=-(x-2)^2+22$

$y\le 22$

289. **State the vertex of** $y=-\frac{1}{3}x^2-8x+2$.

$y=-\frac{1}{3}(x^2+24x+144)+48+2$

$y=-\frac{1}{3}(x+12)^2+50$

$(-12, 50)$

290. **Determine the axis of symmetry of** $y=-\frac{4}{5}x^2-16x-7$.

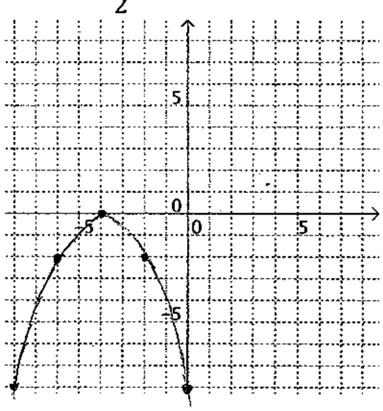
$y=-\frac{4}{5}(x^2+20x+100)+80-7$

$y=-\frac{4}{5}(x+10)^2+73$

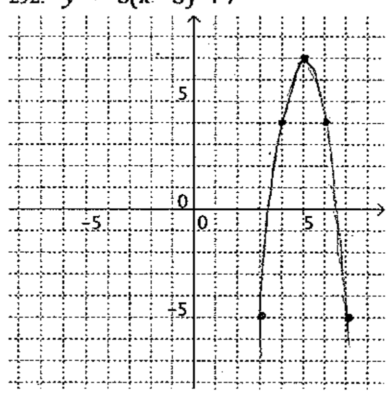
$x=-10$

Graph each of the quadratic functions and plot at least 5 points.

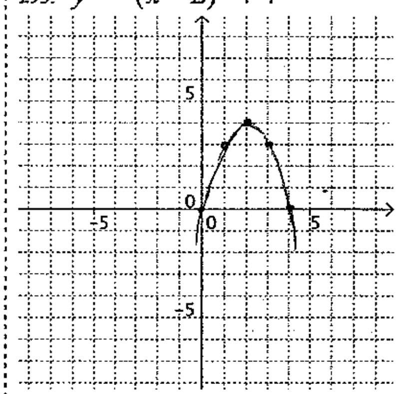
291. $y=-\frac{1}{2}(x+4)^2$



292. $y=-3(x-5)^2+7$



293. $y=-(x-2)^2+4$



Challenge #:

294. A ball is thrown directly upward off a 200 foot cliff with an initial velocity of 96 feet per second. The ball's height in relation to time follows the pathway $h=-16t^2+96t+200$, where h is the height in feet and t is time in seconds. **After how many seconds will the ball reach its maximum height? And, what is the maximum height?**

$h=-16(t^2-6t+9)+144+200$

$h=-16(t-3)^2+344$

3 seconds.

344 feet

Write down the steps to solve the challenge to the left.

Draw a picture

295. A ball is thrown directly upward off a 200 foot cliff with an initial velocity of 96 feet per second. The ball's height in relation to time follows the pathway $h=-16t^2+96t+200$, where h is the height in feet and t is time in seconds. **After how many seconds will the ball reach its maximum height? And, what is the maximum height?**

Draw a picture.

$h=-16t^2+96t+200$

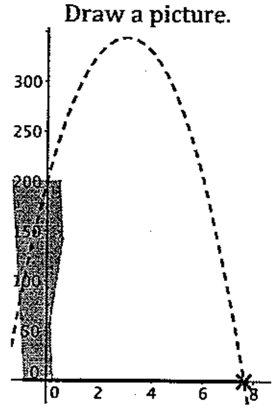
Find the vertex by completing the square.

$h=-16\left(t^2-6t\right)+200$

$h=-16\left(t^2-6t+9-9\right)+200$

$h=-16\left(t-3\right)^2-16\left(-9\right)+200$

$h=-16\left(t-3\right)^2+344$



The vertex is $\left(3,344\right)$.

The ball will reach a maximum height of 344 feet after 3 seconds.

296. A paper airplane is launched upward from a height of 8 feet. The plane's height in relation to time follows the pathway $h=-2t^2+15t+8$. What is the planes maximum height?

$h=-2(t^2-7.5t+14.0625)+28.125+8$

$h=-2(t-3.75)^2+36.125$

36.125 feet

297. A business has conducted an analysis of the pricing of their product and created the following revenue model: $R\left(n\right)=-100n^2+8000n-110000$, where n is the price of item and R is the total revenue generated. **Determine the maximum revenue and the best price.**

$R(n)=-100(n^2-80n+1600)+160000-110000$

$R(n)=-100(n-40)^2+50000$

the best price : 40

the maximum revenue : 50000

298. A twig is tossed upward into the air and follows the pathway $h=-t^2+4t+5$, where h is in feet and t is in seconds. **After it reaches its maximum height, how long will it take the projectile to hit the ground?**

$h=-(t^2-4t+4)+4+5$

$h=-(t-2)^2+9$

$\because h=0$

$(t-2)^2=9$

$t-2=\pm3$

$t=5$, $t=-1$ (can't work)

$\therefore t=5$

$5-2=3$ seconds.

Solving Quadratic Maximization Problems

General format to Quadratic Maximization Questions.

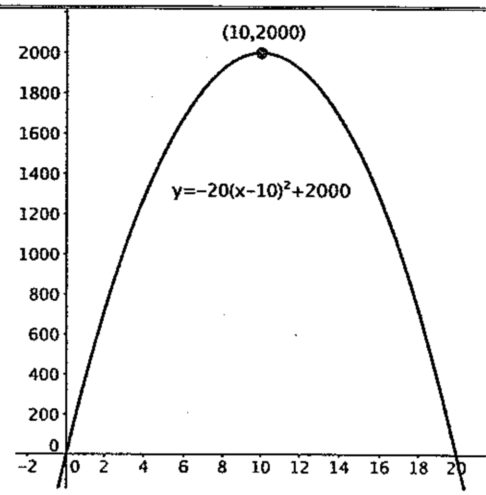
Let's suppose this was a quadratic word problem. It could be about maximizing area or revenue or maximizing anything else. Let's suppose that this word problem when translated takes the following form:

$y = -20x^2 + 40x$

Which can be turned into vertex form:

$y = -20(x-10)^2 + 2000$

299. What is the vertex? (10, 2000)



There two main types of questions asked in Max/Min word problems. In both cases, the vertex is needed to solve the problem

Type 1:	Type 2:
300. What is the maximum value? 2000 301. What is the maximum area? 2000 302. What is the maximum revenue? \$2000	304. What x-value creates the maximum value? 10 305. What side length maximizes the area? 10 306. What incremental change creates the maximum revenue? 10
303. Are type 1 questions asking for the P or Q value? Q	307. Are type 2 questions asking for the P or Q value? P

Challenge #:

308. What is the maximum rectangular area that can be enclosed using 40m of fencing? What are the dimensions that create the maximum area?

$P = 2l + 2w$

$~~20~~\ 40 = 2l + 2w$

$l = 20 - w$

$A = lw = (20-w)w = 20w - w^2$

$A = -(w^2 - 20w + 100) + 100$

$= -(w-10)^2 + 100$

The maximum area is $100\ m^2$, when w is 10m

$\therefore\ w = 10\ m,\ l = 10\ m.$

Strategy pointers

- Draw a picture
- Define variables
- Determine equation
- Complete the square
- State vertex
- Interpret vertex
- Answer original question

Maximizing area

309\. What is the maximum rectangular area that can be enclosed using 40m of fencing? What are the dimensions that create the maximum area?

Solution:

$$P=2l+2w$$
$$40=2l+2w$$

Isolate one of the variables.

$$l=20-w$$

$Max\ Area=lw$

Substitute $(20-w)$ for l.

$$Max=(20-w)w=-w^2+20w$$
$$Max=-(w^2-20w)$$

Complete the square.

$$Max=-(w^2-20w+100-100)$$
$$Max=-(w^2-20w+100)+100$$
$$Max=-(w-10)^2+100$$

The vertex is $(10,100)$

The maximum area $100m^2$.
The best dimensions happen when $w=10$
The length can be found by substituting w=10 into one the perimeter equations.

The best dimensions are 10m by 10m.

310\. You have a 500-foot roll of fencing and a large field. You want to construct a rectangular playground area. What are the dimensions of the largest such yard? What is the largest area?

$P=2l+2W$

$500=2l+2W$

$l=250-W$

$A=lw=(250-W)W$
$=250W-W^2$

$A=-(W^2-250W+15625)+15625$

$=-(W-125)^2+15625$

The max area is $15625\ feet^2$
when w is 125 feet
$\therefore W=125$ feet, $l=125$ feet.

311\. A rectangular field beside a river is to be fenced by 80 meters of fencing. No fence is needed along the riverbank. What are the dimensions of the field that maximize its area?

$80=2W+l$

$W=40-\frac{1}{2}l$

$A=lw=-\frac{1}{2}l^2+40l$

$=-\frac{1}{2}(l^2-80l+1600)+800$

$=-\frac{1}{2}(l-40)^2+800$

The max area is $800\ m^2$, when l is 40 m.

$W=40-\frac{1}{2}\times 40=20m$

312\. A fence is to enclose a field and divide it into 3 equal areas. If 1200 m of fencing is available, find the dimensions of that maximizes the field's area.

$1200=2l+4W$

$l=600-2W$

$A=lw=(600-2W)\cdot W$

$=600W-2W^2$

$=-2(W^2-300W+22500)+45000$

$=-2(W-150)^2+45000$

The max area is $45000\ m^2$, when W is 150 m

$l=600-2\times150=300\ m$

Challenge #:

313\. Jerry runs a solar battery Kiosk at the mall. Each battery is sold for \$10. At this price his kiosk usually sells about 100 batteries per day. A market research company determined that he can increase his revenue by increasing the price. For every \$2.50 price increase, 10 less batteries will be sold per day. Determine the price that maximizes the revenue.

$\$10\times100=\$1000$

$R=(10+2.50X)(100-10X)$
$=1000+150X-25X^2$

$R=-25(X^2-6X+9)+225+1000=-25(X-3)^2+1225$

Strategy pointers

- Use the revenue formula
- Define variables
- Determine equation
- Complete the square
- State vertex
- Interpret vertex
- Answer original question

When $x=3$, it reaches the max revenue

Price will be: $10+2.50\times3=17.50$

Maximizing Revenue

314. Jerry runs a solar battery Kiosk at the mall. Each battery is sold for $10. At this price his kiosk usually sells about 100 batteries per day. A market research company determined that he can increase his revenue by increasing the price. For every $2.50 price increase, 10 less batteries will be sold per day. Determine the price that maximizes the revenue.

Solution:

$Revenue = Price \times Quantity$

$Revenue = \$10 \times 100 = \$1000$

- $2.50 increase equals 10 less sails.

$(\$10,100)$

$(\$12.50,90)$

$(\$15,80)$

$(\$17.50,70)$

$(\$10+2.5x, 100-10x)$

New revenue formula

$R = (\$10 + increase) \times (100 - sales)$

Expand and simplify:

$R = (\$10 + \$2.5x) \times (100 - 10x)$

$R = 1000 - 100x + 250x - 25x^2$

$R = -25x^2 + 150x + 1000$

Complete the square

$R = -25(x^2 - 6x) + 1000$

$R = -25(x^2 - 6x + 9 - 9) + 1000$

$R = -25(x-3)^2 - 25(-9) + 1000$

$R = -25(x-3)^2 + 225 + 1000$

$R = -25(x-3)^2 + 1225$

The vertex is $(3,1225)$.

The maximum revenue is $1250.

The best price occurs when x=3.

$Price = (\$10 + \$2.50x)$

$Price = (\$10 + \$2.50 \times 3)$

$Price = \$17.50$

The best price is $17.50.

315. A sportswear store sells caps. Last year they sold 600 caps at $15 each. The store manager is planning to increase the price. A consumer survey shows that for every $1 increased, there will be a drop of 30 sales a year. Determine the best price and the maximum revenue.

R = Price x Quantity.

$R = (15 + 1x)(600 - 30x)$

$R = 9000 + 150x - 30x^2$

$= -30(x^2 - 5x + 6.25) + 187.5 + 9000$

$= -30(x - 2.5)^2 + 9187.5$

when $x = 2.5$, it reaches max R.

Price will be $15 + 2.5 \times 1 = \$17.5$

Max R will be $9187.5

316. Calculators are sold to students for 20 dollars each. Three hundred students are willing to buy them at that price. For every 5 dollar increase in price, there are 30 fewer students willing to buy the calculator. What selling price will maximize the revenue?

$R = (20 + 5x)(300 - 30x)$

$R = 6000 + 900x - 150x^2$

$= -150(x^2 - 6x + 9) + 1350 + 6000$

$= -150(x-3)^2 + 7350$

Price : $20 + 5 \times 3 = \$35$

317. A magazine has a circulation of 140 000 per month when they charge $2.50 for a magazine. For each $.10 increase in price, 5 thousand sales are lost. How much should be charged per magazine to maximize the revenue?

$R = (2.50 + 0.10x)(140000 - 5000x)$

$R = 350000 + 1500x - 500x^2$

$= -500(x^2 - 3x + 2.25) + 1125 + 350000$

$= -500(x - 1.5)^2 + 351125$

Price : $2.50 + 0.10 \times 1.5 = \$2.65$

Maximum and Minimum word Problems

Challenge:

318. Two numbers have a sum of 10. Their product is a Maximum. Find the two numbers.

$x+y=10$

$x=10-y$

$max = xy$

$= 10y-y^2$

$max = -(y^2-10y+25)+25$

$= -(y-5)^2+25$

$\therefore y=5,\ x=5$

319. Two numbers have a sum of 12. The sum of their squares is a minimum. Find the two numbers.

$x+y=12$

$x=12-y$

$min = y^2+(12-y)^2$

$= y^2+144-24y+y^2$

$= 2y^2-24y+144$

$min = 2(y^2-12y+36)-72+144$

$= 2(y-6)^2+72$

$\therefore y=6,\ x=6$

320. Two numbers have a difference of 6. The sum of their sum and their product is minimum. Find the two numbers.

$x-y=6$

$x=6+y$

$min = 6+y+y+(6+y)y$

$= 6+8y+y^2$

$min = (y^2+8y+16)-16+6$

$= (y+4)^2-10$

$\therefore y=-4,\ x=2$

---

321. Two numbers have a sum of 10. Their product is a Maximum. Find the two numbers.

Two numbers have a sum of ten.

$A+B=10 \rightarrow A=10-B$

There product is a maximum

$max = AB$

Sub $10-B$ in for A.

$max = (10-B)B$

Expand and rearrange.

$max = -B^2+10B$

Complete the Square.

$max = -(B-5)^2+25$

State Vertex.

5,25

Interpret Vertex.

When B=5 the maximum is 25.

B=5 & **A=5** since A=10-B

322. Two numbers have a sum of 12. The sum of their squares is a minimum. Find the two numbers.

Two numbers have a sum of twelve.

$A+B=12 \rightarrow A=12-B$

The sum of their squares is a minimum. $min = A^2+B^2$

Sub $12-B$ in for A.

$min = (12-B)^2+B^2$

Expand and rearrange.

$min = (144-24B+B^2)+B^2$

$min = 2B^2-24B+144$

Complete the Square.

$min = 2(B-6)^2+72$

State Vertex.

6,72

Interpret Vertex

When B=6 the minimum is 72.

B=6 & A=6 since A=12-B

323. Two numbers have a difference of 6. The sum of their sum and their product is minimum. Find the two numbers.

Two numbers have a difference of six.

$A-B=6 \rightarrow A=6+B$

The sum of their sum and their product is minimum.

$min = A+B+AB$

Sub $6+B$ in for A.

$min = (6+B)+B+(6+B)B$

Expand and rearrange.

$min = 6+2B+6B+B^2$

$min = B^2+8B+6$

Complete the Square.

$min = (B+4)^2-10$

State Vertex.

$-4,-10$

Interpret Vertex.

When B=-4 the minimum is -10.

B=-4 & A=2 since A=6+B

324. Two numbers have a difference of 14. Their product is a minimum. Find the two numbers.

$x-y=14$

$x=14+y$

$\min=(14+y)y$

$=14y+y^2$

$\min=(y^2+14y+49)-49$

$=(y+7)^2-49$

$\therefore y=-7, x=7$

325. Two numbers have a difference of 12. The sum of their squares is a minimum. Find the two numbers.

$x-y=12$

$x=12+y$

$\min=(12+y)^2+y^2$

$=144+24y+y^2+y^2$

$\min=2(y^2+12y+36)-72+144$

$=2(y+6)^2+72$

$\therefore y=-6, x=6$

326. Two numbers have a difference of 8. The result of adding their sum and their product is minimum. Find the two numbers.

$x-y=8$

$x=8+y$

$\min=8+y+y+(8+y)y$

$=8+2y+8y+y^2$

$\min=(y^2+10y+25)-25+8$

$=(y+5)^2-17$

$\therefore y=-5, x=3$

327. A rectangular area is enclosed by a fence and divided by another section of fence parallel to two of its sides. You have 600 m of fence to enclose a maximum total area. What are the dimensions of the enclosure?

$600=2l+3w$

$l=300-\frac{3}{2}w$

$\text{max area}=wl=(300-\frac{3}{2}w)w$

$=300w-\frac{3}{2}w^2$

$A=-\frac{3}{2}(w^2-200w+10000)+15000$

$=-\frac{3}{2}(w-100)^2+15000$

$\therefore w=100\text{m}, l=150\text{m}$

328. A large automotive shop sells 300 packages of spark plugs per week for \$6.40 each. For each 10¢ they reduce the price per package, they sell 5 more packages. What price will maximize the revenue?

$R=(6.40-0.10x)(300+5x)$

$R=1920+2x-0.5x^2$

$=-0.5(x^2-4x+4)+2+1920$

$=-0.5(x-2)^2+1922$

Price: $6.40-0.10\times2=\$6.20$

329. The sum of 2 numbers is 16. The sum of their squares is a minimum. Determine the 2 numbers.

$x+y=16$

$x=16-y$

$\min=(16-y)^2+y^2$

$=256-32y+y^2+y^2$

$\min=2(y^2-16y+64)-128+256$

$=2(y-8)^2+128$

$\therefore y=8, x=8$

330. If a new car dealership sets the price of their cars at $28 000 they will sell 54 cars. Every time they drop the price $1000, 2 more cars will be sold. What should the price of the cars be set at to maximize sales?

$R=(28000-1000x)(54+2x)$

$R=1512000+2000x-2000x^2$

$=-2000(x^2-x+0.25)+500+1512000$

$=-2000(x-0.5)^2+1512500$

Price: $28000-1000\times 0.5=\$27500$

331. Joe has 1200 feet of fencing to enclose a large field. He wants to split the rectangular enclosure in half with fencing parallel to two of the other sides. What are the dimensions of the largest enclosure?

$1200=2l+3w$

$l=600-\frac{3}{2}w$

$A=wl=(600-\frac{3}{2}w)w$

$=600w-\frac{3}{2}w^2$

$A=-\frac{3}{2}(w^2-400w+40000)+60000$

$=-\frac{3}{2}(w-200)^2+60000$

$\therefore w=200$ ft $l=300$ feet

332. The sum of two numbers is 60. Their product is a maximum. Determine the two numbers.

$x+y=60$, $x=60-y$

$\max=(60-y)y=60y-y^2$

$\max=-(y^2-60y+900)+900$

$=-(y-30)^2+900$

$\therefore y=30$, $x=30$

333. A rancher has 400 feet of fencing to enclose three adjacent rectangular corrals of equal lengths and widths. What is the maximum area that can be enclosed in the fencing?

$400=2l+4w$

$l=200-2w$

$A=wl=(200-2w)w$

$=200w-2w^2$

$A=-2(w^2-100w+2500)+5000$

$=-2(w-50)^2+5000$

$\therefore w=50$ ft $l=100$ feet

maximum area $=5000$ feet^2

334. Jordan runs a kayak-rental business on a small river in Parksville. She currently charges $12 per kayak and averages 36 rentals a day. An industry journal says that, for every fifty-cent increase in rental price, the average business can expect to lose two rentals a day. Use this information to attempt to maximize her income. What should she charge?

$R=(12+0.50x)(36-2x)$

$R=432-6x-x^2$

$=-(x^2+6x+9)+9+432$

$=-(x+3)^2+441$

Price: $12+0.50\times -3=\$10.50$

335. Two numbers have a difference of 16. The result of adding their sum and their product is a minimum. Determine the numbers.

$x-y=16$

$x=16+y$

$\min=16+y+y+(16+y)y$

$=16+2y+16y+y^2$

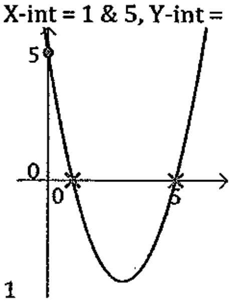
$\min=(y^2+18y+81)-81+16$

$=(y+9)^2-65$

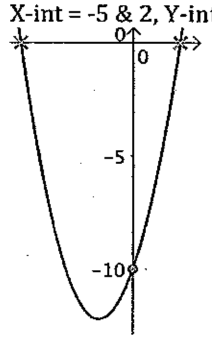
$\therefore y=-9$, $x=7$

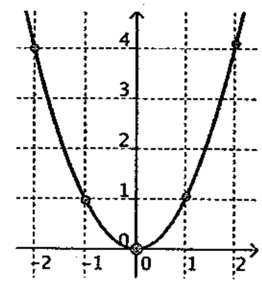
Quadratic Answers

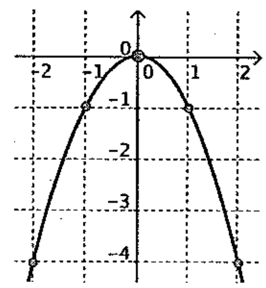
1. 0
2. +
3. X
4. 4 Feet
5. 55 Feet
6. 12 Feet
7. Answered on page
8. Answered on page
9. Answered on page
10. Answered on page
11. Answered on page
12. Answered on page
13. (-3,-2)
14. minimum
15. up
16. x=-3
17. Real
18. $y \geq -2$
19. (2,5)
20. maximum
21. down
22. x=2
23. Real
24. $y \leq 5$
25. The x-coordinate of the vertex is the axis of symmetry.
26. The y-coordinate is either the minimum value or the maximum value.
27. B & E
28. A, B, C, F & G
29. Symmetric about an axis of symmetry. Can open up or down. It is approximately "U" shaped.
30. No term has a degree higher than 2. i.e.
31. (1,1)
32. x=1
33. Max
34. (0,2)
35. Real
36. $y \geq -2$
37. (3,5)
38. $y \leq 5$
39. Domain
40. (3,-4)
41. 1 & 5
42. $y \geq -4$
43. (-2,-1)
44. x=-2
45. Real
46. Answered on the next page.
47. Answered on the page
48. X-int = 1 & 5, Y-int =

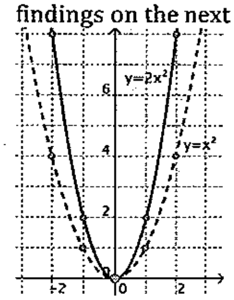


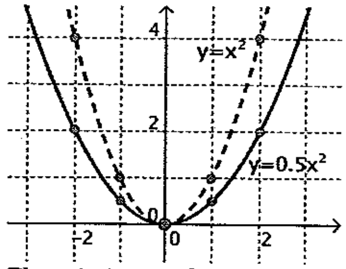
49. X-int = -5 & 2, Y-int = -10

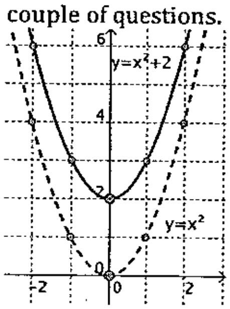


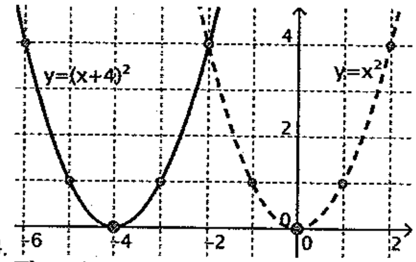
50. 6 seconds
51. Answered page
52. 5 seconds
53. (1,128), (2,144),(3,128) & the maximum is 144.
54. 5 feet
55. (1,19), (2,19), It will take 1.5 seconds because it is halve way between 1 and 2. Both 1 and 2 have the same height.
56. 6 seconds
57. True. The vertex is in the middle and the axis of symmetry always goes through the middle.
58. False. A maximum is at the top of the parabola that opens down.
59. True. There are no restrictions on the domain.
60. 

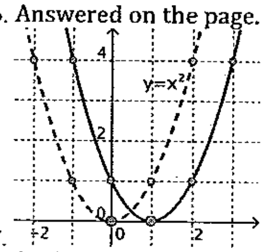
61. 

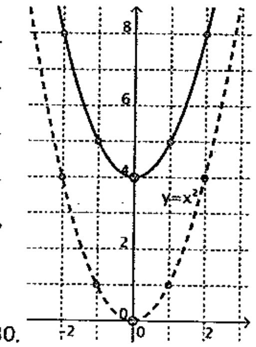
62. The table of values is correct
63. The values are -9, -4, -1, 0, -1, -4, -9
64. (0,0)
65. Min
66. X=0
67. Real
68. $y \geq 0$
69. (0,0)
70. Max
71. X=0
72. Real
73. $y \leq 0$
74. Reflected over the x-axis
75. Confirm with your findings on the next page
76. Confirm with your findings on the next page
77. 

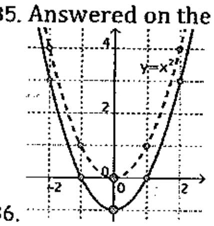
78. 

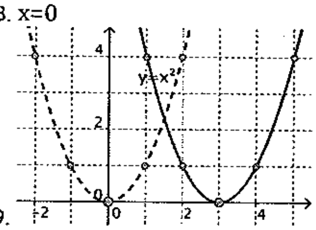
79. The missing y-values are 0, 2, 8 & 18
80. The missing y-values are 0, 0.5, 2 & 4.5
81. (0,0)
82. Min
83. (0,0)
84. Min
85. B
86. C
87. A
88. 8
89. 12
90. 2
91. -4
92. The a-value multiplies the y-value.
93. A,B,C
94. C,A,B
95. C,B,A
96. $y=-3x^2$
97. $y=7x^2$
98. $y=\frac{2}{3}x^2$
99. There are lots of possibilities.
100. Yes, but the spread is very small.
101. Confirm your guess with your findings in the next couple of questions.
102. Confirm your guess with your findings in the next couple of questions.
103. 

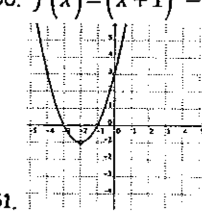
104. 

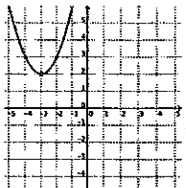
105. The missing y-values are 2, 3, 6 & 11.
106. The missing y-values are 0, 1, 4 & 9.
107. (0,2)
108. x=0
109. (-4,0)
110. x=-4
111. Up
112. Down
113. Right
114. Left
115. True. The a-value is the same in both equations.
116. True. All parabolas of the form $y=a(x-p)^2+q$ have a domain of real values.
117. True. There is no horizontal translation so the axis of symmetry is x=0 for both parabolas.
118. Answered on the next page.
119. Answered on the next page.
120. Answered on the next page.
121. Answered on the next page.
122. Answered on the next page.
123. Answered on the next page.
124. Answered on the page.
125. Answered on the page.
126. Answered on the page.
127. 

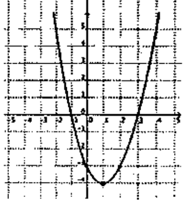
128. (1,0)
129. x=1
130. 

131. (0,4)
132. $y \geq 4$
133. Answered on the page.
134. Answered on the page.
135. Answered on the page.
136. 

137. (0,-1)
138. x=0
139. 

140. (3,0)
141. $y \geq 0$
142. A,B,C
143. C,B,A
144. A,B,C
145. $f(x)=(x-2)^2-5$
146. $f(x)=(x+6)^2-4$
147. $f(x)=(x-3)^2+1$
148. $f(x)=(x-1)^2+1$, (1,1)
149. $f(x)=-(x+1)^2+3$, (-1,3)
150. $f(x)=(x+1)^2-2$, (-1,-2)
151. 

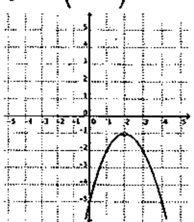
152.


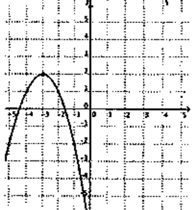
153.


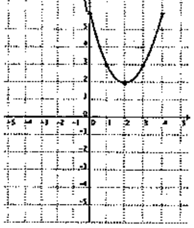
154. $y=-(x-3)^2+5$, x=3

155. $y=(x-3)^2-4$, $y\geq-4$

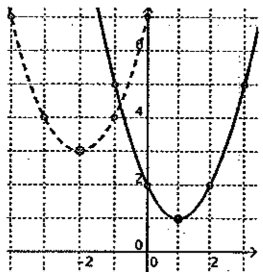
156. $y=-(x+2)^2-1$, Max

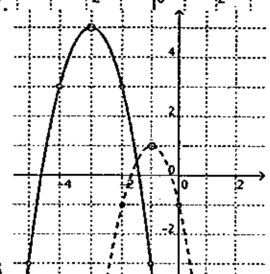
157.


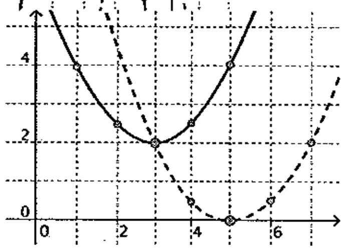
158.


159.


160. y-int=-3, x-int= -1 & -5
161. y-int=-3, x-int= -1 & 3
162. y-int=-16, x-int= 1.2 &-3.2
163. y-int=-6, x-int= 3.6 & -1.6
164. y-int=25, x-int= 5
165. y-int=9, x-int=0.6 & -4.6
166. False. It is the absolute value of the a-value that makes the parabola congruent not the sign of the a-value.

167.


168.


169.


170. A,C
171. A
172. A, B & C

173. $y=3(x-3)^2+4$

174. $y=2(x-7)^2+1$

175. Answered on the page.

176. $y=-\frac{1}{8}(x-2)^2+3$

177. $y=-\frac{1}{2}(x+2)^2+5$

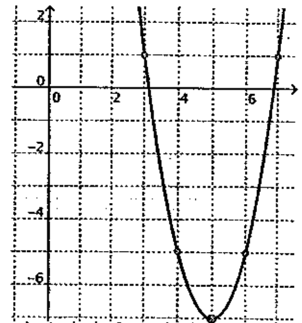
178. Answered on the page.

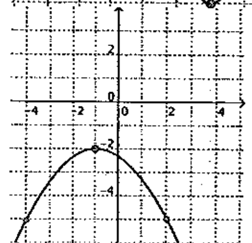
179. $y=-3(x+2)^2+3$

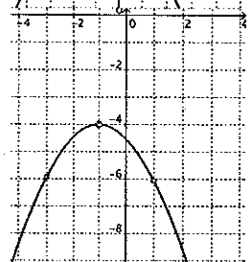
180. $y=-\frac{2}{3}(x-8)^2-2$

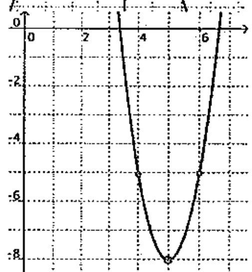
181. Answered on the page.

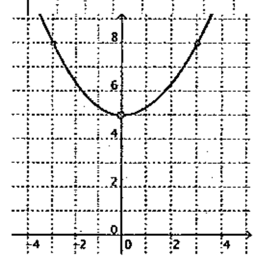
182.


183.


184.


185.


186.


187.


188. $y=2(x-1)^2-7$

189. $y=\frac{1}{2}(x-3)^2+2$

190. $y=-2(x+3)^2+5$

191. $y=3x^2-9$

192. $y=2(x+2)^2-1$

193. $y=-\frac{1}{3}(x-2)^2+6$
194. (10,11), x=10, min (11), real, $y\geq 11$,no
195. (-7,-11), x=-7, max (-11), real, $y\leq -11$,no
196. (-21,-1), x=-21, min (-1), real, $y\geq -1$, yes
197. (7,9), x=7, max (9), real, $y\leq 9$,yes
198. 2 because it opens up and the minimum value is below the x-axis.
199. $y=-\frac{1}{2}(x-4)^2+5$
200. Answered on the page.
201. 2 because it opens down and the maximum value is above the x-axis. $(a\times q=+)$
202. 0 because it opens down and the maximum value is below the x-axis. $(a\times q=0)$
203. 1 the vertex is on the x-axis. $(a\times q=-)$
204. 0 because it opens down and the maximum value is below the x-axis. $(a\times q=0)$
205. 2 because it opens down and the minimum value is below the x-axis. $(a\times q=+)$
206. Answered on the page.
207. $y=\frac{9}{4}(x-3)^2-2$
208. $y=\frac{1}{18}(x+4)^2+3$
209. 0, $(a\times q=-)$, The signs of the a and q are different.
210. 2, $(a\times q=+)$, The signs of the a and q are the same.
211. 1, $(a\times q=0)$, The q-value is zero.
212. 2, $(a\times q=+)$, The signs of the a and q are the same.
213. 1, $(a\times q=0)$, The q-value is zero.
214. 0, $(a\times q=-)$, The signs of the a and q are different.
215. Vertical compression by $\frac{1}{2}$, horizontal translation 2 left, vertical translation 3 down.
216. Reflection over the x-axis, Vertical expansion by 3, horizontal translation 1 right, vertical translation 2 up.
217. Vertical compression by $\frac{1}{4}$, horizontal translation 1 left, vertical translation 1 up.
218. $y=-\frac{7}{100}(x-10)^2+5$
219. $y=1(x-2)^2-1$
220. $y=-1(x-5)^2+2$
221. -7, set x=0 to solve.
222. Cannot answer by observation.
223. Up, the a-value is positive
224. Minimum, the a-value is positive
225. Cannot answer by observation.
226. Expansion, $a>0$
227. Real
228. Cannot answer by observation.
229. B,E
230. C,D,E
231. D
232. C,D
233. A,B,C
234. B,C,D
235. C,A,D,B
236. The missing y-valuesare 1125, 1200, 1225,1200,1125,1000
237. $17.50
238. 70
239. $1225
240. Current revenue is $150000/day, New revenue is $183750/Day. The increased revenue per day is $33750.
241. True
242. True
243. True
244. $y=(x+1)^2$, (-1,0)
245. $y=(x-4)^2$, (4,0)
246. $y=(x+9)^2$, (-9,0)
247. $y=(x^2+8x+16)-16$
 $y=(x+4)^2-16$
248. $y=(x^2+14x+49)-49$
 $y=(x+7)^2-49$
249. $y=(x^2-6x+9)-9$
 $y=(x-3)^2-9$
250. Answered on the next page.
251. Answered on the page.
252. y=(x+4)^2-9 (-4,-9)
253. y=(x+5)^2-15, (-5,-15)
254. y=(x-2)^2-3, (2,-3)
255. y=(x+10)^2-106, (-10,-106)
256. y=(x+6)^2-45, (-6,-45)
257. y=(x+1.5)^2-2.25, (-1.5,-2.25)
258. y=(x-4.5)^2-20.25, (4.5,-20.25)
259. y=(x+7)^2-59, (-7,-59)
260. minimum value is -62
261. minimum value is -44
262. (2.5, 7.75)
263. (-8,-71)
264. $y\geq 7.75$ (4.5,-0.25)
265. $y\geq -0.25$
266. Answered on the next page.
267. $y=4(x+1)^2-11$, (-1,-11)
268. $y=5(x-3)^2-1$, (3,-1)
269. $y=3(x-1)^2+2$, (1,2)

270. $y=-10(x+4)^2+153$, (4, 153)

271. $y=-5(x+6)^2+182$, (-6, 182)

272. $y=7(x-3)^2-66$, (3, -66)

273. (1,-19)
274. (10,2020)
275. (1,1.5)
276. (3,-2)
277. (5,38.5)
278. (4.5,-12.5)
279. (-1,1.25)
280. (-3,99)
281. (-2.5,3.25)

282. $y=-2(x+1)^2+3,\ y\le 3$

283. $y=\frac{1}{2}(x-4)^2-1$, x=4

284. $y=(x+12)^2-244$, -244

285. ii, $y=2(x+1)^2-7$

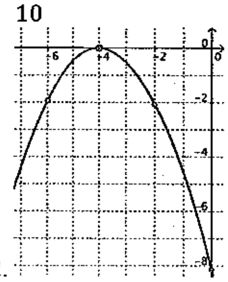
286. iii, $y=-(x+1)^2+5$

287. i, $y=-\frac{1}{3}(x-9)^2+26$

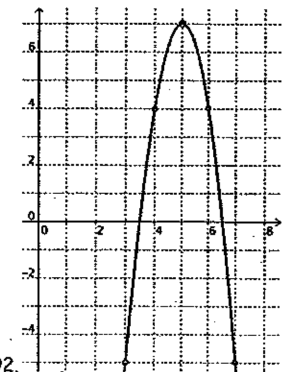
288. $y=-(x-2)^2+22,\ y\le 22$

289. $y=-\frac{1}{3}(x+12)^2+50$, (-12,50)

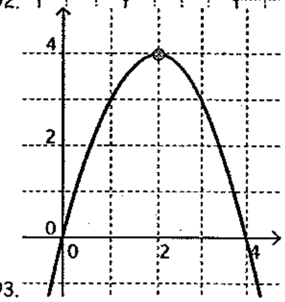
290. $y=-\frac{4}{5}(x+10)^2+73$, x=-10



291.



292.



293.

294. Answered on the next page
295. Answered on the page
296. The maximum height is 36.125 feet.
297. The maximum revenue is \$50000. The best price is \$40.
298. 3 seconds, $h=-(x-2)^2+9$ and the x-intercept is 5. The gap between 2 and 5 is 3.
299. (10,2000)
300. 2000
301. 2000
302. \$2000
303. Q
304. 10
305. 10
306. 10
307. P
308. Answered on the next page.
309. Answered on the page.
310. 125foot by 125 foot, 15625ft^2
311. 40m by 20m, maximum area is 800m^2
312. 150m by 300m, maximum area is 45000m^2
313. Answered on the next page.
314. Answered on the page.
315. The best price is \$17.50 and the maximum revenue is \$9187.50. (The best increment is 2.5.)

$$R=-30(x-2.5)^2+9187.5$$

316. \$35 is the best price. (The best increment is 3.)

$$R=-150(x-3)^2+7350$$

317. \$2.65 is the best price. (The best increment is 1.5),

$$R=-500(x-1.5)^2+351125$$

318. Answered on the page.
319. Answered on the page.
320. Answered on the page.
321. Answered on the page.
322. Answered on the page.
323. Answered on the page.
324. -7 and 7
325. -6 and 6
326. -5 and 3
327. 100m and 150m
328. \$6.20 creates the maximum revenue of \$1922.

$$R=-0.5(x-2)^2+1922$$

Best increment is 2.
329. 8 and 8
330. \$27500
331. 200m by 300m
332. 30 and 30
333. 5000ft^2
334. \$10.50
335. -9 and 7