| HW Mark: 10 9 8 7 6 RE-Submit |
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# Systems of Linear Equations

This booklet belongs to:________________Period____

| LESSON # | DATE | QUESTIONS FROM NOTES | Questions that I find difficult |
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| | | REVIEW | |
| | | **TEST** | |

Your teacher has important instructions for you to write down below.

Systems of Linear Equations

| STRAND Relations & Functions | | DAILY TOPIC | EXAMPLE |
|---|---|---|---|
| C9.<br><br>Solve problems that involve systems of linear equations in two variables, graphically and algebraically | 9.1 | Model a situation, using a system of linear equations. | |
| | 9.2 | Relate a system of linear equations to the context of a problem. | |
| | 9.3 | Determine and verify the solution of a system of linear equations graphically, with and without technology. | |
| | 9.4 | Explain the meaning of a point of intersection of a system of linear equations. | |
| | 9.5 | Determine and verify the solution of a system of linear equations algebraically. | |
| | 9.6 | Explain, using examples, why a system of equations may have no solution, one solution or an infinite number of solutions. | |
| | 9.7 | Explain a strategy to solve a system of linear equations. | |
| | 9.8 | Solve a problem that involves a system of linear equations. | |

**[C]** Communication **[PS]** Problem Solving, **[CN]** Connections **[R]** Reasoning, **[ME]** Mental Mathematics **[T]** Technology, and Estimation, **[V]** Visualization

 

Key Terms

| Term | Definition | Example |
|---|---|---|
| linear equation | | |
| system of linear equations | | |
| solution to a system | | |
| point of intersection | | |
| infinite solutions | | |
| one solution | | |
| no solutions | | |
| consistent | | |
| inconsistent | | |
| parallel | | |
| perpendicular | | |
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# Introduction: Systems of Linear Equations

## Challenge

Jazhon is considering two job offers. Concrete Emporium will pay Jazhon a base monthly salary of $500 plus a commission rate of 5% on all sales each month. All Things Cement offers him a job that pays straight salary, $2500 per month.

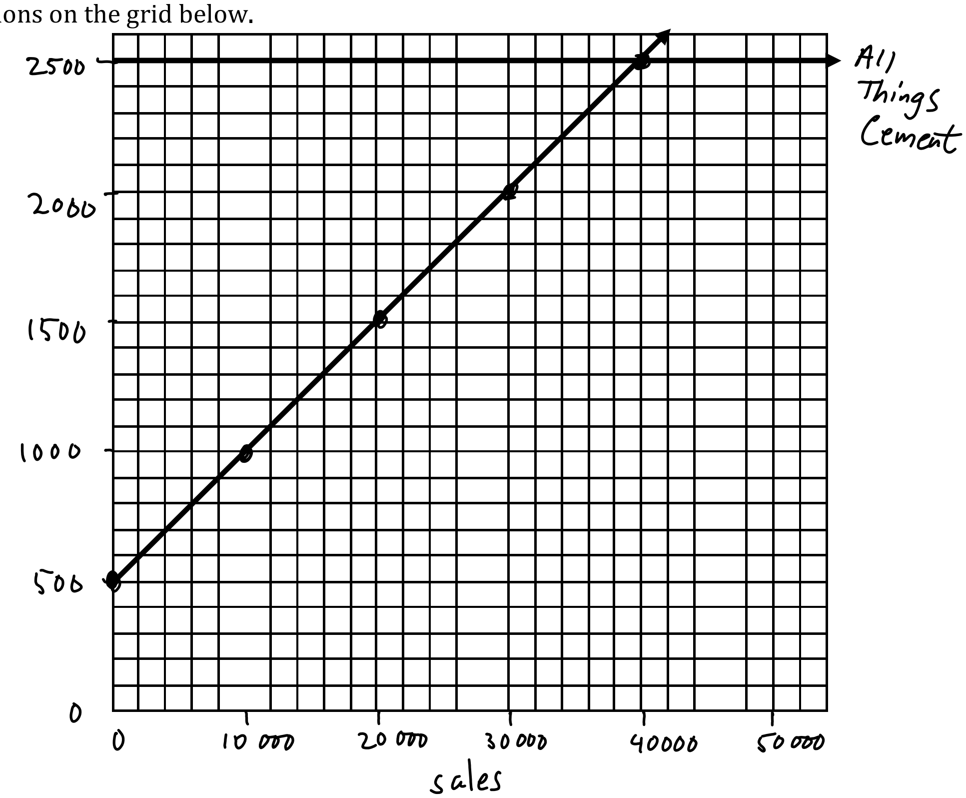
Jazhon wants to consider the two jobs mathematically before he makes his decision. He writes the following equations to represent each job offer.

Concrete Emporium: $E = 0.05s + 500$
All Things Cement: $E = 2500$

1. What does Jazhon need to consider before he can make an educated decision?

   Estimate or research possible sales.

2. Graph the two equations on the grid below.




3. What is the significance of the point where the two lines cross?

   Equal earnings

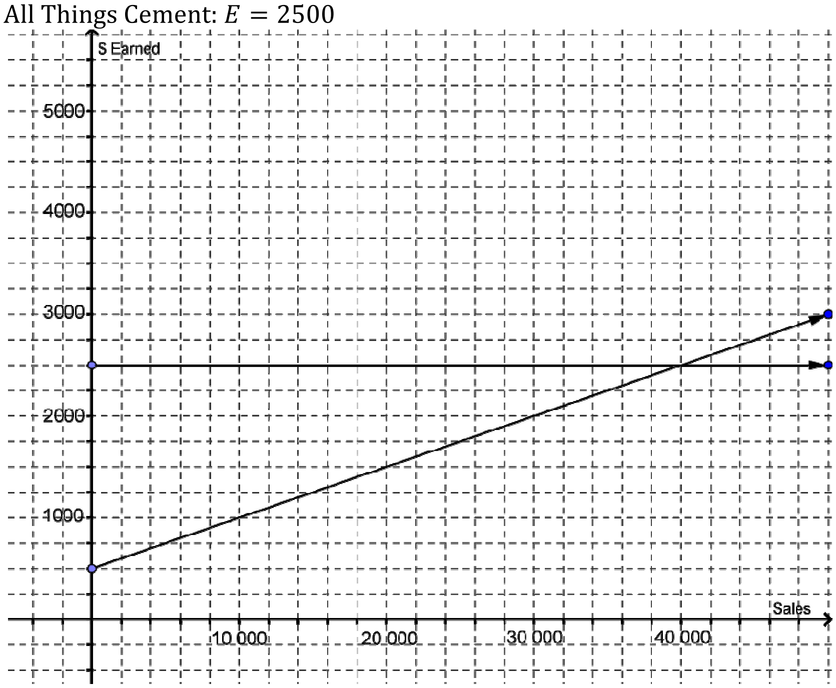
4. When does the job offered by Concrete Emporium pay more?

   Sales over $40 000

## Challenge

Concrete Emporium: $E = 0.05s + 500$
All Things Cement: $E = 2500$




Where the lines cross→ earnings are equal.

Concrete Emporium will pay more if Jazhon sells more than $40 000 worth of concrete.

We call the scenario to the left a **System of Linear Equations.**

The point (40000, 2500) is on both lines.

We say (40000, 2500) is the **solution to the system.**

That is…it is the point that **satisfies both equations.**

5. ## Challenge

Verify that (2,4) is a solution to the following system.

$x + y = 6$

$2x - y = 0$

$2 + 4 = 6$

$6 = 6$

$2(2) - 4 = 0$

$4 - 4 = 0$

$0 = 0$

YES

Explain your reasoning.

See if (2,4) satisfies both equations by substituting in for x and y.

Determine if the given point is a solution to the system of equations. Show your work.

6. Is (1,3) a solution to the following system?
$y = -2x + 5$
$y = x + 2$

3 = -2(1) + 5
3 = -2 + 5
3 = 3

3 = 1 + 2
3 = 3

YES

7. Is (-1,1) a solution to the following system? NO
$5x + 6y = 1$
$6x + 2y = -3$

5(-1) + 6(1) = 1
-5 + 6 = 1
1 = 1 ✓

6(-1) + 2(1) = -3
-6 + 2 = -3
-4 = -3 ✗

8. Is (2,1) a solution to the following system? YES
$x + 2y = 4$
$x - y = 1$

2 + 2(1) = 4
2 + 2 = 4
4 = 4 ✓

2 - 1 = 1
1 = 1 ✓

9. Is (3,3) a solution to the following system? YES
$3y = x + 6$
$3y = -4x + 21$

3(3) = 3 + 6
9 = 9 ✓

3(3) = -4(3) + 21
9 = -12 + 21
9 = 9 ✓

10. Is (1,2) a solution to the following system? YES
$2x + 2y = 6$
$y = 4x - 2$

2(1) + 2(2) = 6
2 + 4 = 6
6 = 6 ✓

2 = 4(1) - 2
2 = 4 - 2
2 = 2 ✓

11. Is (-1,1) a solution to the following system? NO
$7x = 3y + 10$
$6x + 5y = -1$

7(-1) = 3(1) + 10
-7 = 3 + 10 ✗

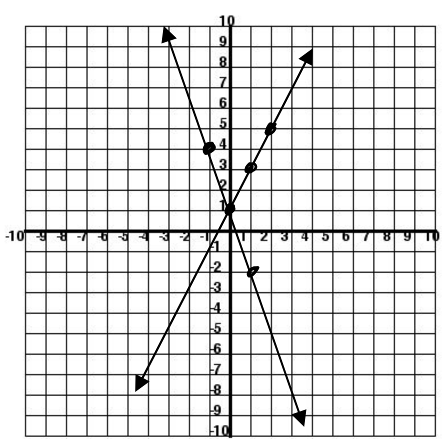
6(-1) + 5(1) = -1
-6 + 5 = -1 ✓

12. Explain how you can determine if a given point is the solution to a system of linear equations.

It must satisfy both equations.

## Challenge

13. Find the solution to the following system of equations.
$y = 2x + 1$
$y = -3x + 1$



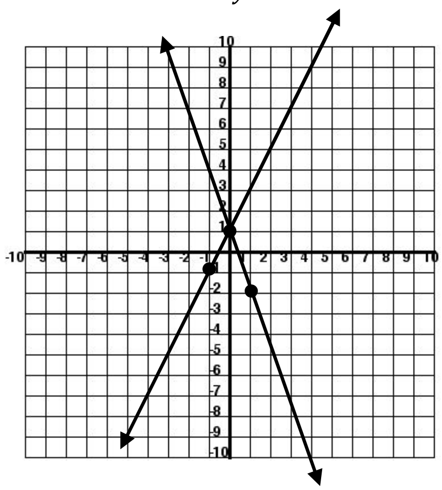
Explain your steps and/or thinking.

① Graph each line.
② Find point of intersection.

(0, 1)

Find the solution to the following system of equations.

$$y = 2x + 1$$
$$y = -3x + 1$$



Explain your steps and/or thinking.

I graphed each of the lines.

I found the coordinates of the point that is on both lines

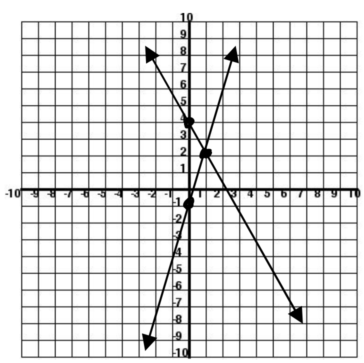
→ where the lines cross!

(0,1)

Solve the following systems by graphing:

14. Solve:

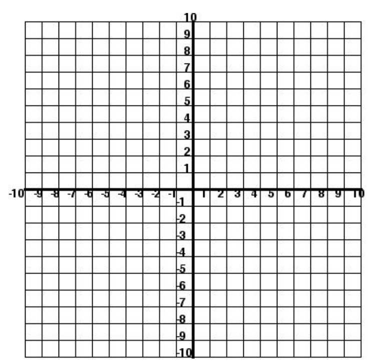
$$y = 3x - 1$$
$$y = -2x + 4$$



(1,2)

15. Solve:

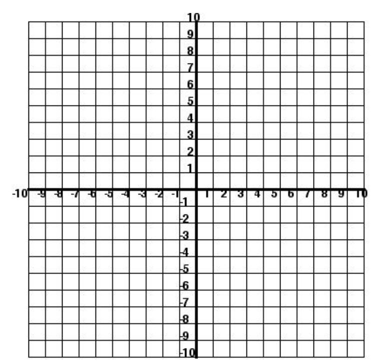
$$x - y = -2$$
$$4x + 2y = 16$$



(2,4)

16. Solve:

$$x + y = 5$$
$$3x - y = 3$$

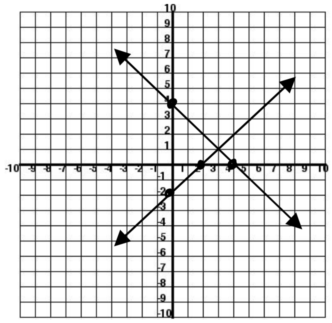


(2,3)

Solve the following systems by graphing:

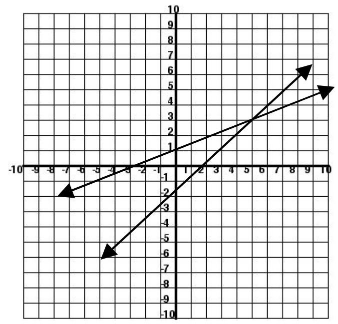
17. Solve: (3,1)

$x + y = 4$ and $x - y = 2$



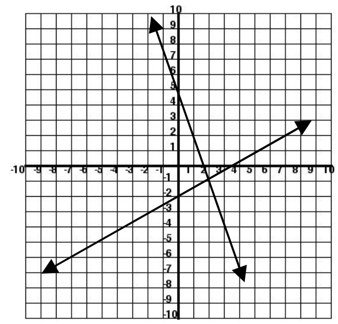
18. Solve: (5,3)

$y = x - 2$ and $y = \frac{2}{5}x + 1$



19. Solve: (2,-1)

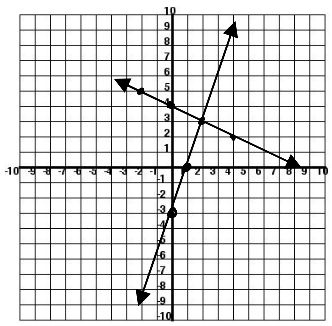
$y = -3x + 5$ and $x - 2y = 4$



20. Solve:

$y = -\frac{1}{2}x + 4$ $y = 3x - 3$

$x + 2y = 8$ and $3x - y = 3$

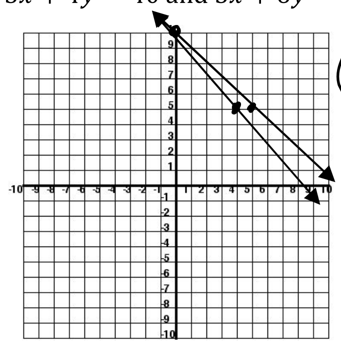


(2,3)

21. Solve:

$y = -\frac{5}{4}x + 10$ $y = \frac{-5}{6}x + 10$

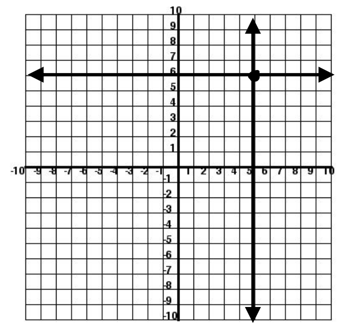
$5x + 4y = 40$ and $5x + 6y = 60$



(0,10)

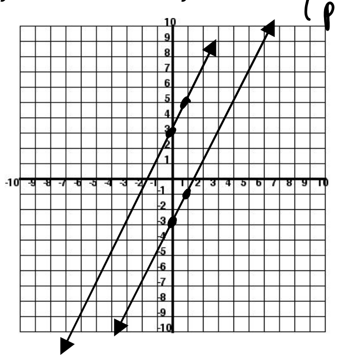
22. Solve:

$x = 5$ and $y + 4 = 10$



23. Solve: NO SOLUTION (parallel)

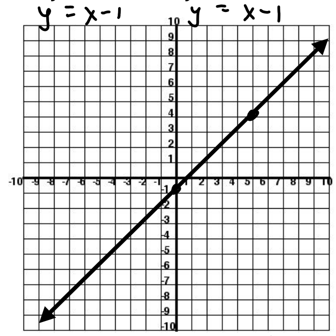
$y = 2x - 3$ and $y = 2x + 3$



24. Solve:

$x - y = 1$ and $3y = 3x - 3$

$y = x - 1$ $y = x - 1$



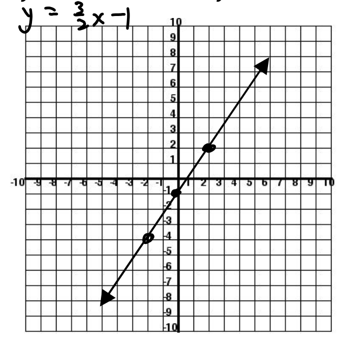
same line ∴ infinite solutions

25. Solve:

$y = \frac{3}{2}x - 1$

$2y = 3x - 2$ and $4y + 4 = 6x$

$y = \frac{3}{2}x - 1$



same line → infinite solutions

26. What do you notice about the equations above?

same slope, diff. y-int.

27. What do you notice about the equations above?

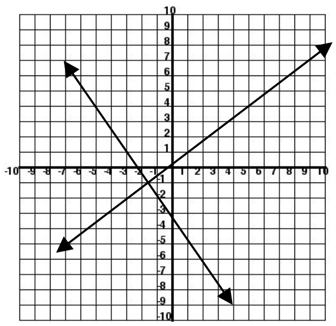
same slope & same y-int.

28. What do you notice about the equations above?

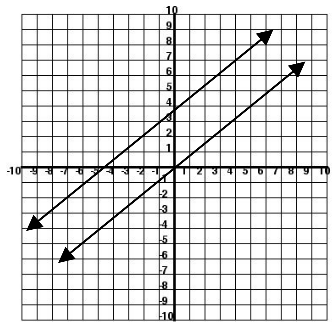
same slope same y-int.

29. ## Challenge

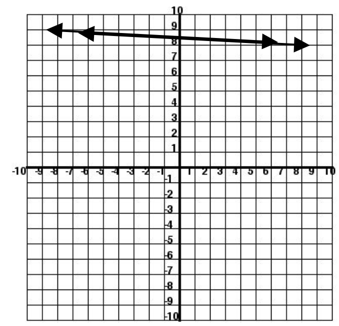
On the three graphs below, draw a system of linear equations with . . .



a) One solution



b) No solutions



c) Infinite Solutions

30. ## Challenge

How many solutions are there to the system

$y = 3x + 3$

$y = x + 1$

one

Explain your reasoning.

different slopes ∴ lines will intersect at one point.

| Types of Solution Sets: | | |
|---|---|---|
| **One solution** | **No Solutions** | **Infinite Solutions** |
| • Lines intersect once.<br>• Different Slopes. | • Parallel Lines<br>• Same Slopes<br>• Different y-intercepts | • Same Lines<br>• Same Slopes<br>• Same y-intercepts |
| We say the system is **CONSISTENT** | We say the system is **INCONSISTENT** (no solution) | We say the system is **CONSISTENT** |

Determine if the following systems have one solution, no solutions, or infinite solutions.

31. $y = 3x + 3$
$y = x + 1$

One solution because the slopes are different.

Lines will intersect once.

32. $y = 2x + 5$
$y = 3x - 5$

one

33. $3y = 9x + 12$
$3x - 9y = 12$

$y = 3x + 4$

$y = \frac{1}{3}x - \frac{4}{3}$

one

34. $6x + 4y = 1$
$3x - 2y = 4$

$y = \frac{-6}{4}x + \frac{1}{4}$

$y = \frac{3}{2}x - 2$

one

35. $2x + y = 5$
$y = -2x - 5$

$y = -2x + 5$

None

parallel

36. $y = \frac{2}{3}x + 5$
$3y = 2x - 5$

$y = \frac{2}{3}x - \frac{5}{3}$

NONE

parallel

Find the value of $k$ that makes each system **inconsistent**. parallel

37. $y = kx - 3$
$2y = 2x + 6$

$y = x + 3$

$kx = x$

$k = 1$

38. $2y = kx + 1 \Rightarrow y = \frac{k}{2}x + \frac{1}{2}$
$2x - y = 7 \quad y = 2x - 7$

$\frac{k}{2} = \frac{2}{1}$

$k = 4$

39. $4kx = y - 2 \quad y = 4kx + 2$
$5x + 3y - 12 = 0$

$y = \frac{-5}{3}x + 4$

$\frac{4k}{1} = \frac{-5}{3}$

$12k = -5$

$k = -\frac{5}{12}$

Find the value of $b$ that will produce a system with **infinite solutions**.

40. $y = x - b$
$2y = 2x - 4$

$y = x - 2$

$b = 2$

41. $3x - y = 7$
$4y = 12x + b$

$y = 3x + 7$

$y = 3x + \frac{b}{4}$

$7 = \frac{b}{4} \quad b = 28$

42. $2x + 3y - 2b = 0$
$y = -\frac{2}{3}x + 1$

$y = -\frac{2}{3}x - \frac{2b}{3}$

$\frac{-2b}{3} = 1$

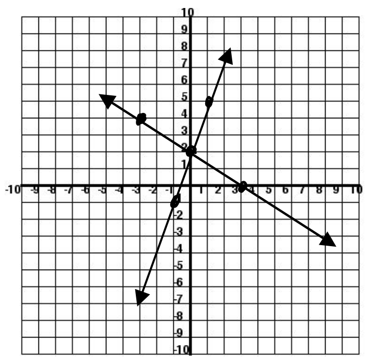
$-2b = 3$

$b = -\frac{3}{2}$

43. Solve:

$2x + 3y - 6 = 0$ $\quad y = -\frac{2}{3}x + 2$

$3x - y + 2 = 0$ $\quad y = 3x + 2$



$\boxed{(0,2)}$

44. The system above is

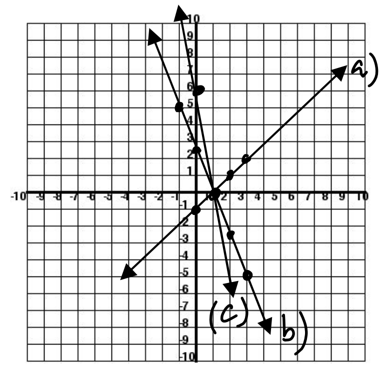
a) Consistent — has a solution

b) Inconsistent

45. Solve:

$x - y = 1$ $\quad$ a) $y = x - 1$

$5x + 2y = 5$ $\quad$ b) $y = -\frac{5}{2}x + \frac{5}{2}$




$\boxed{(1,0)}$

46. Add the two equations above and graph the new equation. $6x + y = 6$

(c) $y = -6x + 6$

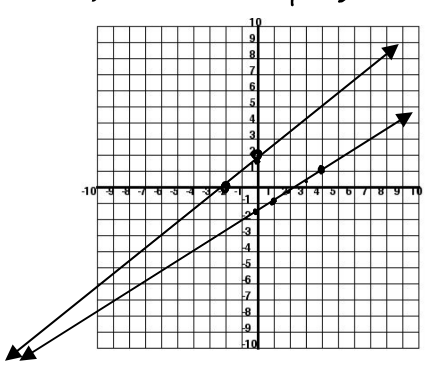
47. What do you notice?

It also passes through (1,0).

48. Graph the system of equations:

$y = x + 2$

$3y = 2x - 5$ $\quad y = \frac{2}{3}x - \frac{5}{3}$



49. What is the problem when solving this system by graphing?

Tough to accurately plot (quickly) and solution is off the provided graph.

50. Challenge

Solve the system of linear equations: $y = x + 2$ and $3y = 2x - 5$.

$y = x + 2$ } if $y$ is equal to $(x+2)$, I can substitute it into
$3y = 2x - 5$ the other equation.

$3(x+2) = 2x - 5$

$3x + 6 = 2x - 5$

$-2x - 6 \quad -2x - 6$

$x = -11$

if $x = -11$, I can find $y$.

$y = x + 2$

$y = -11 + 2$

$y = -9$

$\boxed{\therefore (-11, -9)}$

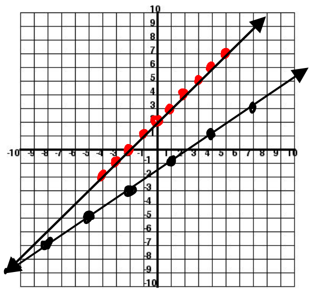
# Solving Systems of Equations (without graphing)

## Part 1: Solving By substitution.

Graph the system of equations:

$y = x + 2$

$3y = 2x - 5$



My thoughts…

If I graph each of these lines, I notice that they do not cross at a point that I can easily read on **this** graph.

Also, the second equation is not easily graphed.

I can use a different method.

**Algebra**! See My Solution Below.

51. What is the solution to a system of linear equations?

The point that satisfies both equations.

52. If a point is present on two lines, what values of that point are equal:

 a. x-values

 b. y-values

 c. both x- and y-values

Solve the system of equations:

"1" $y = x + 2$

"2" $3y = 2x - 5$

I will substitute $(x+2)$ in to equation "2" for $y$.

$3(x+2) = 2x - 5$

$3x+6 = 2x - 5$

$x = -11$

Then substitute $x = -11$ into equation "1".

$y = (-11) + 2$

$y = -9$

Therefore the solution is $(-11, -9)$

53. Solve the following system of equation without graphing, consider the answers to the previous questions to guide you.

$$y = 2x - 1$$
$$y = -x + 1$$

$$2x - 1 = -x + 1$$
$$+x \quad +1 \qquad +x \quad +1$$
$$\frac{3x}{3} = \frac{2}{3}$$
$$x = \frac{2}{3}$$
$$\therefore y = -\left(\frac{2}{3}\right) + 1$$
$$= -\frac{2}{3} + \frac{3}{3}$$
$$= \frac{1}{3}$$

$$\left(\frac{2}{3}, \frac{1}{3}\right)$$

54. Verify your solution above.

$$2\left(\frac{2}{3}\right) - 1 = \frac{1}{3}$$
$$\frac{4}{3} - 1 = \frac{1}{3}$$
$$\frac{4}{3} - \frac{3}{3} = \frac{1}{3}$$
$$\frac{1}{3} = \frac{1}{3}$$
✓

$$\frac{1}{3} = -\frac{2}{3} + 1$$
$$\frac{1}{3} = \frac{1}{3}$$
✓

YES, it satisfies both equations.

Solve the following systems of equations **by substitution**.

55. Solve.

$y = 2x - 1$

$y = -x + 1$

Since both $(2x - 1)$ and $(-x + 1)$ are equal to 'y', then they must be equal to each other.

$2x - 1 = -x + 1$

$3x = 2$

$x = \frac{2}{3}$

To find 'y', substitute your known 'x' into either equation.

$$y = -\left(\frac{2}{3}\right) + 1$$

$$y = \frac{1}{3}$$

Solution $\left(\frac{2}{3}, \frac{1}{3}\right)$

56. How can I check the solution to the right?

Substitute $\left(\frac{2}{3}, \frac{1}{3}\right)$ into
x , y
both equations to see if the point satisfies the equations

57. Check the solution to the right.

see previous page

58. Solve.

$3x + y = 1$  $y = -3x + 1$  $\left(-\frac{8}{7}, \frac{31}{7}\right)$

$2x + 3y = 11$

$2x + 3(-3x + 1) = 11$

$2x - 9x + 3 = 11$

$-7x = 8$

$x = -\frac{8}{7}$

$\therefore y = -3\left(-\frac{8}{7}\right) + 1$

$= \frac{24}{7} + 1$

$= \frac{24}{7} + \frac{7}{7}$

$= \frac{31}{7}$

59. Solve.

$a + c = 9$  $c = 9 - a$

$2a + c = 11$

$2a + 9 - a = 11$

$a + 9 = 11$

$a = 2$

$\therefore c = 9 - 2$

$= 7$

$(a, c)$

$(2, 7)$

60. Solve.

$3x - 4y = -15$

$5x + y = -2$  $y = -5x - 2$

$3x - 4(-5x - 2) = -15$

$3x + 20x + 8 = -15$

$23x = -23$

$x = -1$

$\therefore y = -5(-1) - 2$

$5 - 2$

$= 3$

$(x, y)$

$(-1, 3)$

61. Solve.

$d + e = 1$  $e = 1 - d$

$3d - e = 11$

$3d - (1 - d) = 11$

$3d - 1 + d = 11$

$4d = 12$

$d = 3$

$\therefore e = 1 - d$

$= 1 - 3$

$= -2$

$(d, e)$

$(3, -2)$

Solve the following systems of equations **by substitution**.

62. Solve.

$a + 6b = 9$ $\quad a = 9 - 6b$

$3a - 2b = -23$

$3(9-6b) - 2b = -23$

$27 - 18b - 2b = -23$

$27 - 20b = -23$

$-20b = -50$

$b = \frac{-50}{-20}$

$= \frac{5}{2}$

$\therefore a = 9 - 6\left(\frac{5}{2}\right)$

$= 9 - 15$

$= -6$

$\left(-6, \frac{5}{2}\right)$

63. Solve.

$2t - w = 13$ $\quad w = 2t - 13$

$4t + 3w = 1$

$4t + 3(2t - 13) = 1$

$4t + 6t - 39 = 1$

$10t = 40$

$t = 4$

$\therefore w = 2(4) - 13$

$= -5$

$(4, -5)$

64. Solve.

$3y = -6x + 15$ $\quad y = -2x + 5$

$5y = 5x + 10$

$5(-2x + 5) = 5x + 10$

$-10x + 25 = 5x + 10$

$15 = 15x$

$1 = x$

$\therefore y = -2(1) + 5$

$= 3$

$(1, 3)$

65. Solve.

$y = \frac{x}{3} + 2$

$3y + 4x = 21$

$3\left(\frac{x}{3} + 2\right) + 4x = 21$

$\frac{3x}{3} + 6 + 4x = 21$

$5x + 6 = 21$

$5x = 15$

$x = 3$

$\therefore y = \frac{3}{3} + 2$

$y = 3$

$(3, 3)$

66. Solve.

$3x - 2y = 4$ $\quad -2y = -3x + 4$

$3x + 4y = 10$ $\quad y = \frac{3}{2}x - 2$

$3x + 4\left(\frac{3}{2}x - 2\right) = 10$

$3x + 6x - 8 = 10$

$9x = 18$

$x = 2$

$\therefore y = \frac{3}{2}(2) - 2$

$= 3 - 2$

$= 1$

$(2, 1)$

67. Solve.

$\frac{1}{4}x + \frac{1}{2}y = 10$ $\quad 2\left(\frac{1}{2}y = -\frac{1}{4}x + 10\right)$

$\frac{1}{4}x - \frac{1}{2}y = 0$ $\quad y = -\frac{1}{2}x + 20$

$\frac{1}{4}x - \frac{1}{2}\left(-\frac{1}{2}x + 20\right) = 0$

$\frac{1}{4}x + \frac{1}{4}x - 10 = 0$

$\frac{1}{2}x = 10$

$x = 20$

$\therefore y = -\frac{1}{2}(20) + 20$

$= -10 + 20$

$= 10$

$(20, 10)$

68. Write a system of 2 linear equations for the following problem.

The sum of two numbers is 65. The first number is 17 greater than the second.

let x = first number
y = second number

$x + y = 65$
$x = y + 17$

69. Find the numbers in the problem to the left.

$x + y = 65$
$x = y + 17$
$(y+17) + y = 65$
$2y + 17 = 65$
$2y = 48$
$y = 24$

$\therefore x = 24 + 17$

70. Write a system of 2 linear equations for the following problem.

One number is 12 less than another number. Their sum is 102.

$x = y - 12$
$x + y = 102$

71. Find the numbers in the problem to the left.

$x + y = 102$
$x = y - 12$
$(y-12) + y = 102$
$2y - 12 = 102$
$2y = 114$
$y = 57$

$\therefore x = 57 - 12$
$= 45$

$(45, 57)$

72. Write a system of 2 linear equations for the following problem.

Mr. J bought a total of 12 pairs of socks. Athletic socks cost $5 per pair and dress socks cost $7 per pair. He spent $70 in total.

$a + d = 12$
$5a + 7d = 70$

73. How many pairs of each type of socks did he buy?

$a = 12 - d$
$5(12-d) + 7d = 70$
$60 - 5d + 7d = 70$
$2d = 10$
$d = 5$

$\therefore a = 12 - 5$
$= 7$

He bought
5 pair dress
7 pair athletic.

## Part 2: Solving By Elimination (Addition or Subtraction)

### Challenge Questions

74. Is (3,1) a solution to the system $2x - y = 5$ and $2x - 4y = 2$ ?

$2(3) - 1 = 5$ $\quad$ $2(3) - 4(1) = 2$
$6 - 1 = 5$ ✓ $\quad$ $6 - 4 = 2$ ✓ $\quad$ YES

75. Multiply each of the equations above by 2.

$2(2x - y = 5) \rightarrow$ $4x - 2y = 10$ $\quad$ $2(2x - 4y = 2) \rightarrow$ $4x - 8y = 4$

76. Is (3,1) still a solution to each of the equations above?

$4(3) - 2(1) = 10$ $\quad$ $4(3) - 8(1) = 4$
$12 - 2 = 10$ ✓ $\quad$ $12 - 8 = 4$ ✓ $\quad$ YES

77. Add the two original equations together:

$2x - y = 5$
$\underline{2x - 4y = 2}$
$4x - 5y = 7$

78. Is (3,1) a solution to the new equation?

$4(3) - 5(1) = 7$
$12 - 5 = 7$ ✓
$7 = 7$ $\quad$ YES

79. What conclusions can you draw about adding/subtracting equations together?

Does not affect the solution. That is, the solution will satisfy the newly created equation.

80. What conclusions can you draw about multiplying equations in a system by a constant?

Same as question above.

81. Can you multiply the equations by different numbers without affecting the solution?

Yes, as long as you are consistent within each equation.

Eg. $3(x + 2y = 6)$ $\quad$ $3x + 6y = 18$ ✓ } YES

$^{3}(x) + ^{2}(2y) = ^{1}(6)$ $\quad$ $3x + 6y = 6$ ✗ } NO

82. Graph equation ①:

① $2x + y = 8$

$y = -2x + 8$

83. Graph equation ②:

② $y = 4x - 4$

84. Add equations ① and ②. Call this equation ③.

$y = -2x + 8$

$+ \quad y = 4x - 4$

③ $2y = 2x + 4$

③ $2y = 2x + 4$

or $y = x + 2$

85. Graph equation ③.

86. Multiply ③ × 3 and call this equation ④.

$3(y = x + 2)$

$3y = 3x + 6$

④ $3y = 3x + 6$

or $y = x + 2$

87. Graph equation ④.

same as ③

88. Add ③ and ④, call this equation ⑤.

③ $y = x + 2$

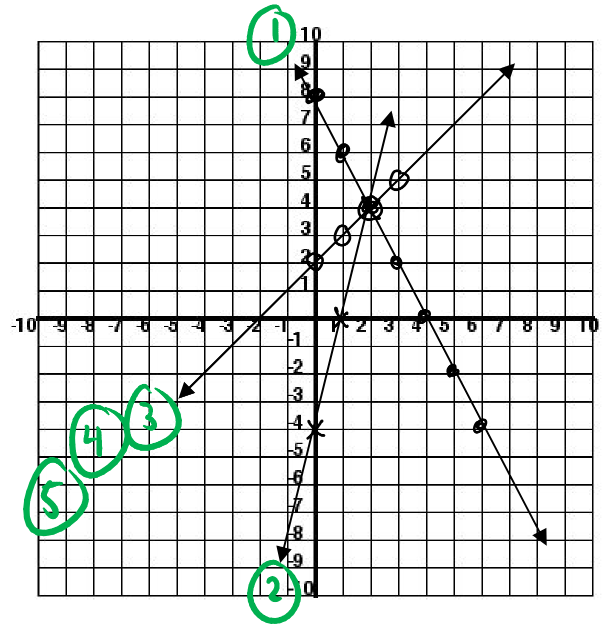
④ $y = x + 2$

$2y = 2x + 4$

⑤ $2y = 2x + 4$

$y = x + 2$

89. Graph equation ⑤.



I could have used the original ③ and ④ instead

$\Rightarrow 5y = 5x + 10$

90. Describe what you see happening above.

All lines cross at (2,4), the common solution.

91. Write a set of rules describing what you may do to a system of equations in order to find the solution. That is, how can you manipulate the equations without affecting the solution?

92. Add the two equations together, then solve.

$3x - 6y = 21$
$\underline{-3x - 4y = -1}$

-10y = 20

y = - 2 → 3x – 6(- 2) = 21

3x + 12 = 21

3x = 9

x = 3

Solution: (3, - 2)

93. Solve.

$2x + 3y = 18$
$2x - 3y = -6$

$4x + 0 = 12$

$\div 4 \quad \div 4$

$x = 3$

$\therefore 2(3) + 3y = 18$

$6 + 3y = 18$

$3y = 12$

$y = 4$

$(3, 4)$

94. Solve.

$8x + 2y = -20$
$2x - 2y = -30$

$10x = -50$

$\div 10 \quad \div 10$

$x = -5$

$\therefore 8(-5) + 2y = -20$

$-40 + 2y = -20$

$2y = 20$

$y = 10$

$(-5, 10)$

95. Solve.

$(-4t + 3s = 2) \times 2 \rightarrow -8t + 6s = 4$
$8t - 6s = -4 \longrightarrow 8t - 6s = -4$

$0 = 0$

same line!

$\therefore$ Infinite Solutions

96. Solve.

$6x - 3y = 24$
$3(x + y = -2)$

$6x - 3y = 24$
$3x + 3y = -6$

$9x = 18$

$x = 2$

$\therefore 2 + y = -2$

$y = -4$

$(2, -4)$

97. Solve.

$4(3b - a = 1)$
$-12b + 4a = -4$

$12b - 4a = 4$
$-12b + 4a = -4$

$0 = 0$

same line $\therefore$

infinite solutions

98. Solve.

$100(0.05x + 0.07y = 19)$

$-5(x + y = 300)$

$5x + 7y = 1900$

$-5x - 5y = -1500$

$2y = 400$

$y = 200$

$\therefore x + 200 = 300$

$x = 100$

$(100, 200)$

99. Solve.

$-2(x + y = 1200)$

$10(0.20x + 0.40y = 36)$

$-2x - 2y = -2400$

$2x + 4y = 360$

$2y = -2040$

$y = -1020$

$\therefore x + (-1020) = 1200 \quad x = 2220$

$(2220, -1020)$

100. Two numbers have a sum of 25 and a difference if 7. What are the two numbers?

$x + y = 25$

$x - y = 7$

$2x = 32$

$x = 16$

$\therefore 16 + y = 25$

$y = 9$

The numbers are 16 and 9.

101. Anya has a pocket full of loonies ($1 coins) and toonies ($2 coins). She has $41 in total. If she has 29 coins, how many of each does she have? $x$ = # toonies / $y$ = # loonies

# coins: $x + y = 29$

value coins: $-(2x + 1y = 41)$

$-x = -12$

$x = 12$

$\therefore 12 + y = 29$

$y = 17$

17 toonies + 12 loonies.

102. When three times one number is added to two times another number, the sum is 21. When 4 times the second number is subtracted from 10 times the first number, the difference is 38. What are the numbers?

let $x$ = first number $y$ = second #

$2(3x + 2y = 21)$

$10x - 4y = 38$

$6x + 4y = 42$

$10x - 4y = 38$

$16x = 80$

$x = 5$

$\therefore 3(5) + 2y = 21$

$15 + 2y = 21$

$2y = 6$

$y = 3$

5 is first #
3 is second #

103. The total cost (before taxes) for three coffees and two cookies is $10.05. The cost for five coffees and three cookies is $16.10. Find the individual cost for each item.

# coffees = $x$

# cookies = $y$

$3(3x + 2y = 10.05)$

$-2(5x + 3y = 16.10)$

$9x + 6y = 30.15$

$-10x - 6y = 32.20$

$-x = -2.05$

$\therefore x = 2.05$

$3(2.05) + 2y = 10.05$

$2y = 10.05 - 6.15$

$2y = 3.90$

$y = 1.95$

Coffee $2.05

Cookie $1.95

Solving Problems with Systems of Equations. Use the method of your choice.

104. A job offered to Mr. Xu will pay straight commission at a rate of 6% on all sales. A second job offer will pay a monthly salary of $400 and 2% commission. How much would Mr. Xu have to sell so that both jobs would pay him the same amount.

(equal)

$E = 0.06s$

$E = 0.02s + 400$

Substitution:

$0.06s = 0.02s + 400$

$0.04s = 400$

$s = 10000$

When sales are $10,000 both jobs pay the same.

When would the job paying straight commission be a better choice?

When monthly sales were over $10,000.

105. In his 2004-05 season, Steve Nash scored 524 total baskets (not including free throws). He scored 336 more two point baskets than three point baskets. Write and solve a system of linear equations that represents this problem.

x = 3pt shots

y = 2pt shots

$$\left.\begin{aligned} x + y &= 524 \\ x - 336 &= y \end{aligned}\right\}$$

substitution:

$x + (x - 336) = 524$

$2x - 336 = 524$

$2x = 860$

$x = 430$

Interpret your solution: $\therefore y = 430 - 336$

$= 94$

Steve scored 430 2-pt baskets and 94 3-pt baskets.

106. Mr. J has a class with 30 students in it. 22 of those students own a cell phone. $\frac{4}{5}$ of the girls owned a cell phone and $\frac{3}{5}$ of the boys owned a cell phone. How many girls were in this class? x = # boys y = # girls

$x + y = 30 \rightarrow x = 30 - y$

$5\left(\frac{3}{5}x + \frac{4}{5}y = 22\right) \rightarrow 3x + 4y = 110$

Substitute:

$3(30 - y) + 4y = 110$

$90 - 3y + 4y = 110$

$y = 20$

20 girls in the class

107. Daiki invested a total of $12 000 in two stocks in 2009. One stock earned 4% interest and the other earned 7% interest. Daiki earned a total of $615 in interest in 2009. How much did he invest in each stock? Let x = amount in 4% stock

y = " 7% "

$x + y = 12000 \rightarrow y = 12000 - x$

$100(0.04x + 0.07y = 615) \rightarrow 4x + 7y = 61500$

Substitute:

$4x + 7(12000 - x) = 61500$

$4x + 84000 - 7x = 61500$

$-3x = -22500$

$x = 7500$

$\therefore 7500 + y = 12000$

$y = 4500$

$\therefore$ $7500 @ 4% and $4500 @ 7%

For each of the following problems, write and solve a system of equations. Interpret solutions!

108. Breakers Volleyball sold 570 tickets to their home opener, some tickets cost \$2 and some cost \$5. The total revenue was \$1950. How many of each type of ticket were sold?

let $x$ = # sold at \$2 and $y$ = # sold at \$5

$x + y = 570 \longrightarrow x = 570 - y$

$2x + 5y = 1950$

substitute...

$2(570 - y) + 5y = 1950$

$1140 - 2y + 5y = 1950$

$3y = 810$

$y = 270$

$\therefore x + 270 = 570$

$x = 300$

300 at \$2
270 at \$5

109. Mr. J is doing routine maintenance on his old farm truck. This month he spent \$26.50 on 6 litres of oil and 2 gaskets. Last month he spent \$25.00 on 4 litres of oil and 4 gaskets. Find the price of each gasket and one litre of oil.

Let $x$ = price oil
$y$ = price gaskets

$[6x + 2y = 26.50] \to y = -3x + 13.25$

$4x + 4y = 25.00$

$4x + 4(-3x + 13.25) = 25.00$

$4x - 12x + 53 = 25$

$-8x = -28$

$x = 3.5$

$\therefore 4(3.5) + 4y = 25 \,//\, 14 + 4y = 25 \,//\, 4y = 11$

$y = \frac{11}{4} = 2.75$

Oil: \$3.50
Gasket: \$2.75

110. Anya makes a trip to the local grocery store to buy some bulk candy. She chooses two of her favourite candies, gummy frogs and gummy penguins. Gummy frogs sell for \$1.10 per 100g and penguins sell for \$1.75 per 100g. Anya buys a total of 500g of candy for \$7.84 (no taxes). How much of each type did she buy?

$x$ = # units of frogs
$y$ = # units of penguins

work in 100g units

$x + y = 5 \} \to x = 5 - y$

$1.10x + 1.75y = 7.84$

Substitute

$1.10(5 - y) + 1.75y = 7.84$

$5.5 - 1.10y + 1.75y = 7.84$

$0.65y = 2.34$

$y = 3.6$

$\therefore x = 5 - 3.6, \; x = 1.4$

360g penguins
140g frogs

111. For his Christmas party, Teems Prey is making a bowl of exotic punch for the kid's table. Imported leechi juice sells for \$12.50 per litre and guava nectar sells for \$18 per litre. He is making 8 litres and will need to pay \$\$126.40 for the perfect blend. How much of each type does he use?

let $x$ = volume leechi
$y$ = " guava

$x + y = 8 \} \; x = 8 - y$

$12.5x + 18y = 126.40$

$12.5(8 - y) + 18y = 126.4$

$100 - 12.5y + 18y = 126.4$

$5.5y = 26.4$

$y = 4.8$

$\therefore x + 4.8 = 8$

$x = 3.2$

4.8 l guava
3.2 l leechi

$d = st$

112. Jay Maholl swam 12 km downstream in Englishman River in two hours. The return trip upstream took 6 hours. Find the speed of the current in Englishman River.

x = speed of Jay in still water
y = speed current

$12 = 2(x+y) \rightarrow 6 = x+y$
$12 = 6(x-y) \quad\quad 2 = x-y$ Elimination
$8 = 2x$
$4 = x \quad \therefore y = 2$

Jay 4 km/h
current 2 km/h

113. (What assumption must you make?)

current & Jay's speed are constant

114. The Lucky-Lady dinghy travels 25 km upstream in five hours. The return trip takes only half an hour. Find the speed of the boat and the speed of the current.

$d = st$

$5(x-y) = 25 \rightarrow x-y = 5$
$\frac{1}{2}(x+y) = 25 \rightarrow x+y = 50$
Elimination
$2x = 55$
$x = 27.5$

$\therefore 27.5 + y = 50$
$y = 22.5$

boat speed is 27.5 km/h
current speed is 22.5 km/h

115. A bumble bee travels 4.5 km into a headwind in 45 minutes. The return trip with the wind only takes 15 minutes. Assuming speeds are constant, find the speed of the bumble bee in still air.

x - bee speed
y - wind speed

$\frac{3}{4}(x+y) = 4.5 \quad 0.75(x+y) = 4.5$
$\frac{1}{4}(x-y) = 4.5 \quad 0.25(x-y) = 4.5$

$x+y = 6$
$x-y = 18$
$2x = 24$
$x = 12$

Bee's speed is 12 km/h

116. A plane flew a distance of 650 km in 3.25 hours when travelling in a tailwind. The return trip took 6.5 hours against the same wind. Assume both speeds are constant. Find the speed of the plane and the wind speed.

$3.25(x+y) = 650 \quad x+y = 200$
$6.5(x-y) = 650 \quad x-y = 100$
$2x = 300$
$x = 150$

$\therefore 150 + y = 200$
$y = 50$

plane speed = 150 km/h
wind speed = 50 km/h

117. A 50% acid solution is required for a chemistry lab. The instructor has a 20% stock solution and a 70% stock solution. She needs to make 20 litres of the 50% acid solution. How much of each stock solution should she use?

Let x = volume of 20% solution

Let y = volume of 70% solution.

x + y = 20

0.2x + 0.7y =(0.5)(20)

$0.2x + 0.7y = 10$

Solve the System: I Used Elimination

$-2(x+y=20) \longrightarrow -2x-2y=-40$

$10(0.2x+0.7y=10) \rightarrow 2x+7y=100$

$5y=60$

$y=12$

$\therefore x+12=20$

$x=8$

8 l of 20% acid
12 l of 70% acid

118. A 65% acid solution is required for a chemistry lab. The instructor has a 20% stock solution and a 70% stock solution. She needs to make 20 litres of the 65% acid solution. How much of each stock solution should she use?

$x+y=20 \rightarrow x=20-y$

$0.2x+0.7y=(0.65)(20)$

substitution:

$0.2(20-y)+0.7y=13$

$4-0.2y+0.7y=13$

$0.5y=9$

$y=18$

$\therefore x+18=20$

$x=2$

18 l of 70% acid
2 l of 20% acid

119. The karat (or carat) is a measure of the purity of gold in gold alloy. 18K gold is approximately 75% pure and 14K gold is approximately 58.5% pure. Using 18K and 14K stock, a goldsmith needs to produce 40g of gold alloy that is 70% pure. How much of each stock will he need to use? (round to nearest hundredth)

Let:
x = grams 18K
y = grams 14K

$x+y=40 \rightarrow x=40-y$

$0.75x+0.585y=0.70(40)$

substitute

$0.75(40-y)+0.585y=28$

$30-0.75y+0.585y=28$

$-0.165y=-2$

$y=12.12\text{ g}$

$\therefore x+12.12=40$

$x=27.88\text{ g}$

Use 12.12g 14K ; 27.88g 18K

120. A goldsmith needs to make 50g of 14K gold (58.5%) from 18K (75%) and 10K (41.7%) stock alloys. How much of each does she need? (round to nearest hundredth)

Let x = grams 18K
y = grams 10K

$x+y=50 \rightarrow x=50-y$

$0.75x+0.417y=0.585(50)$

$0.75(50-y)+0.417y=29.25$

$37.5-0.75y+0.417y=29.25$

$-0.333y=-8.25$

$y=24.77$

$\therefore x+24.77=50$

$x=25.23$

Use 25.23g 18k and 24.77g 10k