HW Mark:	10	9	8	7	6	RE-Submit

Systems of Equations & Inequalities

This book belongs to ______________________ Period___

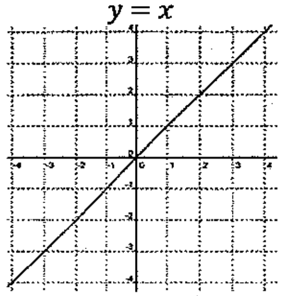
Lesson	Topics / Pages from Notes	Questions I find difficult...
	Review: Graphing Linear Functions	
	Review: Graphing Quadratic Functions	
	Review: Solving Linear Systems	
	Linear-Quadratic or Quad-Quad Systems	
	Linear Inequalities	
	Quadratic Inequalities	
	Systems of Inequalities	

You will need: Quadratic Formula: $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Graphing Equations in Two Variables

Linear Functions

$y = x$



A linear function will produce a straight line when graphed on a Cartesian plane (x-y grid).

Examples:

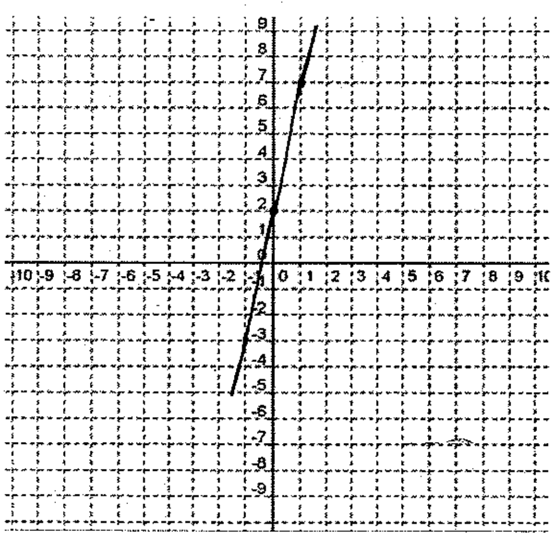
$$y = 3x - 2$$
$$5x - 4y = 20$$
$$y - 7 = \frac{3}{4}(x + 2)$$
$$2y - 7 = 0$$

1. What do you notice about all the linear equations listed above?
   That is, what makes an equation "linear"?

All linear equations have variables that are degree 1. There may be 2 variables present or only one variable.

2. Challenge.

Graph $y = 5x + 2$

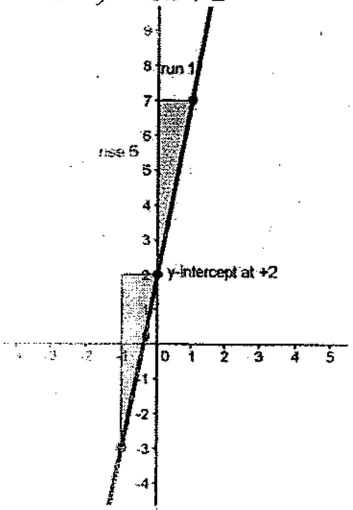


Explain your steps for graphing the line to the left.

- find the y-intercept.
- put a number into ~~the~~ equation
  when $x=1$, $y=5+2=7$
  we got another point.
- creat the line.

Graph the following Linear Functions

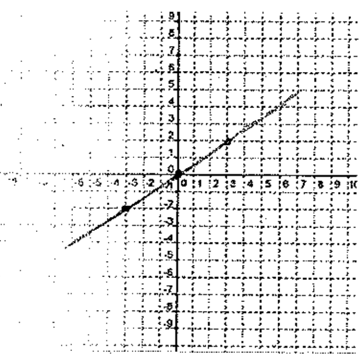
3. $y = 5x + 2$



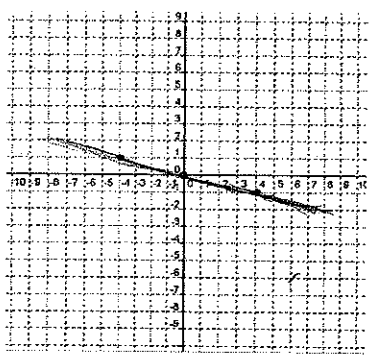
When written in the form $y = mx + b$, we use b as the starting point on the y-axis and m represents the rate of change, or slope of the line. This allows us to "walk" to another point, creating the line.

What if b is 0 such as $= \frac{2}{3}x$?
Answer: Start at the origin!

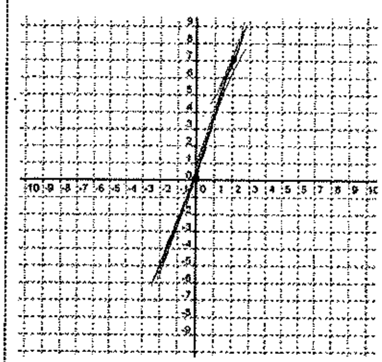
4. $y = \frac{2}{3}x$



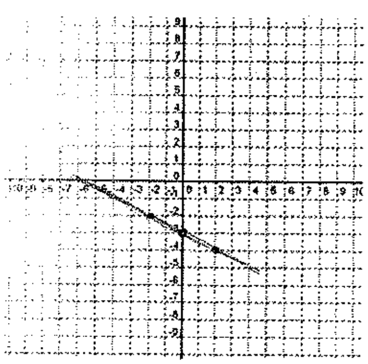
5. $y = -\frac{1}{4}x$



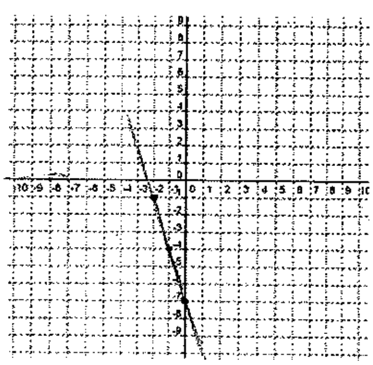
6. $y = 3.5x$



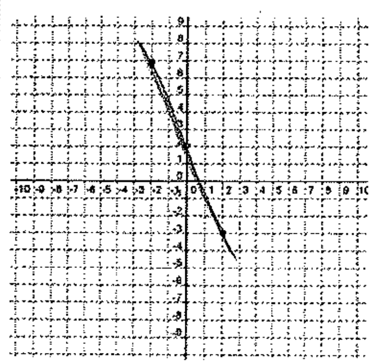
7. $y = -\frac{1}{2}x - 3$



8. $y = -3x - 7$

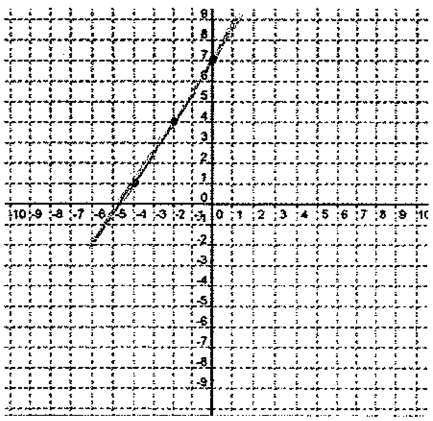


9. $y = -\frac{5}{2}x + 2$



10. Challenge.

Graph $3x - 2y + 14 = 0$



Explain your steps for graphing the line to the left.

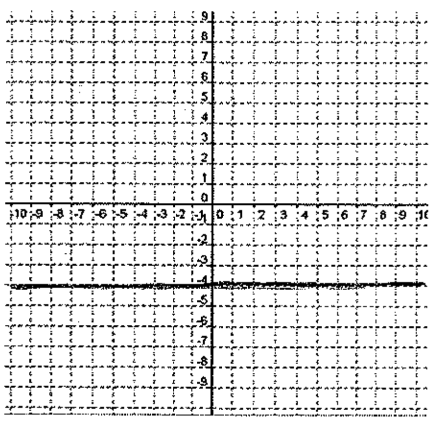
- Rewrite the equation in slope-intercept form
- Graph using slope & y-intercept

$-2y = -3x - 14$

$y = \frac{3}{2}x + 7$

11. Challenge.

Graph $2y + 8 = 0$



Explain your steps for graphing the line to the left.

- Rearrange the equation

$2y = -8$

$y = -4$

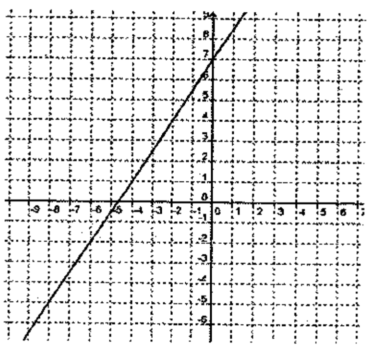
Graph each of the following linear equations.

12. $3x - 2y + 14 = 0$

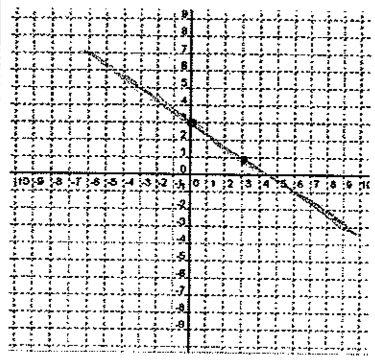
Rewrite the equation in slope-intercept form.

$$-2y = -3x - 14$$
$$y = \frac{3}{2}x + 7$$

Graph using slope & y-intercept.



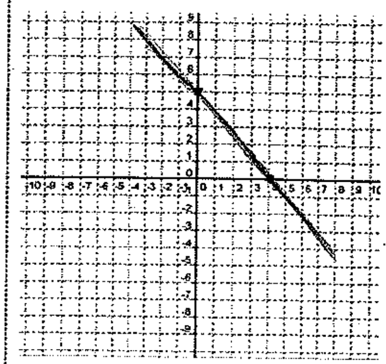
13. $2x + 3y - 9 = 0$



$3y = -2x + 9$

$y = -\frac{2}{3}x + 3$

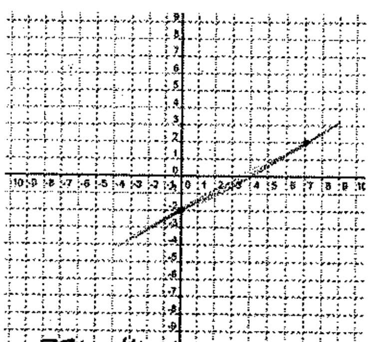
14. $5x + 4y - 20 = 0$



$4y = -5x + 20$

$y = -\frac{5}{4}x + 5$

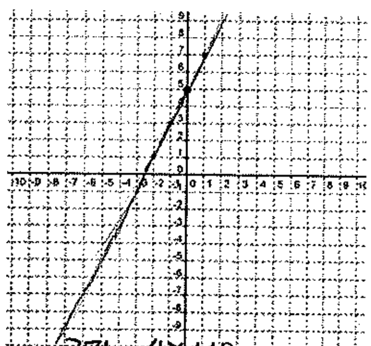
15. $0 = 14 + 7y - 4x$



$-7y = 14 - 4x$

$y = \frac{4}{7}x - 2$

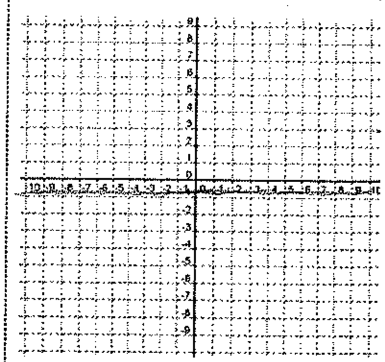
16. $2y - 7x = 10 - 3x$



$2y = 4x + 10$

$y = 2x + 5$

17. $3y - 2x = 5 + 9y - 2x$



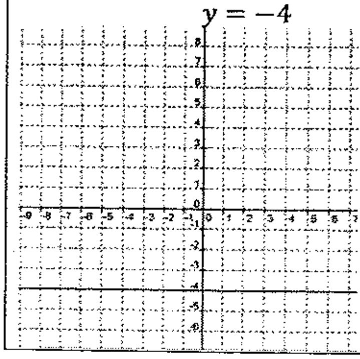
$-6y = 5$

$y = -\frac{5}{6}$

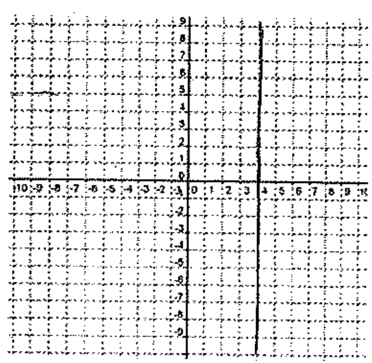
18. $2y + 8 = 0$

Rearrange: $2y = -8$

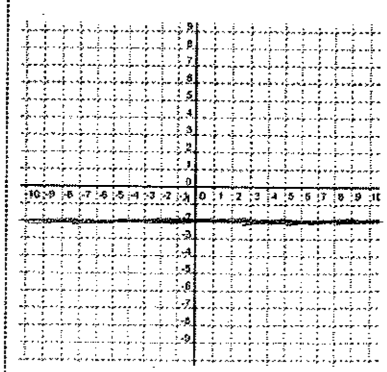
$y = -4$



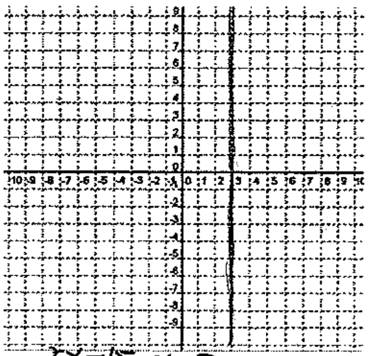
19. $x = 4$



20. $y = -2$

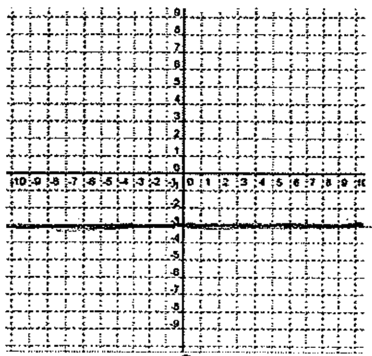


21. $5x - 6 = 9$



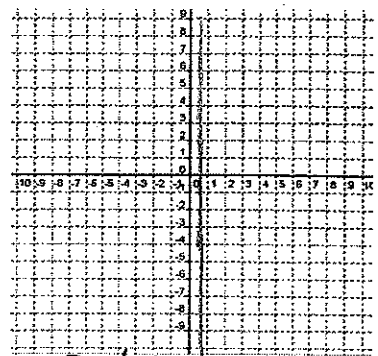
$5x=15 \quad x=3$

22. $18 = -6y$



$y=-3$

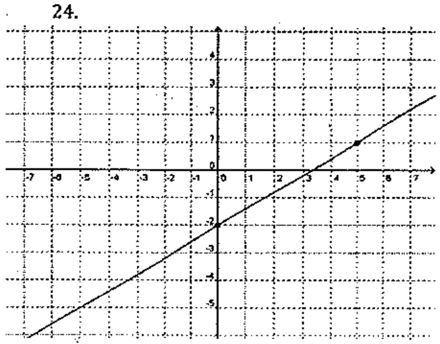
23. $12 - 4x = 9 + x$



$3=5x \quad x=\frac{3}{5}$

Write a linear equation for each of the following lines.

24.



$y=mx+b$

$y=mx-2$

$1=5m-2$

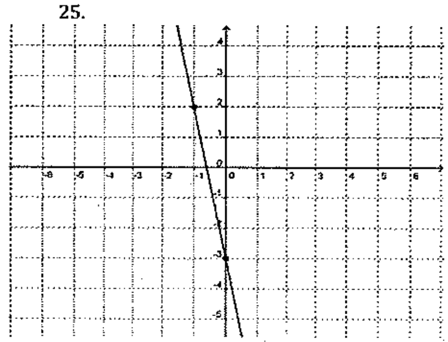
$5m=3$

$m=\frac{3}{5}$

$3x-5y=10$

$y=\frac{3}{5}x-2$

25.



$y=mx-3$

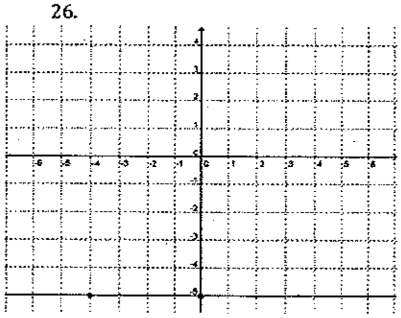
$2=-m-3$

$m=-5$

$y=-5x-3$

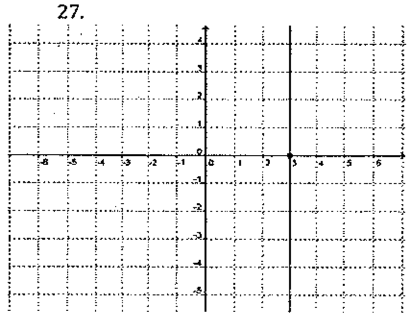
$5x+y=-3$

26.



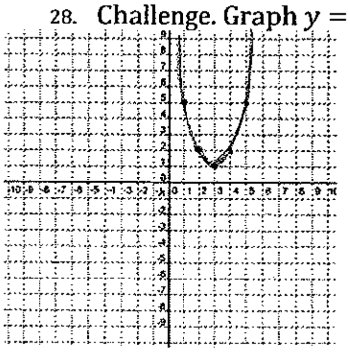
$y=-5$

27.



$x=3$

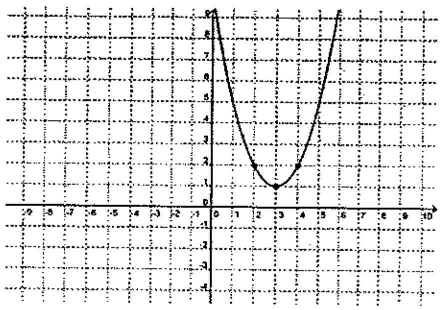
28. Challenge. Graph $y = (x-3)^2 + 1$.



Quadratic Equations and Their Graphs

Graph each of the following quadratic equations.

29. $y = (x-3)^2 + 1$



The equation is given to us in "vertex form"

$$y = a(x-p)^2 + q$$

We know the vertex is at (p,q) and we can graph subsequent points using the increase intervals.

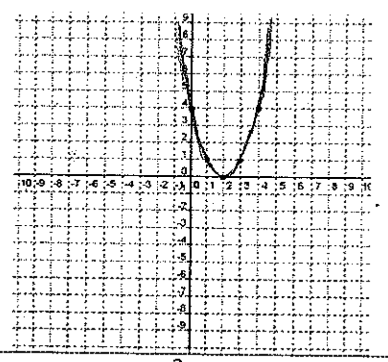
For the example to the left...

Vertex: $(3,1) \rightarrow$ opening upwards + a-value

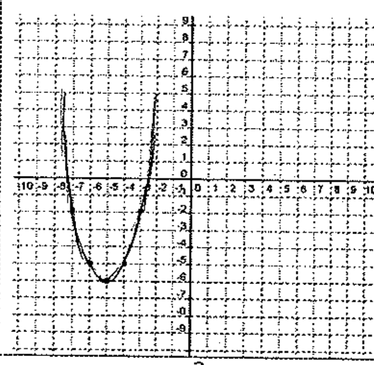
Increase Intervals: $\{1,3,5,7,\ldots\}$

[For more detail on graphing parabolas, refer to the booklet on Quadratic Functions.]

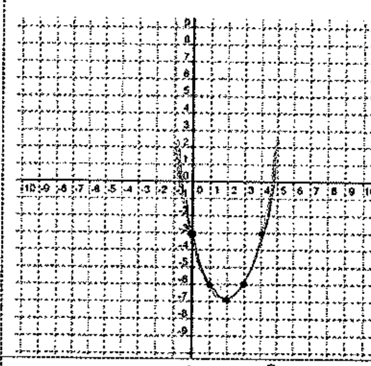
30. $y = (x-2)^2$



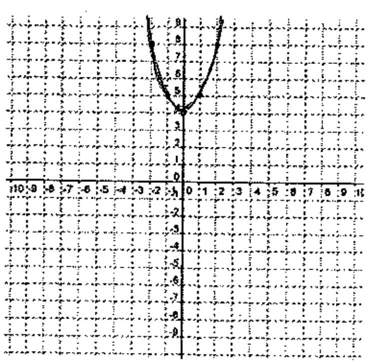
31. $y = (x+5)^2 - 6$



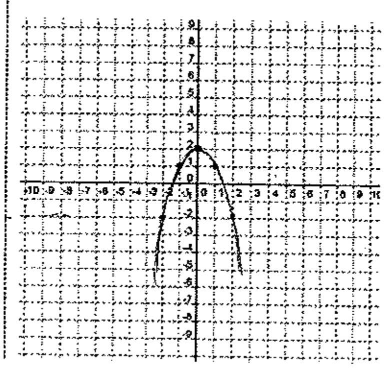
32. $y = (x-2)^2 - 7$



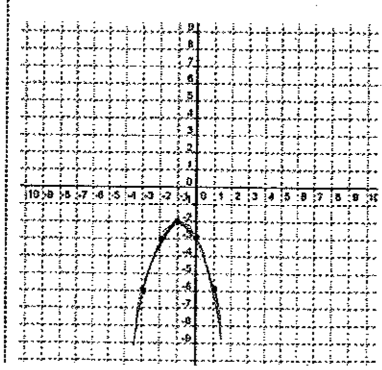
33. $y = x^2 + 4$



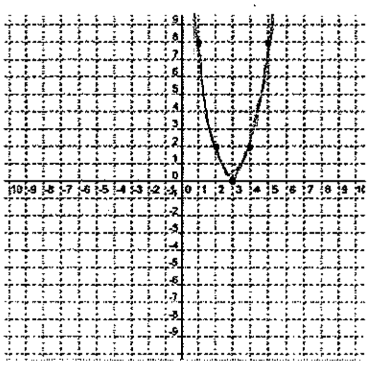
34. $y = -x^2 + 2$



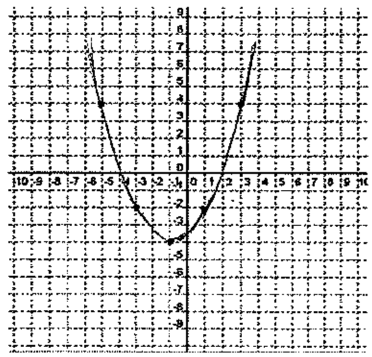
35. $y = -(x+1)^2 - 2$



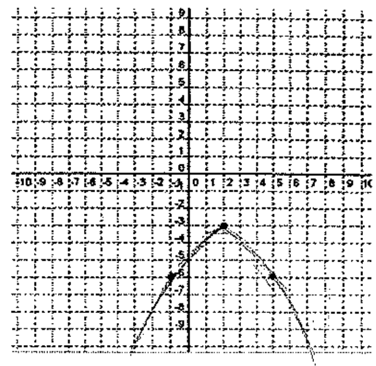
36. $y = 2(x-3)^2$



37. $y = \frac{1}{2}(x+1)^2 - 4$



38. $y = -\frac{1}{3}(x-2)^2 - 3$

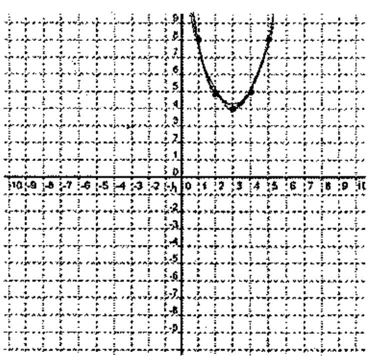


39. $y = x^2 - 6x + 13$

Write in vertex form by completing the square first.

$y = (x^2 - 6x + \underline{9}) - \underline{9} + 13$

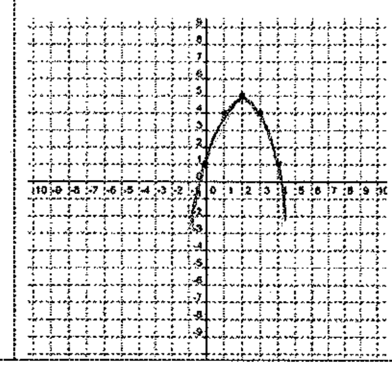
$y = (x - \underline{3})^2 + \underline{4}$



40. $y = -x^2 + 4x + 1$

$y = -(x^2 - 4x + 4) + 4 + 1$

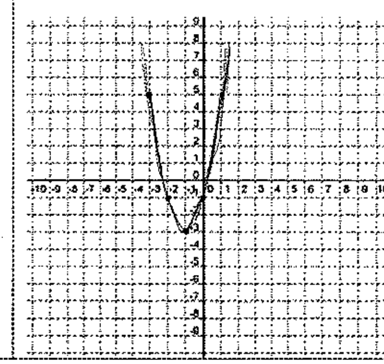
$y = -(x-2)^2 + 5$



41. $y = 2x^2 + 4x - 1$

$y = 2(x^2 + 2x + 1) - 2 - 1$

$y = 2(x+1)^2 - 3$



42. CHALLENGE. Is (1,1) a solution to the following system?

$$y = 2x - 1$$
$$2x - 3y + 5 = 0$$

$1 = 2 - 1 \qquad 2 - 3 + 5 = 0$

$4 \neq 0$

$\therefore$ (1, 1) is not a solution because it does not satisfy $2x - 3y + 5 = 0$

Systems of Equations

43. What does it mean to "find the solution" to a system of equations?

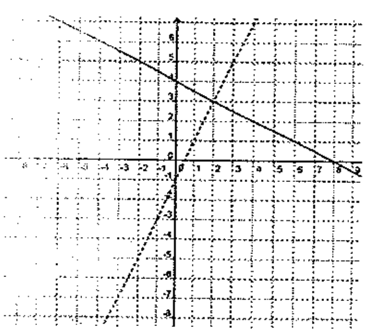
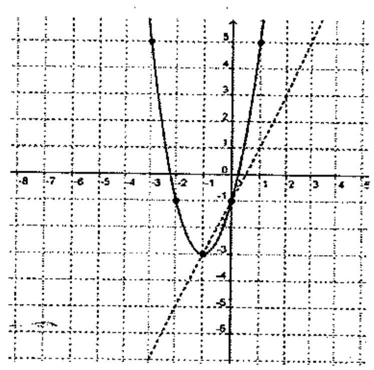
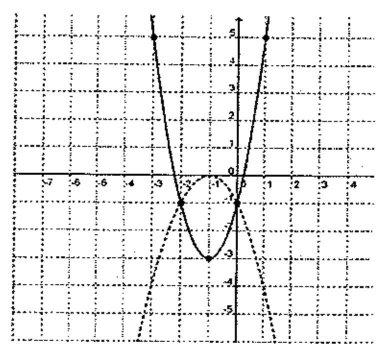
It means to find point that satisfies all the equations of the system.

Which of the following systems have (1,1) as a solution? Show how you know.

44. $y = 2x - 1$ $2x - 3y + 5 = 0$	45. $y = -2x + 3$ $y = -x^2 + 2$	46. $y + 4x = 5$ $y - 2 = -(x - 2)^2$
If (1,1) is a solution, it will satisfy both equations. $y = 2x - 1$ $1 = 2(1) - 1$ $1 = 2 - 1$ $1 = 1$ $2x - 3y + 5 = 0$ $2(1) - 3(1) + 5 = 0$ $2 - 3 + 5 = 0$ $4 = 0$ ∴ (1,1) is not a solution, it does not satisfy $2x - 3y + 5 = 0$.	$1 = -2 + 3$ $1 = -1^2 + 2$ $1 = 1$ $1 = 1$ ∴ (1,1) is a solution.	$1 + 4 = 5$ $1 - 2 = -(1-2)^2$ $5 = 5$ $-1 = -1$ ∴ (1,1) is a solution

We will consider three types of systems.

- Linear-Linear systems.
- Linear-Quadratic systems.
- Quadratic-Quadratic systems.

Linear-Linear	Linear-Quadratic	Quadratic-Quadratic
		
47. Find the solution(s) to the system above. (2, 3)	48. Find the solution(s) to the system above. (−1, −3), (0, −1)	49. Find the solution(s) to the system above. (−2, −1), (0, −1)

50. A system of two linear equations can have which of the following:

Zero solutions	One solution	Two solutions	Infinite solutions
✓	✓		✓

51. A system of a linear and a quadratic equation can have which of the following:

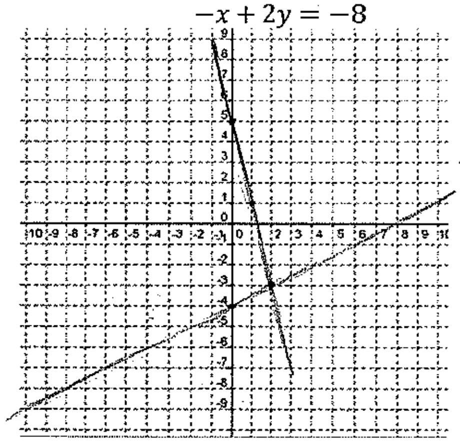
Zero solutions	One solution	Two solutions	Infinite solutions
✓	✓	✓	

52. A system of two quadratic equations can have which of the following:

Zero solutions	One solution	Two solutions	Infinite solutions
✓	✓	✓	✓

53. CHALLENGE.
Solve the system by graphing.

$$4x + y = 5$$
$$-x + 2y = -8$$



$y = -4x + 5$ $\quad 2y = x - 8$

$y = \frac{1}{2}x - 4$

∴ $(2, -3)$ is the solution.

Explain your steps.

- Graph each line
- Look for the point of intersection.

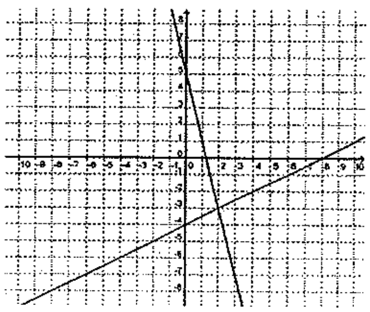
Finding solutions using graphs

Solve the following systems of linear equations by graphing.

54.

$$4x + y = 5$$
$$-x + 2y = -8$$

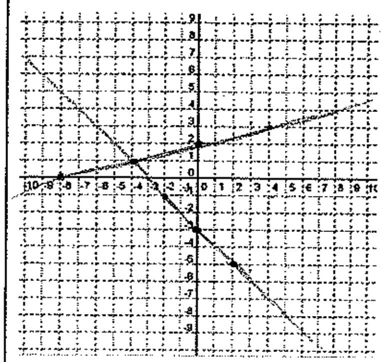
Graph each line.
Look for the point of intersection.



Solution: $(2, -3)$

55.

$$x + y = -3$$
$$x - 4y = -8$$



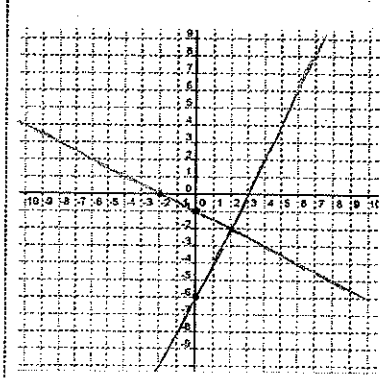
$y = -x - 3$ $\quad 4y = x + 8$

$y = \frac{1}{4}x + 2$

$\therefore (-4, 1)$

56.

$$y = 2x - 6$$
$$x + 2y + 2 = 0$$



$2y = -x - 2$

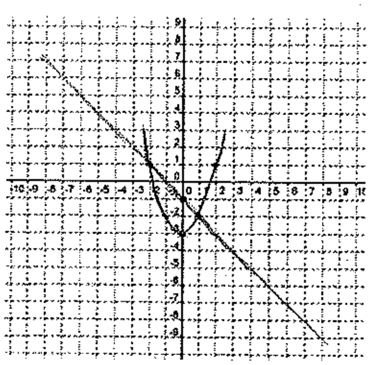
$y = -\frac{1}{2}x - 1$

$\therefore (2, -2)$

Solve the following linear-quadratic systems of equations by graphing.

57.

$$y = x^2 - 3$$
$$x + y = -1$$

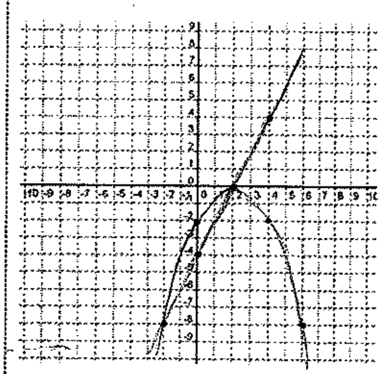


$y = -x - 1$

$\therefore (-2, 1)$ and $(1, -2)$

58.

$$2x - y = 4$$
$$y = -\frac{1}{2}(x - 2)^2$$

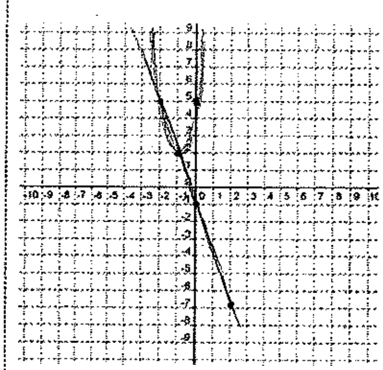


$y = 2x - 4$

$\therefore (2, 0)$ and $(-2, -8)$

59.

$$3x + y + 1 = 0$$
$$y - 2 = 3(x + 1)^2$$



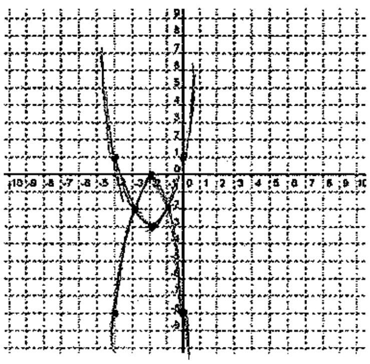
$y = -3x - 1$ $\quad y = 3(x+1)^2 + 2$

$\therefore (-1, 2)$ and $(-2, 5)$

Solve the following quadratic-quadratic systems of equations by graphing.

60.

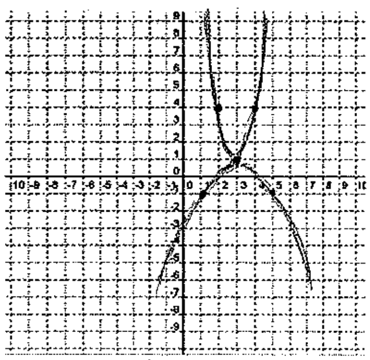
$$y = (x+2)^2 - 3$$
$$y = -2(x+2)^2$$



$\therefore (-3,-2)$ and $(-1,-2)$

61.

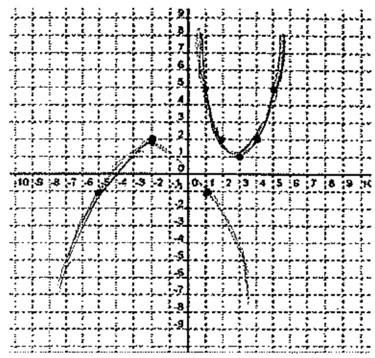
$$y = 3(x-3)^2 + 1$$
$$y = -\frac{1}{2}(x-3)^2 + 1$$



$\therefore (3,1)$

62.

$$y = -\frac{1}{3}(x+2)^2 + 2$$
$$y = (x-3)^2 + 1$$



$\therefore$ no solution.

63. Challenge.
Find the solution to the following system **without** graphing.

$$y - x = 1$$
$$2y - x = 4$$

$2y - 2x = 2$

$2y - x = 4$

$\therefore x = 2$

$y = 3$

$\therefore (2,3)$

64. Challenge.
Find the solution to the following system **without** graphing.

$$y = 2x + 2$$
$$y = -x^2 + 2$$

$2x + 2 = -x^2 + 2$

$2x + x^2 = 0$

$x(2 + x) = 0$

$\therefore x = 0,\ x = -2$

$y = 2(0) + 2 = 2$

$y = 2(-2) + 2 = -2$

$\therefore (-2,-2)$ and $(0,2)$

Finding solutions to systems using algebra.

Solve the following systems of equations using substitution or elimination (SEE NEXT PAGE).

65. Solve.
(1) $y - x = 1$
(2) $2y - x = 4$

We will use the process known as **SUBSTITUTION** shown on the right.

Rearrange (1) and substitute into (2).

(1) $x = y - 1$ *rearrange the first equation to isolate x*

(2) $2y - (y - 1) = 4$ *substitute new expression for x into second equation*

$2y - y + 1 = 4$ *solve for y*
$y + 1 = 4$
$\therefore y = 3$

Substitute 3 in for y into an original equation to solve for x.

$$3 - x = 1$$
$$\therefore x = 2$$

Solution: (2,3)

66. Solve.
$y = -3x + 2$
$y = -2x + 1$

$-3x+2=-2x+1$
$2=x+1$
$x=1$
$y=-1$
$\therefore (1,-1)$

67. Solve.
$2y - 6x = -10$
$y - 2x = -5$

$2y-4x=-10$
$2x=0$
$x=0$
$y=-5$
$\therefore (0,-5)$

68. Solve.
$3x - 2y + 6 = 0$
$6y = 4x - 6$

$9x-6y+18=0$
$4x-6y-6=0$
$5x+24=0$
$x=-\frac{24}{5}$, $y=-\frac{21}{5}$
$\therefore (-\frac{24}{5}, -\frac{21}{5})$

69. Solve.
$y = 2x + 2$
$y = -x^2 + 2$

Method: SUBSTITUTION

$2x + 2 = -x^2 + 2$
$x^2 + 2x = 0$
$x(x + 2) = 0$
$\therefore x = 0$ or $x = -2$

Use these x-values to find y-values.

$y = 2(0) + 2$
$y = 2$

$y = 2(-2) + 2$
$y = -2$

Solution: $(-2,-2)$ and $(0,2)$

70. Solve.
$3x - y = -8$
$y = 3x^2 + 2$

$3x-3x^2-2+8=0$
$3x-3x^2+6=0$
$x-x^2+2=0$
$x^2-x-2=0$
$(x-2)(x+1)=0$
$x=2$, $x=-1$
$y=3\times2^2+2=14$
$y=3\times(-1)^2+2=5$
$\therefore (2,14)$ and $(-1,5)$

71. Solve.
$x + y = 6$
$y = \frac{1}{2}x^2 + 2$

$\frac{1}{2}x^2+2-6+x=0$
$x^2-8+2x=0$
$(x-2)(x+4)=0$
$x=2$, $x=-4$
$y=6-2=4$
$y=6-(-4)=10$
$\therefore (2,4)$ and $(-4,10)$

Solve the following systems using substitution or elimination.

72. Solve.

$$3x - 2y = 12$$
$$5x + 3y = 1$$

$9x - 6y = 36$

$10x + 6y = 2$

$19x = 38$

$x = 2$

$y = -3$

$\therefore (2, -3)$

73. Solve.

Method: ELIMINATION

$$3x - 2y = 12$$
$$5x + 3y = 1$$

Create opposite coefficients of y using multiplication.

$$3(3x - 2y = 12)$$
$$2(5x + 3y = 1)$$

Add the two equations to eliminate y.

$$9x - 6y = 36$$
$$\underline{+\,10x + 6y = 2}$$
$$19x = 38$$

$$\therefore x = 2$$

Substitute to calculate y.

$$3(2) - 2y = 12$$
$$-2y = 6$$
$$\therefore y = -3$$

Solution: $(2, -3)$

74. Solve.

$$2x + 3y = 7$$
$$3x + 5y = 11$$

$6x + 9y = 21$

$6x + 10y = 22$

$y = 9$

$x = -10$

$\therefore (-10, 9)$

75. Solve.

$$3y = 12x + 6$$
$$8x = 2y - 4$$

$4x + 2 - y = 0$

$y - 2 - 4x = 0$

infinite solutions.

76. Solve.

$$5y - 10x = 5$$
$$-4y + 8x = 8$$

$y - 2x = 1$

$y - 2x = -2$

no solution

77. Solve.

$$2x + 3y = 1$$
$$5x - 4y = 14$$

$10x + 15y = 5$

$10x - 8y = 28$

$23y = -23$

$y = -1$

$x = 2$

$\therefore (2, -1)$

Solve the following systems using substitution or elimination.

78. $y = 4$
$y = -(x+4)^2 + 4$

$-(x+4)^2 = 0$

$x = -4$

$\therefore (-4, 4)$

79. $y - 4 = 0$
$y = -2(x-1)^2 + 6$

$y = 4$

$-2(x-1)^2 = -2$

$(x-1)^2 = 1$

$x = 2, 0$

$\therefore (2, 4)$

$(0, 4)$

80. $y = -2x - 8$
$y = 2(x+1)^2 - 4$

$2x^2 + 4x + 2 - 4 + 2x + 8 = 0$

$2x^2 + 6x + 6 = 0$

$x^2 + 3x + 3 = 0$

no solution.

81. $-x + y = 3$
$y = x^2 - 4x + 7$

$x^2 - 4x + 7 - 3 - x = 0$

$x^2 - 5x + 4 = 0$

$(x-1)(x-4) = 0$

$x = 1, 4$

$y = 4, 7$

$\therefore (1, 4)$

$(4, 7)$

82. $y = x$
$y = -\frac{1}{2}x^2 - 2x$

$-x^2 - 4x - 2x = 0$

$x^2 + 6x = 0$

$x(x+6) = 0$

$x = 0, -6$

$\therefore (0, 0)$

$(-6, -6)$

83. $\frac{1}{2}x - \frac{2}{3}y = 6$
$\frac{1}{4}x + \frac{1}{3}y = -1$

$3x - 4y = 36$

$3x + 4y = -12$

$8y = -48$

$y = -6$

$x = 4$

$\therefore (4, -6)$

84. CHALLENGE.
Solve the following quadratic system.
$y = x^2 - 4x + 1$
$y = -x^2 + 4x - 5$

$2x^2 - 8x + 6 = 0$

$x^2 - 4x + 3 = 0$

$(x-1)(x-3) = 0$

$x = 1, 3$

$y = -2, -2$

$\therefore (1, -2)$ and $(3, -2)$

Solve the following systems using substitution.

85. Solve.

$y = x^2 - 4x + 1$
$y = -x^2 + 4x - 5$

$$x^2 - 4x + 1 = -x^2 + 4x - 5$$

$$2x^2 - 8x + 6 = 0$$

Divide all terms by 2.

$$x^2 - 4x + 3 = 0$$

Factor.

$$(x-1)(x-3) = 0$$

Solve using the Zero Product Rule.

$\therefore x = 1$ or $x = 3$

Use x-values and an original equation to calculate y-values.

Solution: $(1, -2)$ and $(3, -2)$

86. Solve.

$y = 3x^2 + 4x - 20$
$y = 2x^2 - 3x + 10$

$x^2 + 7x - 30 = 0$

$(x+10)(x-3) = 0$

$x = -10, 3$

$y = 240, 19$

$\therefore (-10, 240)$

$(3, 19)$

87. Solve.

$y = 5x^2 + 3x$
$y = 6x^2 + 2$

$x^2 - 3x + 2 = 0$

$(x-1)(x-2) = 0$

$x = 1, 2$

$y = 8, 26$

$\therefore (1, 8)$

$(2, 26)$

88. Solve.

$y = -\frac{1}{3}(x+2)^2 + 2$
$y = -\frac{1}{2}(x-3)^2 + 1$

$-\frac{1}{3}(x+2)^2 + 2 = -\frac{1}{2}(x-3)^2 + 1$

$2(x+2)^2 - 12 = 3(x-3)^2 - 6$

$2x^2 + 8x + 8 - 12 = 3x^2 - 18x + 27 - 6$

$2x^2 + 8x - 4 = 3x^2 - 18x + 21$

$x^2 - 26x + 25 = 0$

$(x-25)(x-1) = 0$

$x = 25, 1$

$y = -241, -1$

$\therefore (25, -241)$

$(1, -1)$

89. Solve.

$y = x^2 + 6x + 8$
$y = -x^2 - x + 2$

$2x^2 + 7x + 6 = 0$

$(2x+3)(x+2) = 0$

$x = -\frac{3}{2}, -2$

$y = \frac{5}{4}, 0$

$\therefore \left(-\frac{3}{2}, \frac{5}{4}\right)$

$(-2, 0)$

90. Solve.

$y = 10x^2 - x - 10$
$y = 8x^2 - x + 6$

$2x^2 - 16 = 0$

$x^2 = 8$

$x = \pm 2\sqrt{2}$

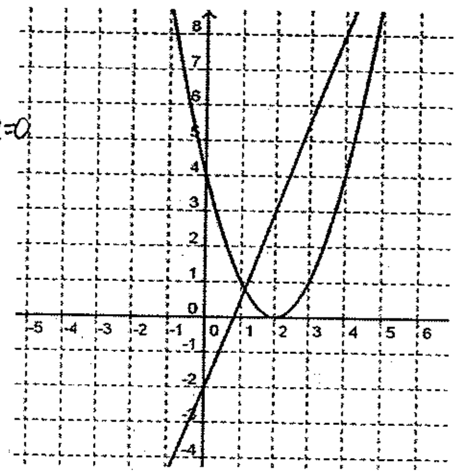
$y = 70 - 2\sqrt{2}, 70 + 2\sqrt{2}$

$\therefore (2\sqrt{2}, 70 - 2\sqrt{2})$

$(-2\sqrt{2}, 70 + 2\sqrt{2})$

91. The following graph shows part of the solution to $y = \frac{5}{2}x - 2$ and $y = (x-2)^2$. Calculate the values of x that satisfy both equations. Answer to the nearest tenth.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$



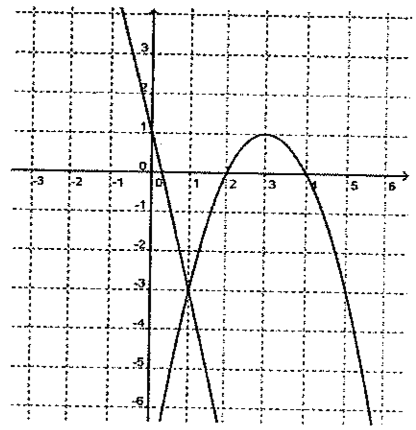
$x^2 - 4x + 4 - 2.5x + 2 = 0$

$x^2 - 6.5x + 6 = 0$

$x = 5.4, 1.1$

92. The following graph shows part of the solution to $y = -4x + 1$ and $y = -(x-3)^2 + 1$. Calculate the values of x that satisfy both equations. Answer to the nearest tenth.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$



$-x^2 + 6x - 9 + 1 + 4x - 1 = 0$

$x^2 - 10x + 9 = 0$

$(x-1)(x-9) = 0$ $\quad x = 1.0, 9.0$

93. CHALLENGE.

A squirrel launches itself from an oak tree at the instant a nut falls from a branch. The nut falls on a path that can be approximated by the equation $h(t) = -5t^2 + 80$. The squirrel's path approximates a straight line given by $h(t) = -5t + 50$. How long after launch will the squirrel intercept the nut?

$-5t^2 + 80 = -5t + 50$

$-5t^2 + 5t + 30 = 0$

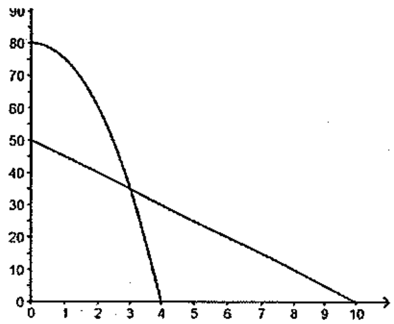
$t^2 - t - 6 = 0$

$(t-3)(t+2) = 0$

$t = 3, -2$ (can't work)

$\therefore$ 3 seconds after launch.

94. A squirrel launches itself from an oak tree at the instant a nut falls from a branch. The nut falls on a path that can be approximated by the equation $h(t) = -5t^2 + 80$, where height is in feet and time is in seconds. The squirrel's path approximates a straight line given by $h(t) = -5t + 50$. How long after launch will the squirrel intercept the nut?



$$-5t + 50 = -5t^2 + 80$$
$$5t^2 - 5t - 30 = 0$$
$$5(t-3)(t+2) = 0$$

3 seconds after launch, the squirrel has the nut.

95. Kai and some friends set up a zipline at Top Bridge. The zip line can be modelled by the equation $h(t) = -t + 45$, where height is in feet and time is in seconds. A water balloon is launched from a sling shot at the same time Kai leaves on the zipline. The water balloon can be modelled by $h(t) = -10t^2 + 54t$. Kai dodges the balloon on the way up but it hits him on the way down. How long after launch does Kai get hit?

$-t+45 = -10t^2+54t$

$-10t^2+t+54t-45=0$

$10t^2-55t+45=0$

$2t^2-11t+9=0$

$(2t-9)(t-1)=0$

$t=\frac{9}{2}, t=1$

$\therefore$ 4.5 seconds after launch.

96. Finnley tosses a teddy bear out of his playpen. The bear travels on a parabolic path modelled by the equation $h(t) = -\frac{1}{4}t^2 + 2t$, where height is in metres and time is in seconds. When will the bear first reach a height of 3 metres?

$t^2-8t+12=0$

$(t-2)(t-6)=0$

$t=2,\ 6$

$\therefore$ 2 seconds.

97. At an apple orchard near Parksville, Bob and Trevor are competing in child-like form of skeet shooting. One child throws an apple in the air, the other waits then tries to hit the apple out of the air with a second apple. If the first apple's height can be modelled by $h(t) = -15t^2 + 44.25t + 2.25$ and the second apple can be modelled by $h(t) = -8t^2 + 24.8t + 2.2$, find the time (in seconds) at which the apples collide.

$-7t^2+19.45t+0.05=0$

$t=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$

$t=0.00018,\ 2.78$

$\therefore$ 2.8 seconds.

98. The profit for a business venture can be modelled as a function of the number of items sold, $P(n) = -20n^2 + 1500n - 2500$. Solve an inequality that shows the possible number of sales that produce a profit greater than \$10 000.

$-20n^2+1500n-2500-10000=0$

$n^2-75n+625=0$

$n=9.5,\ 65.45$

$\therefore 9.5<n<65.5$

99. An amusement park game for children has the child launch small rubber balls from an air cannon in an attempt to land the balls in a water slide. The slide can be modelled by the equation $h(t) = \frac{3}{2}t - 6$ where height is in metres and time is in seconds. The rubber pall travels in a path modelled by the equation $h(t) = -5t^2 + 40t$. Will the ball reach the slide? If so, how long does it take to reach the slide?

$1.5t + 5t^2 - 6 - 40t = 0$

$5t^2 - 38.5t - 6 = 0$

$t^2 - 7.7t - 1.2 = 0$

$t = 7.9s$

$\therefore$ It takes 7.9 seconds.

100. A motorcycle takes off from a jump travelling on a path modelled by the function $h(d) = -0.2d^2 + 30$, where height and horizontal distance are in feet. A landing ramp that can be modelled by the equation $h(d) = -\frac{13}{8}d + 30$ is in place. Will the motorcycle land on the ramp? If so, at what height will it first make contact with the ramp?

$-0.2d^2 + \frac{13}{8}d = 0$

$0.2d^2 - \frac{13}{8}d = 0$

$d = 0,\ 8.125$

$h = -0.2d^2 + 30$

$= 16.8$

$\therefore$ at 16.8 feet.

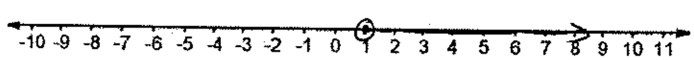
101. CHALLENGE.

Solve the following inequality and graph your solution on the number line provided.

$$-3x + 8 < 5$$

$-3x < -3$

$x > 1$



Inequalities in One Variable

Reminders:

- Solve an inequality like you would solve an equation, isolate the variable.
- Dividing (or multiplying) both sides by a negative requires that you reverse the symbol.
- Plotting: Solid dot with $\geq$ or $\leq$. Hollow dot with $>$ or $<$.

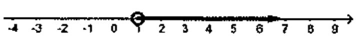
Solve the following inequality and graph your solution on the number line provided.

102. $-3x + 8 < 5$

Isolate the variable

$$-3x < -3$$
$$x > 1$$

Remember to reverse the symbol when dividing by a negative!

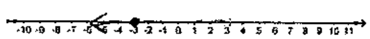


Note the hollow circle indicating that 1 is NOT part of the solution.

103. $-3x + 2 \geq 11$

$-3x \geq 9$

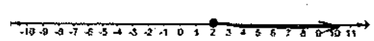
$x \leq -3$



104. $3x + 1 \geq 7$

$3x \geq 6$

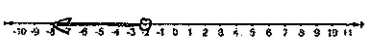
$x \geq 2$



105. $-4x - 2 > x + 8$

$-5x > 10$

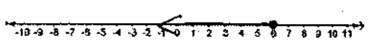
$x < -2$



106. $2x + 1 \geq 13 + 4x$

$-2x \geq 12$

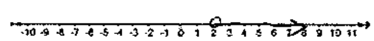
$x \leq 6$



107. $2x - 5 < 7x - 15$

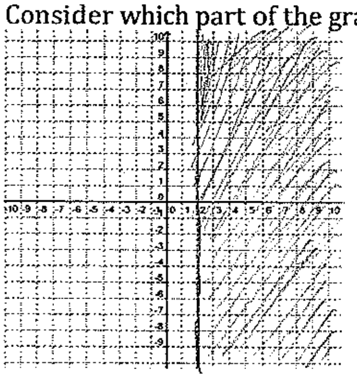
$-5x < -10$

$x > 2$



108. CHALLENGE.

Graph $x \geq 2$ on the coordinate plane below.
Consider which part of the graph satisfies $x \geq 2$



109. CHALLENGE.

How many ordered pairs satisfy the inequality $y \leq 3x + 2$?

infinite.

List 5 ordered pairs that satisfy $y \leq 3x + 2$.

1. (0, 0)
2. (1, 4)
3. (2, 6)
4. (3, 9)
5. (4, 10)

Inequalities In Two Variables [on the Coordinate Plane]

Boundary Line: The border of by a solution formed the corresponding equation (function).

- Solid: boundary line is part of the solution → for use with $\leq$ or $\geq$.
- Dotted: boundary line is NOT part of the solution → for use with $<$ or $>$.

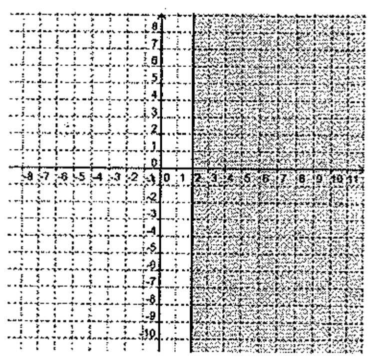
Solution Region: The boundary line "cuts" the coordinate plane into two regions one of which will represent the solution area for the linear inequality.

Graph the following inequalities on the coordinate plane.

110. Graph $x \geq 2$ on the coordinate plane.

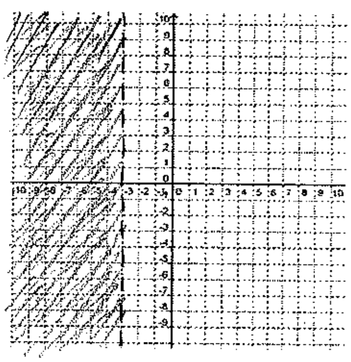
Ordered pairs in the region to the right of the vertical line $x = 2$ satisfy the above inequality. That is, they all have x-values equal to or greater than 2.

BOUNDARY LINE: $x = 2$



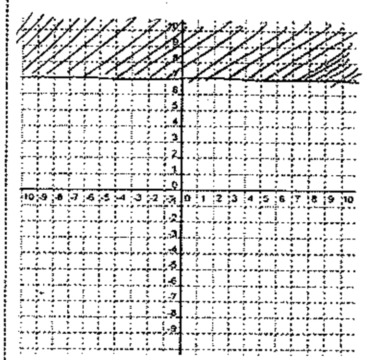
111. Graph $x < -3$ on the coordinate plane.

Boundary Line: $x = -3$



112. Graph $y \geq 7$ on the coordinate plane.

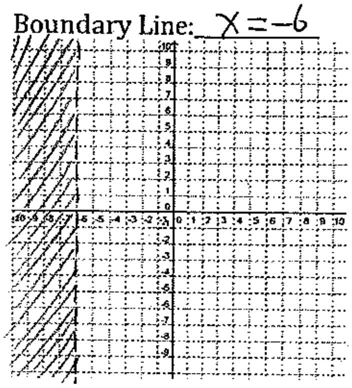
Boundary Line: $y = 7$



113. Graph.

$2x < -12$

Boundary Line: $x = -6$

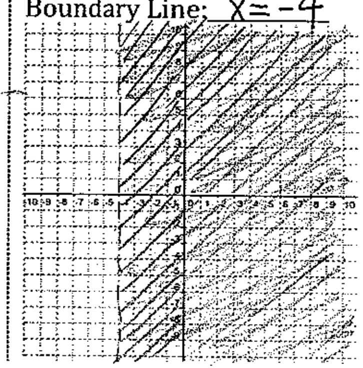


114. Graph.

$-3 - x < 1$

$-x < 4$ $\quad x > -4$

Boundary Line: $x = -4$

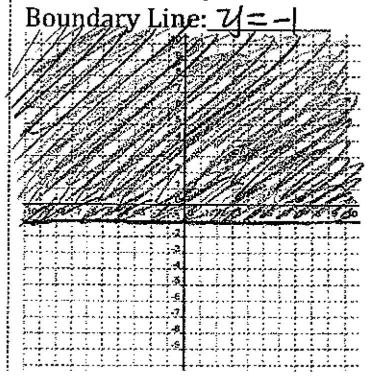


115. Graph.

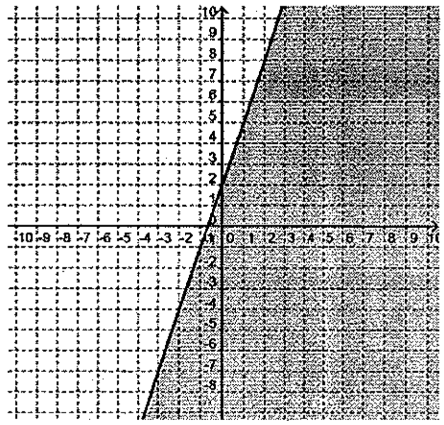
$5 - 2y \leq 8 + y.$ $\quad 3y \geq -3$

$5 \leq 8 + 3y$ $\quad y \geq -1$

Boundary Line: $y = -1$



116. How many ordered pairs satisfy the inequality $y \leq 3x + 2$?



There are an infinite number of ordered pairs that satisfy $y \leq 3x + 2$.

Eg.
(0,0), (2,1), (3,1), (0, −8), (−2, −9)

All the possible solutions are shown in the shaded region to the left.

117. Find 3 ordered pairs that satisfy the following inequality.
$y > 2x - 3$

Possible strategy…a quick graph like the one to the left.

(0, 0)

(1, 4)

(3, 5)

118. Find 3 ordered pairs that satisfy the following inequality.
$3x - y > -3$

(0,0) (1,2) (2,4)

119. Find 3 ordered pairs that satisfy the following inequality.
$6y - 4x > 2x - 30$

(0,0) (1,2) (2,4)

120. Find 3 ordered pairs that satisfy the following inequality.
$-2y \leq 2x + 6$

(0,0) (1,2) (2,4)

121. Is it possible to use a list of ordered pairs to show the entire solution to any of the inequalities above? Explain.

No. Because there are an infinite number of points that satisfy the inequalities.

122. How is the **solution** to the third question above similar to the first two?

The solution is a half-plane created by a boundary line.

How is it different?

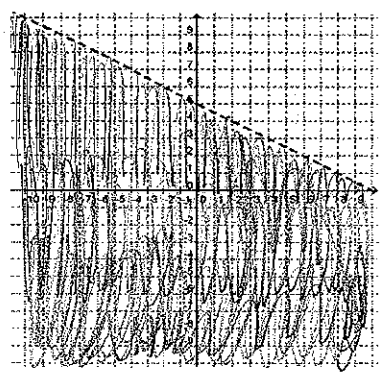
The boundary line on the third inequalities is part of the solution but it was not in the first two.

123. What do we know about all the points on the boundary line?

All the points on the boundary line make left side = right side. That's why points on the boundary line are only included in the solution if the inequality is either ≥ or ≤.

124. CHALLENGE.
Graph the inequality $y < -\frac{1}{2}x + 5$

The boundary line $y = -\frac{1}{2}x + 5$ has been provided.



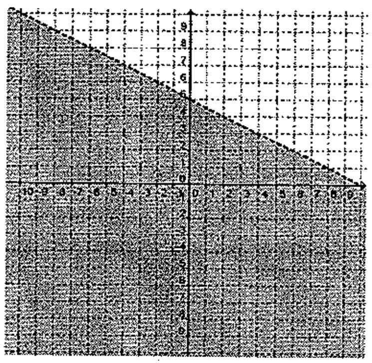
Graph the following linear inequalities.

125. Graph.

$$y < -\frac{1}{2}x + 5$$

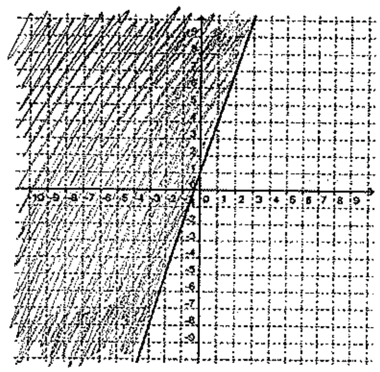
We need to decide which region to shade.

- Pick a point and test it.
- Shade below because the equation reads *"y is less than"*



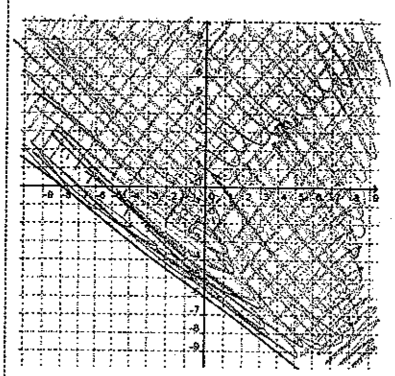
126. Given the boundary line, graph

$$y \geq 3x + 1$$



127. Given the boundary line, graph

$$y \geq -\frac{3}{4}x - 6$$



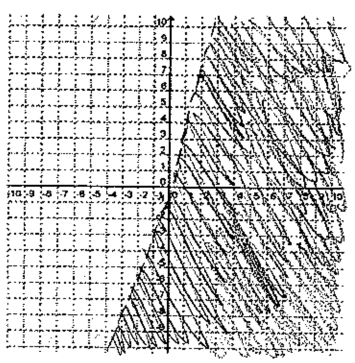
Shading

Method 1:
Pick a point on the graph and test in the original inequality. If true…shade the region of the graph including your test point.

Method 2:
If the equation has been rearranged to isolate y…
Shade below if $<$ or $\leq$.
Shade above if $>$ or $\geq$.

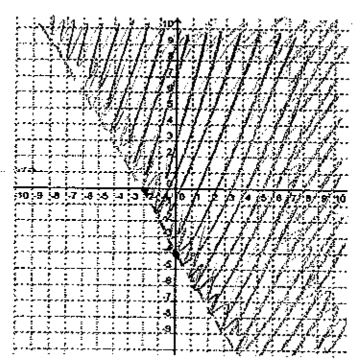
128. .Graph.

$$y < 4x - 1$$



129. Graph.

$$-2y - 3x < x + 8$$

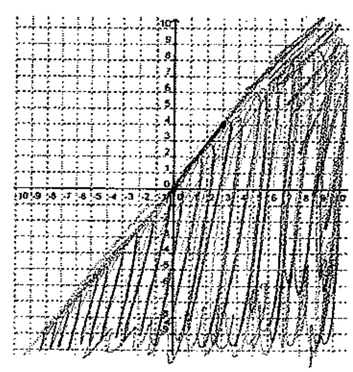


$-2y < 4x + 8$

$y > -2x - 4$

130. Graph.

$$4x - 3y \geq 0$$

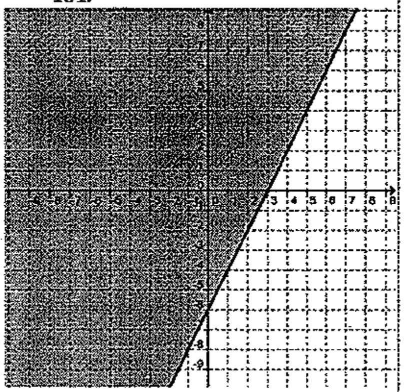


$4x \geq 3y$

$\frac{4}{3}x \geq y$

Write an inequality to represent each of the following graphs.

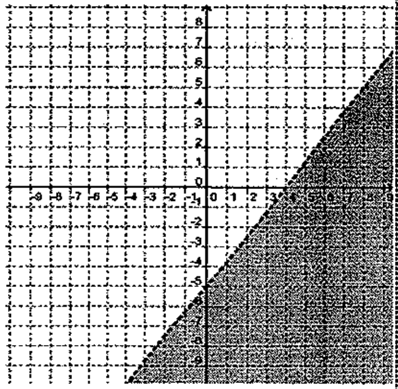
131.



$y=mx+b$
$y=mx-6$
$y=2x-6$

$y \geq 2x-6$

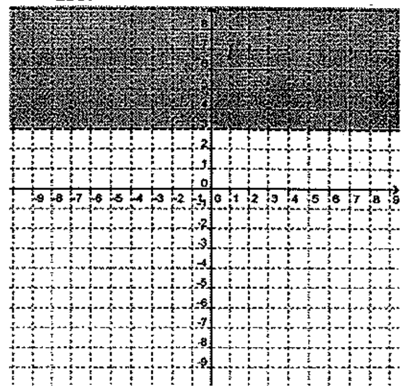
132.



$y=mx-5$
$y=\frac{5}{4}x-5$

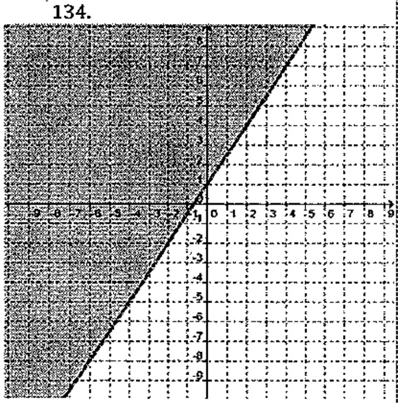
$y < \frac{5}{4}x-5$

133.



$y>3$

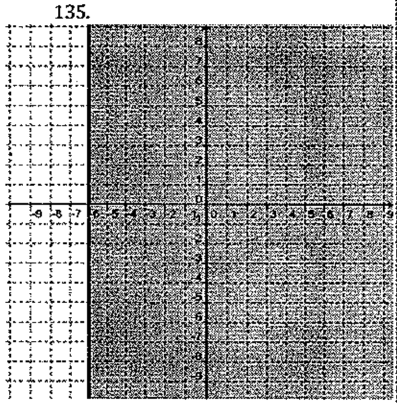
134.



$y=mx+1$
$y=\frac{3}{2}x+1$

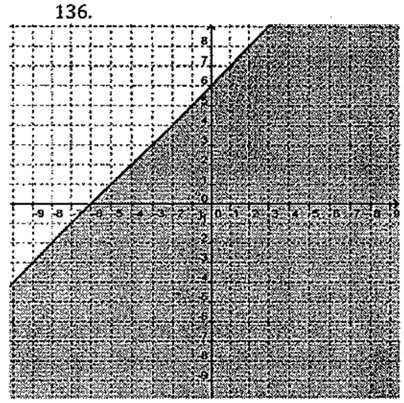
$y>\frac{3}{2}x+1$

135.



$x \geq -6$

136.



$y=mx+6$
$y=x+6$

$y \leq x+6$

137. Write an inequality for: The car holds up to 6 people.

$n \leq 6$

138. Write an inequality The volleyball team can have up to 12 girls.

$n \leq 12$

139. Write an inequality for: The golf team can have a combination of boys and girls but no more than 7 total people.

$b+g \leq 7$

140. Write an inequality for: My wage plus tips must be more than \$250.

$w+t > 250$

141. Write an inequality for: Sales for ties (\$5 each) and socks (\$3 each) must be at least \$90.

$5t+3s \geq 90$

142. Write an inequality for: Income from A dollars in an investment (10% return) must be at least \$2000.

$0.1A \geq 2000$

Applications of Linear Inequalities & Their Graphs

Discrete Data or Continuous Data

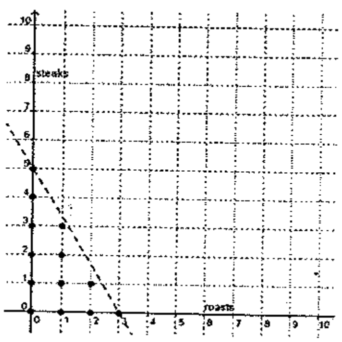
Data where not all numbers are valid. Often this excludes fractions.
Eg. Data about manufacturing items involves whole numbers only.
GRAPHS → Dotted Line / Dotted Region

Data that includes all values, or all values over a specific range.
Eg. Time or height would be continuous.
Graphs → Solid Line / Shaded Region

Answer each of the following questions about linear inequalities..

143. A person is to put 5kg roasts and 3kg steaks into a backpack. The backpack can hold up to 15 kg.

Write an Inequality: $5r+3s \leq 15$



144. Why do you think the graph to the left is filled in with a series of dots, not shaded?

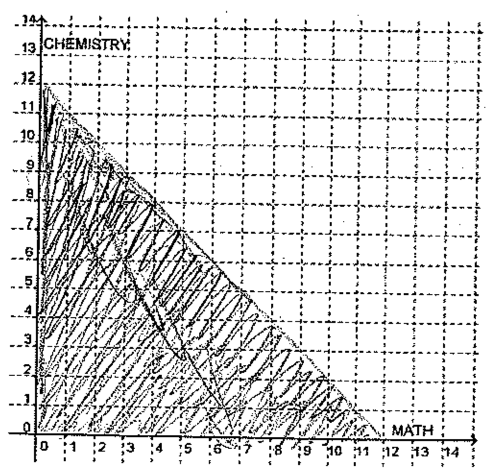
Only whole roasts or steaks will be carried in the backpack.

145. How many possible combinations of roasts and steaks can the person carry in the backpack?

13.

146. Upcoming exams are forcing Finnley to set a study schedule for chemistry and math. He has up to 12 hours available. Write and graph the inequality to show how he could divide his time.

$m+c \leq 12$

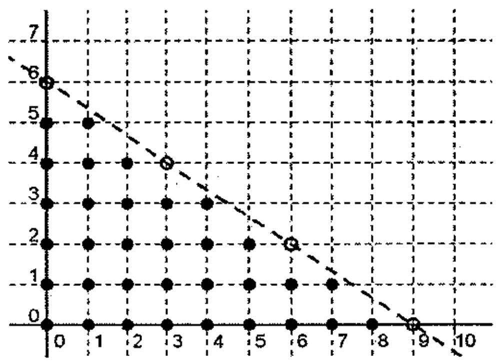


147. Write an inequality for the following graph.

Give a possible scenario that could be represented by the graph/inequality.

Note, only Quadrant 1 is shown indicating only positive quantities are to be considered.

$2x+3y<18$

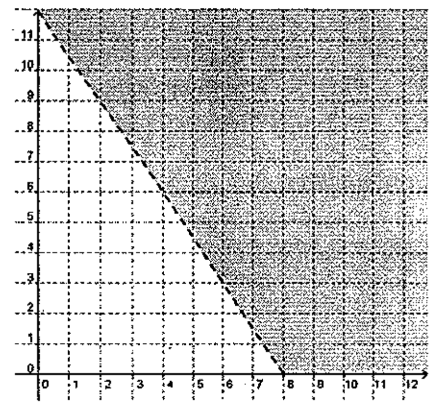


148. Write an inequality for the following graph.

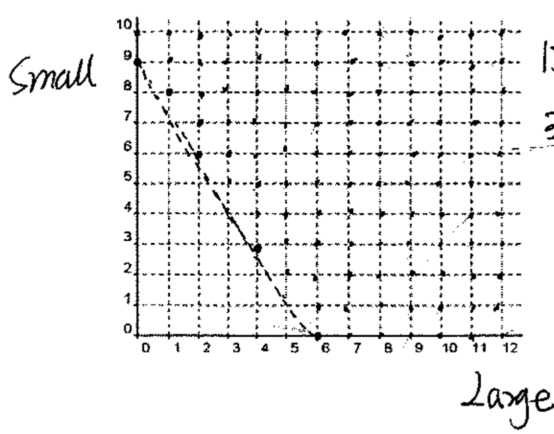
Give a possible scenario that could be represented by the graph/inequality.

Note, only Quadrant 1 is shown indicating only positive quantities are to be considered.

$2y+3x>24$



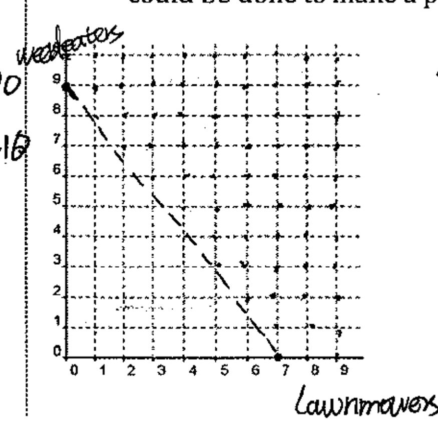
149. DianaLynn sells photos at the Sidney Market. Large photos sell for 15 dollars and small photos sell for 10 dollars. She pays $90 for the permit each night. Draw a graph to show how many of each type she needs to sell to cover her permit costs.



$15L+10S\geq 90$

$3L+2S\geq 18$

150. MowerManipulators repairs lawnmowers and weedeaters. They charge $45 to service lawnmowers and $35 for weed eaters. To make a profit, they need to earn more than $315 per day. Draw a graph to show what repairs could be done to make a profit.

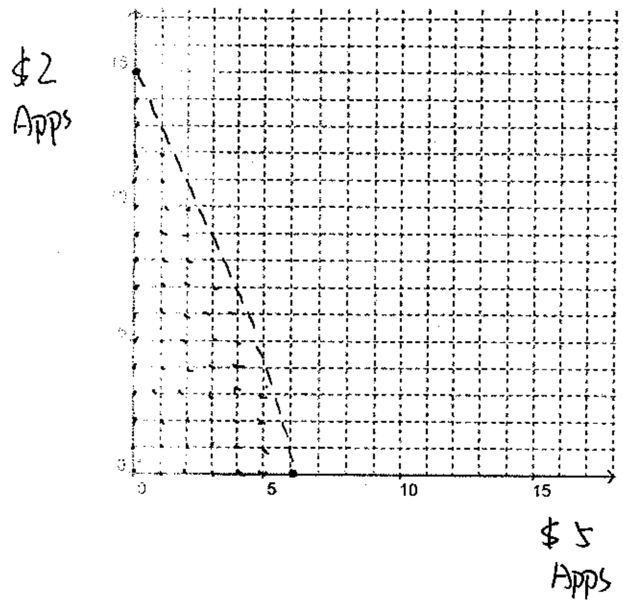


$45L+35W>315$

$9L+7W>63$

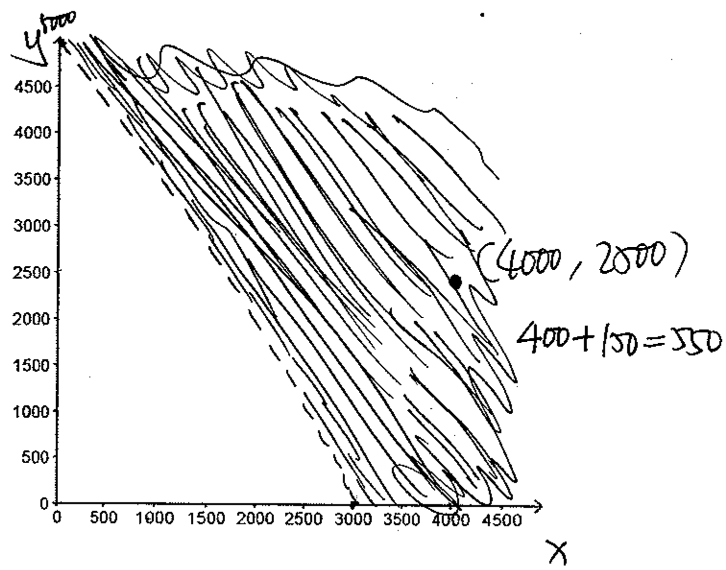
151. Smartphone Apps at a new internet site cost either \$5 or \$2. Amy was given a gift card for \$30. Write and graph an inequality that shows how she could spend her gift card.

Inequality $2x+5y \leq 30$



152. Tariq invests in two different types of bonds. One bond (x) earns 10% per year, the other (y) earns 6% per year. If he must invest in both bonds, and he needs to earn more than \$300 this year show the possible investments he could make on the graph below.

Inequality $0.10x+0.06y>300$



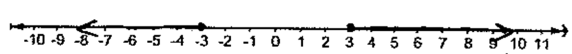
153. CHALLENGE.

Solve the following quadratic inequality and plot your solution on the number line.

$$x^2 - 9 \geq 0$$

$x^2 \geq 9$

$x \geq 3$, $x \leq -3$



Quadratic Inequalities in one variable.

Solve the following quadratic inequalities. Plot your solution on the number line.

154. $x^2 - 9 \geq 0$

Use the corresponding equation.
We could isolate the x^2, then square root both sides.

$$x^2 = 9$$
$$x = \pm 3$$

3 *and* -3 are our key points.
Use some number logic or test some values on number line to conclude that $x \geq 3$ *or* $x \leq -3$.

Eg. $4^2 - 9 \geq 0$...TRUE $2^2 - 9 \geq 0$ FALSE

155. $2x^2 > 32$

$x^2 > 16$
$x > 4$
$x < -4$

156. $x^2 - 32 \leq 32$

$x^2 \leq 64$
$-8 \leq x \leq 8$

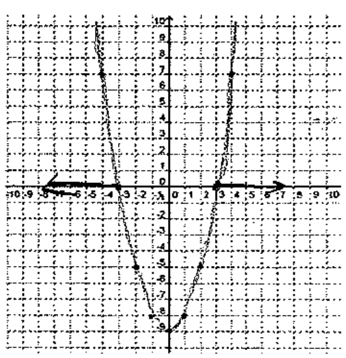
157. $3x^2 - 2 > 25$

$3x^2 > 27$
$x^2 > 9$
$x > 3$, $x < -3$

158. $(x - 3)^2 < 25$

$-2 < x < 8$

159. CHALLENGE.
Solve $x^2 - 9 \geq 0$ using the graph of $y = x^2 - 9$.

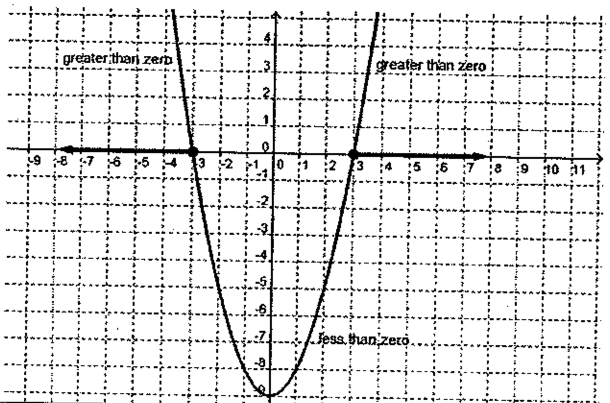


$x \leq -3$
$x \geq 3$

160. Solve $x^2 - 9 \geq 0$ using the graph of $y = x^2 - 9$.

The inequality is asking us when $x^2 - 9$ is above zero.

If we were to consider where $y = x^2 - 9$ was "above zero", we would be looking for the points on the graph above the x-axis.

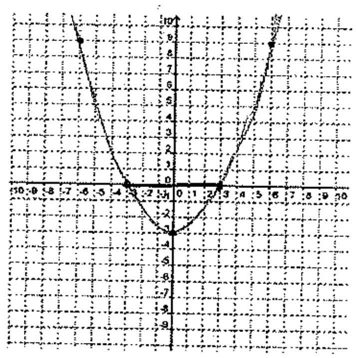


Shown on the x-axis are the values where the parabola is "greater than zero".

We get our solution from there.

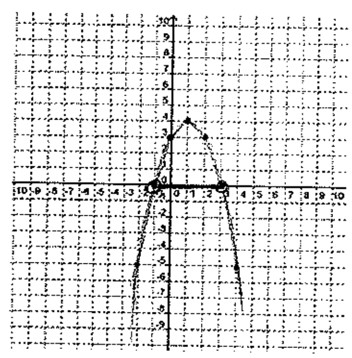
$x \leq -3$ or $x \geq 3$

161. Solve $\frac{1}{3}x^2 - 3 \leq 0$ using the graph of $y = \frac{1}{3}x^2 - 3$.



$-3 \leq x \leq 3$

162. Solve $-(x-1)^2 + 4 > 0$ using the graph of $y = -(x-1)^2 + 4$



$-1 < x < 3$

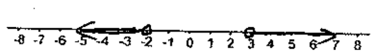
163. CHALLENGE.

Solve the following quadratic inequality. Plot your solution on the number line.

$x^2 - x - 6 > 0$

$(x-3)(x+2) = 0$

$x = 3, -2$



$x < -2,\ x > 3$

Solve the following quadratic inequalities.

164. **CHALLENGE.**

Solve the following quadratic inequality.

Plot your solution on the number line.

$$x^2 - x - 6 > 0$$

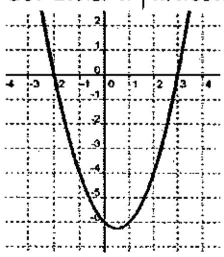
POSSIBLE SOLUTION:

Solve the corresponding equation.

$$x^2 - x - 6 = 0$$
$$(x-3)(x+2) = 0$$
$$x = -2,3$$

These are key values (intercepts).

Consider a parabola with those x-intercepts opening up.



Where is the parabola "greater than zero"?

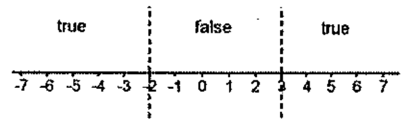
Solution: $x < -2$ or $x > 3$

165. $x^2 - x - 6 > 0$

ALTERNATE SOLUTION:

$$x^2 - x - 6 = 0$$
$$(x-3)(x+2) = 0$$
$$x = -2,3$$

These are key values, they divide the x-axis into intervals. Test a value from each interval to see if it satisfies the inequality.



$\boxed{\text{Try } -5}$: $(-5)^2 - (-5) - 6 > 0$
$25 + 5 - 6 > 0$ ...True

$\boxed{\text{Try } 0}$: $0^2 - 0 - 6 > 0$
$-6 > 0$ ... False

$\boxed{\text{Try } 4}$: $4^2 - 4 - 6 > 0$
$16 - 4 - 6 > 0$ ... True

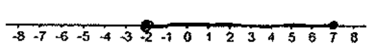
From the "true" intervals we get:
$x < -2$ or $x > 3$

166. $x^2 - 5x - 14 \le 0$

$(x-7)(x+2)=0$

$x=7,-2$

$-2 \le x \le 7$

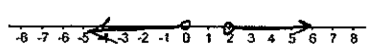


167. $2x^2 - 4x > 0$

$2x(x-2)=0$

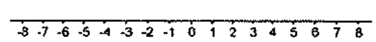
$x=0, 2$

$x<0, x>2$



168. $-2(x-3)^2 > 0$

no solution.



169. Solve.

$2x^2 - 8x - 10 < 0$

$x^2-4x-5<0$

$(x-5)(x+1)=0$

$x=5,-1$

$-1<x<5$

170. Solve.

$-x^2 + 5x \ge -6$

$x^2-5x-6 \le 0$

$(x-6)(x+1)=0$

$x=6,-1$

$-1 \le x \le 6$

171. Solve.

$4x^2 + 8x + 6 < 0$

$2x^2+4x+3<0$

no solution.

Recall, not all quadratic expressions are factorable.

Use the quadratic formula when necessary.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Solve the following quadratic inequalities. Answer to the nearest tenth if necessary.

172. $2x^2 - 5x + 1 > 0$

$x = \frac{5 \pm \sqrt{25-8}}{4}$

$= 2.3,\ 0.2$

$x < 0.2,\ x > 2.3$

173. $3x^2 - 8x \leq 0$

$x(3x-8)=0$

$x = 0,\ 2.7$

174. $\frac{x^2}{2} - \frac{x}{3} > 4$

$3x^2 - 2x - 24 > 0$

$x = \frac{2 \pm \sqrt{4+288}}{6}$

$= 3.2,\ -2.5$

$x < -2.5,\ x > 3.2$

175. $5x^2 - 6 < 2x + 14$

$5x^2 - 2x - 20 < 0$

$x = \frac{2 \pm \sqrt{4+400}}{10}$

$= 2.2,\ -1.8$

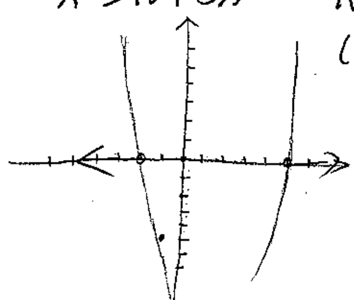
$-1.8 < x < 2.2$

176. CHALLENGE

Consider the following relationship between two numbers...

The "square of a number" is greater than "ten more than triple the same number." Graph the relation to show the possible numbers that would satisfy this relationship.

$x^2 > 10 + 3x$



$x^2 - 3x - 10 > 0$

$(x-5)(x+2) > 0$

$x = 5,\ -2$

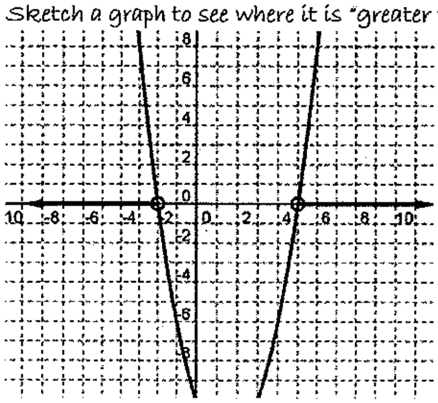
$x < -2,\ x > 5$

177. The "square of a number" is greater than "ten more than triple the same number." Graph the relation to show the possible numbers that would satisfy this relationship.

$$x^2 > 3x + 10$$
$$x^2 - 3x - 10 > 0$$
$$(x+2)(x-5) > 0$$

Intercepts: $x = -2, 5$

Sketch a graph to see where it is "greater than zero".



Solution: $x < -2$ or $x > 5$.

178. Five more than a number is greater than four times the square of the same number. Write and solve an inequality to show the possible values of the number.

$5 + x > 4x^2$

$5 + x - 4x^2 > 0$

$4x^2 - x - 5 < 0$

$(4x-5)(x+1) < 0$

$x = \frac{5}{4}, -1$

$-1 < x < \frac{5}{4}$

179. An object is flung through the air on a parabolic path modelled by the equation $h = -\frac{1}{2}t^2 + 4t$, where height is measured in metres and time in seconds. For how long is the object at or above 6 metres?

$-\frac{1}{2}t^2 + 4t \geq 6$

$-t^2 + 8t - 12 \geq 0$

$t^2 - 8t + 12 \leq 0$

$(t-2)(t-6) \leq 0$

$t = 2, 6$

$2 \leq t \leq 6$ ∴ for 4 seconds.

180. Sammy is watching Canada Day fireworks from his backyard. A fence obstructs his view for part of the flight path of each flare. The path of the flares can be modelled by the function $h(t) = -30t^2 + 170t + 2$, where height is in metres and time is in seconds. Sammy's line of sight can be modelled by the equation $h(t) = -5t + 40$ where he can only see above this line. For how long is the flare visible to Sammy?

$-30t^2 + 170t + 2 = -5t + 40$

$-30t^2 + 175t - 38 = 0$

$30t^2 - 175t + 38 = 0$

$t = \frac{175 \pm \sqrt{30625 - 4560}}{60}$

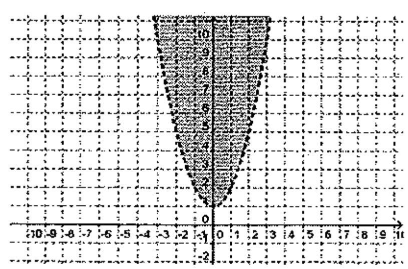
$= 5.6, 0.2$

$5.6 - 0.2 = 5.4$

∴ For 5.4 seconds.

181. CHALLENGE.

Write an inequality for the graph.

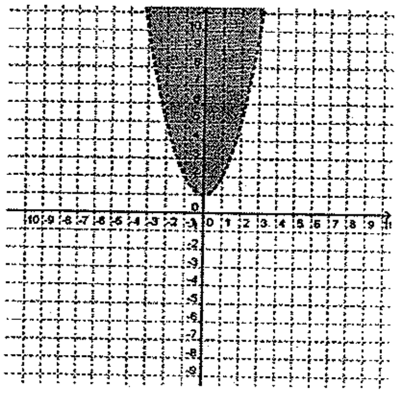


$y = x^2 + 1$

$y > x^2 + 1$

Write quadratic inequalities for the following graphs.

182.



The boundary line is a parabola that has the quadratic equation

$$y = x^2 + 1$$

Shading occurs above the <u>dotted</u> line

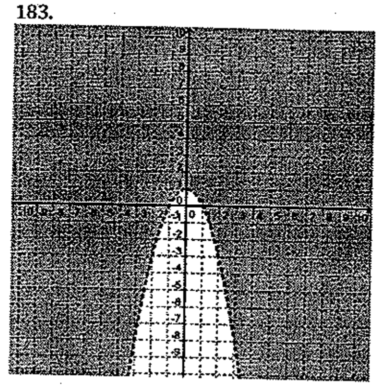
$$\therefore y > x^2 + 1$$

Check test point: (0,4)

$$4 > 0^2 + 1$$
$$4 > 1$$

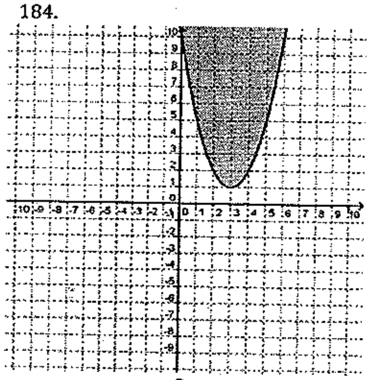
True, therefore correct inequality.

183.



$y = -x^2 + 1$ $\quad y > -x^2 + 1$

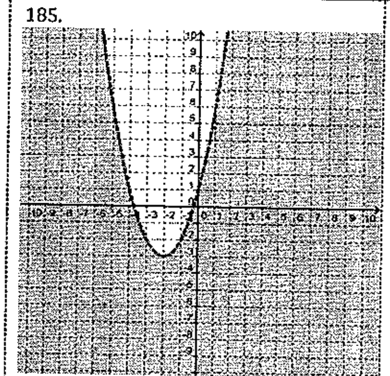
184.



$y = (x-3)^2 + 1$

$y \geq (x-3)^2 + 1$

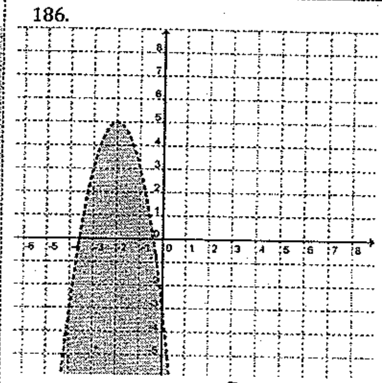
185.



$y = (x+2)^2 - 3$

$y < (x+2)^2 - 3$

186.

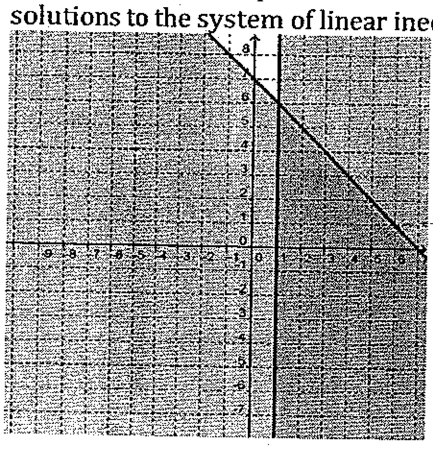


$y = -2(x+2)^2 + 5$

$y < -2(x+2)^2 + 5$

187. CHALLENGE

Name three ordered pairs that would be possible solutions to the system of linear inequalities below.



$x \geq 1$

$y = -x + 7$

$y \leq -x + 7$

$y + x \leq 7$

(2,0)

(3,0)

(4,0)

188. CHALLENGE

Write a system of inequalities for the graph to the left.

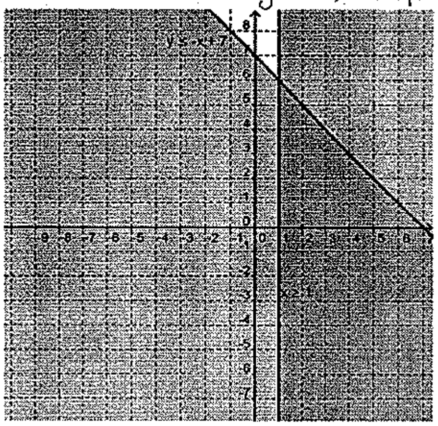
Systems of Linear Inequalities

A system of inequalities exists when two or more inequalities are considered together to model a situation or problem.

Write a system of inequalities for each of the following graphs.

189.

There are two boundary lines, therefore there will be two inequalities.



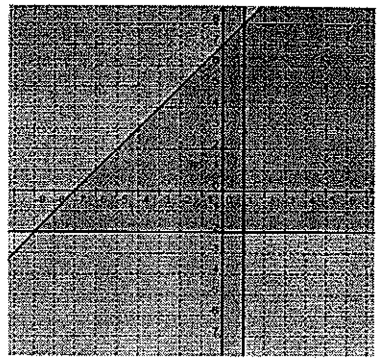
$x = 1$...*shaded "greater than"*

$x + y = 7$... *shaded "less than"*

System:

$$x \geq 1$$
$$x + y \leq 7$$

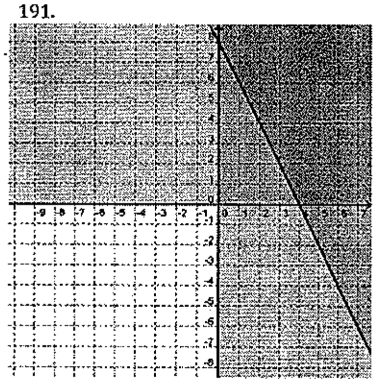
190.



~~x~~ $y=x+7$ $y \geq -2$

$y \leq x+7$ $x \leq 1$

191.

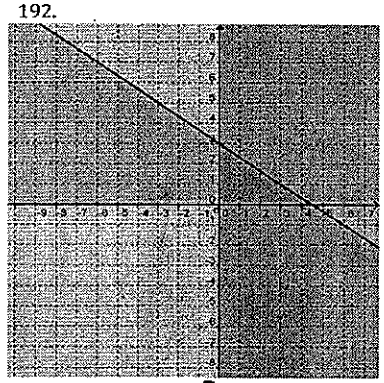


$x \geq 0$ $y=-2x+8$

$y \geq 0$ $y \geq -2x+8$

$2x+y \geq 8$

192.

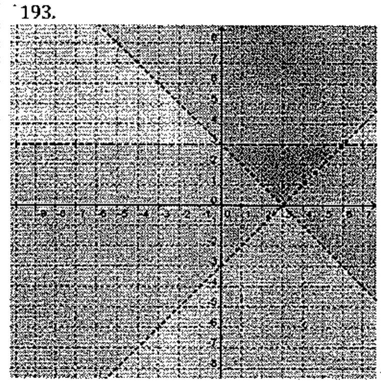


$x \geq 0$ $y=-\frac{2}{3}x+3$

$y \geq 0$ $3y<-2x+9$

$2x+3y \leq 9$

193.



$y<3$ $y=x-3$ $y=-x+3$

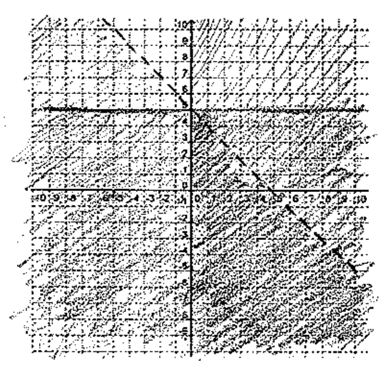
$y-x>-3$ $y+x>3$

$x-y<3$

194. CHALLENGE

Graph the following system. Clearly identify the solution region.

$$x \geq 0$$
$$y \leq 5$$
$$x + y < 5$$



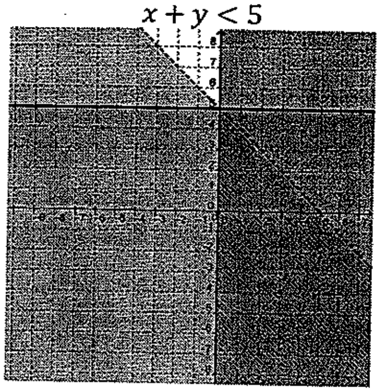
List 3 ordered pairs that would satisfy this system.

(1,0) (2,0) (3,0)

Graph the following system. Clearly identify the solution region.

195. Graph.

$x \geq 0$
$y \leq 5$
$x + y < 5$



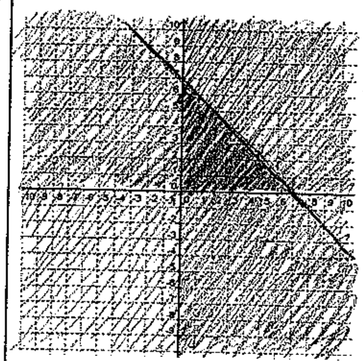
Graph each corresponding equation.
Remember <u>solid</u> or <u>dotted</u> line.

Chose the correct "side" of each line to shade.

The solution is the region shaded by all 3 inequalities.

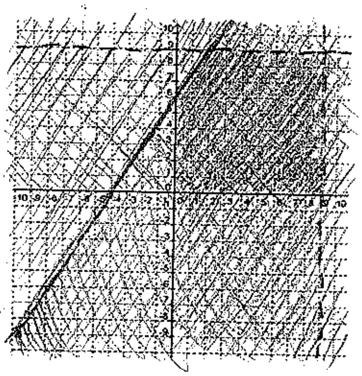
196. Graph.

$x \geq 0$
$y \geq 0$
$x + y \leq 7$



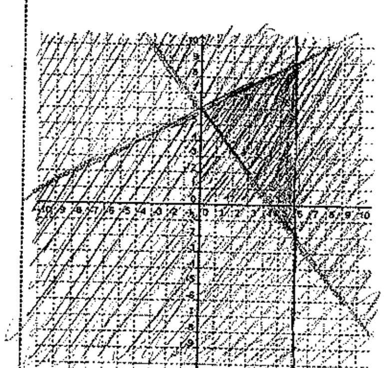
197. Graph.

$x, y \geq 0$
$x < 9$
$y < 9$
$-3x + 2y \leq 12$



198. Graph.

$4x + 3y \geq 18$
$-x + 2y \leq 12$
$x \leq 6$

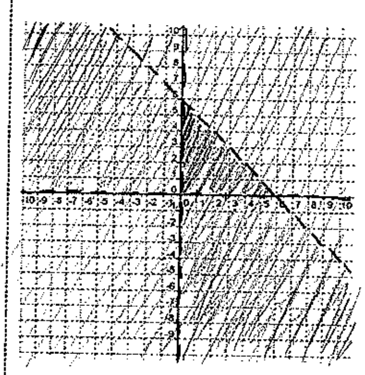


$A = \frac{1}{2}(11 \times 6) = 33$ units.

Find the area of the solution region to the nearest square unit.

199. Graph.

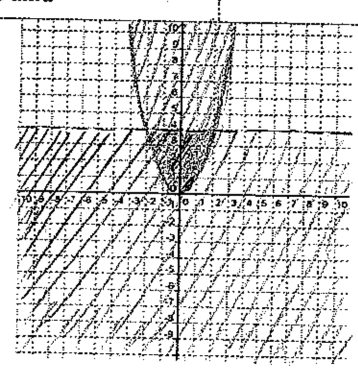
$x > 0$
$y > 0$
$y < -x + 6$



Is (2,4) a solution to this system? No.

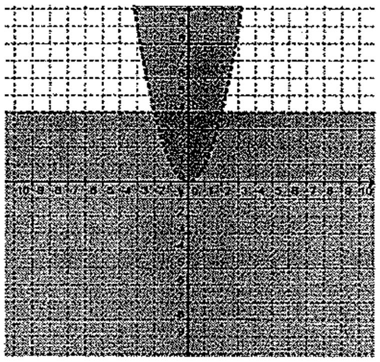
200. CHALLENGE.
Graph the system of inequalities.

$y > x^2$
$y < 4$



Graph the following systems of linear and quadratic inequalities. Clearly identify the solution region. Test a point to check your solution.

201. $y > x^2$



Graph a dotted parabola for $y = x^2$ using the method of intervals or some other method. Shade above.

Graph the dotted boundary line $y = 4$ Shade below.

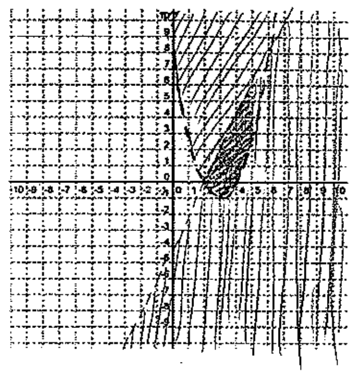
Test a point such as $(0,2)$

$y > x^2$ $\quad$ $y < 4$

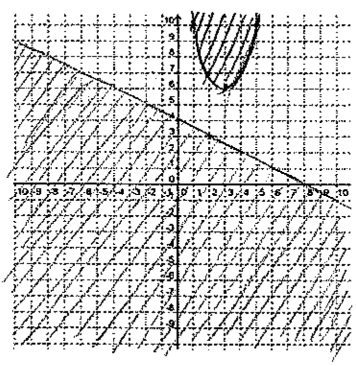
$2 > 0^2$ $\quad$ $2 < 4$

$(0,2)$ satisfies both.

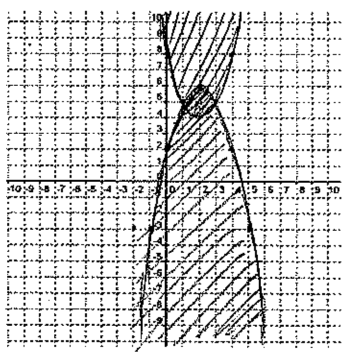
202. $y < 2x - 4$
$y > (x-3)^2 - 1$



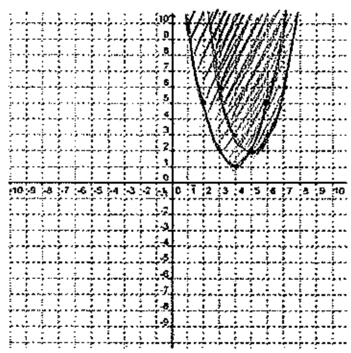
203. $y \le -\frac{1}{2}x + 4$
$y \ge (x-3)^2 + 6$



204. $y \le -(x-2)^2 + 6$
$y \ge (x-2)^2 + 4$



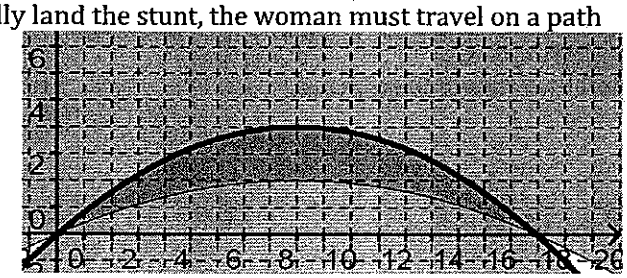
205. $y \ge (x-4)^2 + 1$
$y \ge (x-5)^2 + 2$



206. CHALLENGE.

At a circus, a trapeze act has a woman "fly" from one tower to the next in a parabolic path. The two towers are 18 m apart. To successfully land the stunt, the woman must travel on a path defined by the quadratic equations below.

$y = -0.05(x-9)^2 + 4$
$y = -0.025(x-9)^2 + 2$



Write a system of inequalities that represents the safe flight path for the trapeze artist.

$y \le -0.05(x-9)^2 + 4$

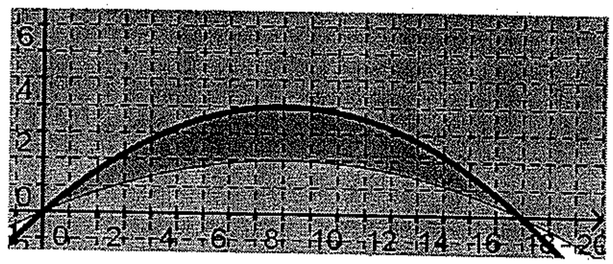
$y \geq -0.025(x-9)^2 + 2$

Applications of Systems of Inequalities.

207. CHALLENGE.

At a circus, a trapeze act has a woman "fly" from one tower to the next in a parabolic path. The two towers are 18 m apart. To successfully land the stunt, the woman must travel on a path defined by the quadratic equations below.

$y = -0.05(x-9)^2 + 4$
$y = -0.025(x-9)^2 + 2$

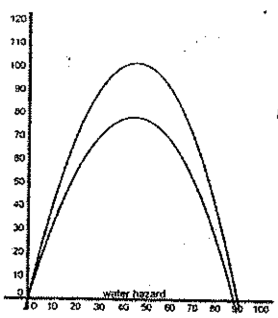


Write a system of inequalities that represents the safe flight path for the trapeze artist.

She must fly above the lower path and below the upper path.

$y \leq -0.05(x-9)^2 + 4$ *and* $y \geq -0.025(x-9)^2 + 2$

208. Porter has a short chip shot over the water on the 18^{th} hole at Yellowville GC. To land safely on the green the ball flight should be somewhere between the paths defined by two quadratic equations
$h(x) = -0.04x^2 + 3.53x$ and
$h(x) = -0.05x^2 + 4.5x$.
Use the equations to write a system of inequalities for the safe chip shot.



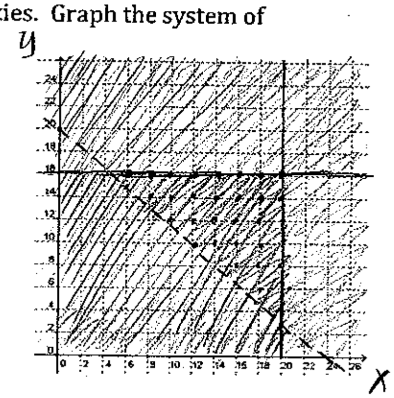
$h(x) \leq -0.05x^2 + 4.5x$
$h(x) \geq -0.04x^2 + 3.53x$

209. Z-Games officials regulate the shape of the half pipe for snowboard events. A cross-section of the course approximates the shape of a parabola. Competition regulations require the course is somewhere on or between the parabolas of $y = \frac{1}{40}x^2$ and $y = \frac{1}{60}x^2$. Use the equations to write a system of inequalities for a regulation course.

$y \leq \frac{1}{40}x^2$
$y \geq \frac{1}{60}x^2$

210. CHALLENGE.

Anya's lemonade stand sells lemonade for \$0.50 and cookies for \$0.60. To break even she must sell at least \$12. Anya has enough lemonade to sell up to 20 cups and has made 16 cookies. Graph the system of inequalities to find combinations that would enable her to make a profit.



Let x be the number of lemonades sold
y be the number of cookies sold.

$x \geq 0,\ y \geq 0$
$x \leq 20,\ y \leq 16$
$0.5x + 0.6y > 12$ $\quad$ $0.6y = -0.5x + 12$
$y = -\frac{5}{6}x + 20$

211. Anya's lemonade stand sells lemonade for $0.50 and cookies for $0.60. To break even she must earn more than $12. Anya has enough lemonade to sell up to 20 cups and has made 16 cookies. Graph the system of inequalities to find combinations that would enable her to make a profit.

Let x= number of lemonades sold
Let y= number of cookies sold

Since we are talking about selling items, negative quantities don't apply.

$$x \geq 0, y \geq 0$$

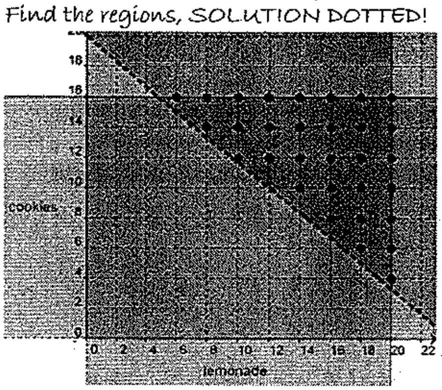
From her limits on supplies...

$$x \leq 20, y \leq 16$$

To make a profit, sales must be over $12.

$$0.5x + 0.6y > 12$$

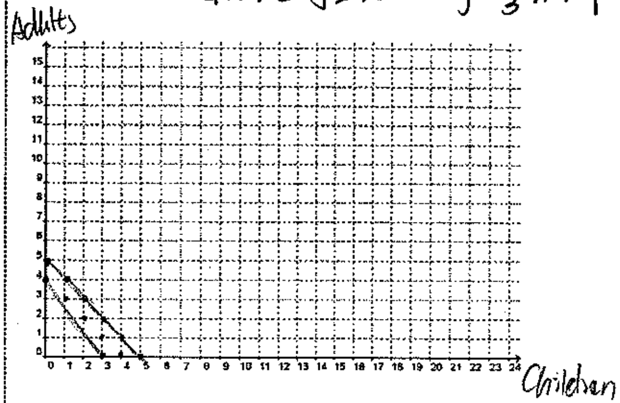
Find the regions, SOLUTION DOTTED!



212. A pedi-cab will hold up to 5 people. The cab has a flat rate of $4 for each adult and $3 for each child. The cab must make at least $12 per ride to cover his costs. Write and graph inequalities to show possible combinations of adults and children for each ride.

$x \geq 0$ $\quad x+y \leq 5$

$y \geq 0$ $\quad 4x+3y \geq 12$ $\quad y=\frac{4}{3}x+4$



What combination of adults and children maximizes revenue?

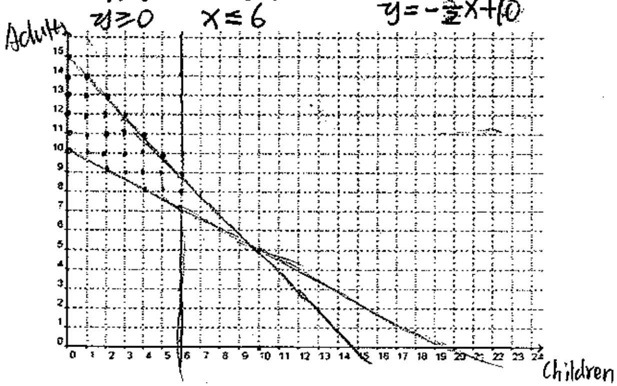
5 adults and 0 children.

How many combinations are there that allow the cab to reach its $12 goal? 12.

213. A whale watching boat seats a maximum of 15 people. By law, the boat cannot have more than 6 children on board. To make a profit, the boat must sell at least 100 dollars in tickets. Adults pay $10 and children pay $5. Write and graph a system of inequalities to show possible combinations of adults and children required to turn a profit.

$x \geq 0$ $\quad x+y \leq 15$ $\quad 5x+10y \geq 100$

$y \geq 0$ $\quad x \leq 6$ $\quad y=-\frac{1}{2}x+10$



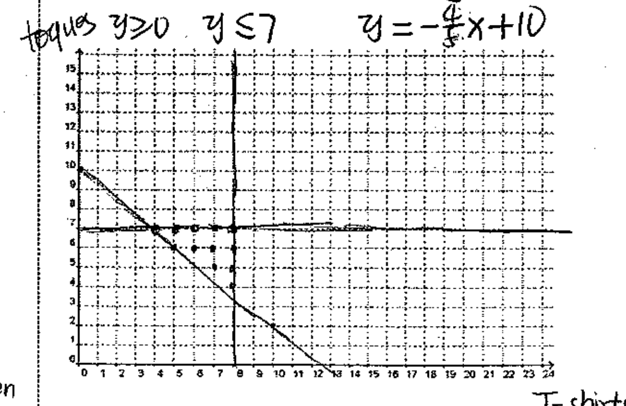
How many possible combinations are there?

30

214. OBVB sells t-shirts and toques to fundraise for a trip to Hawaii. T-shirts sell for $12 and toques for $15. The daily goal is to sell at least $150 in merchandise. Production limits daily sales of t-shirts to 8 and toques to 7. Write and graph a system of inequalities to show possible combinations of sales to reach their goal.

$x \geq 0$ $\quad x \leq 8$ $\quad 12x+15y \geq 150$

$y \geq 0$ $\quad y \leq 7$ $\quad y=-\frac{4}{5}x+10$



What combination maximizes revenue?

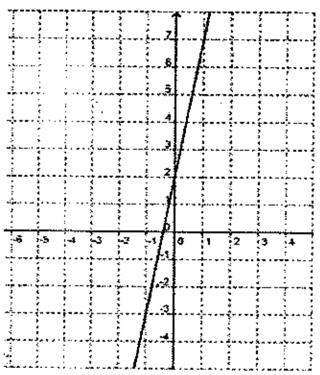
8 T-shirts and 7 toques.

SYSTEMS OF EQUATIONS & INEQUALITIES

Answer Key

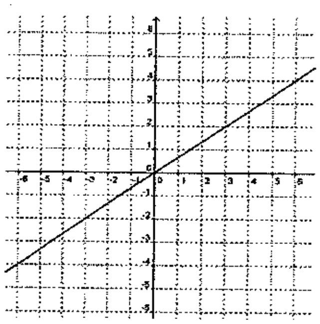
1. All linear equations have variables that are degree 1. There may be two variables present or only one variable. Note: $x = 3$ produces a graph that is linear but it is not a function.

2.



3. Answered in booklet.

4.



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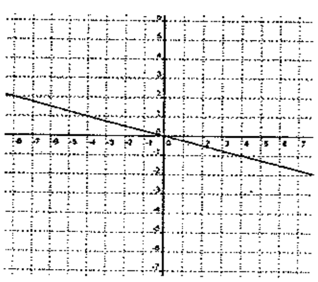

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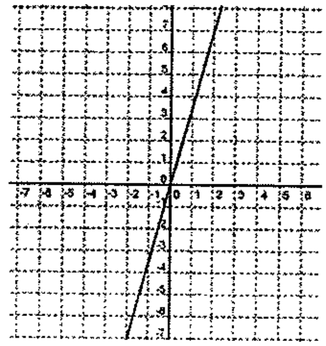

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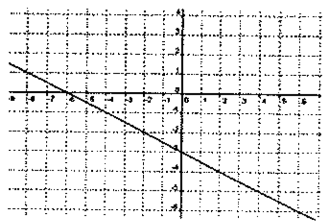

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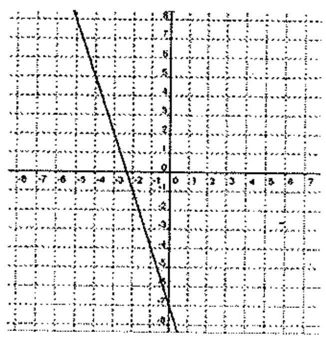

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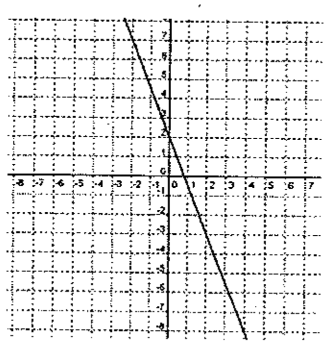

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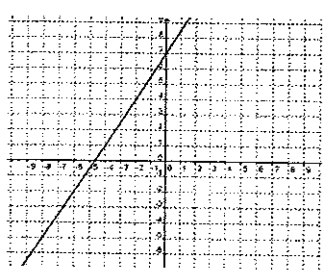
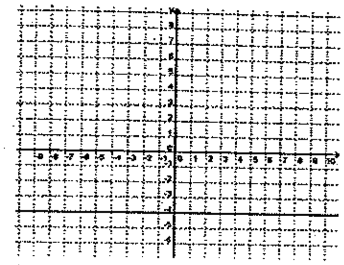
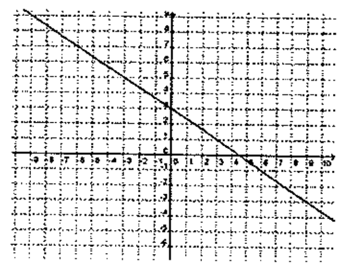


11.



12. Answered on page.

13.



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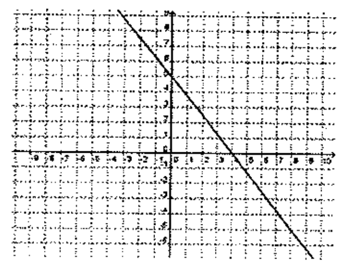

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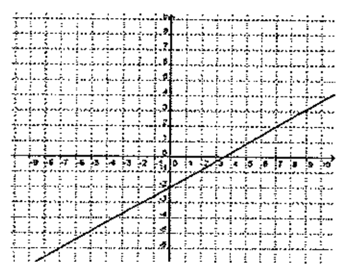

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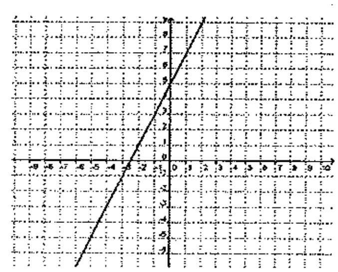
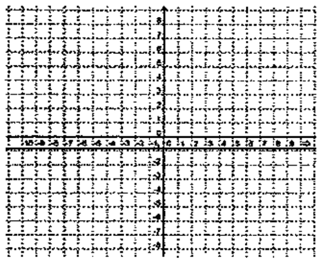
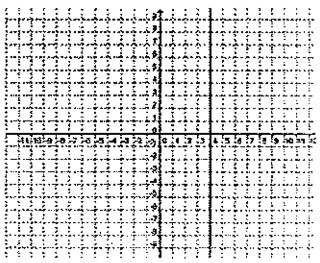


17.



18. Answered on page.

19.



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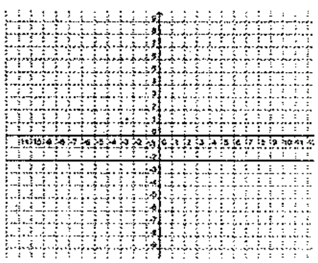

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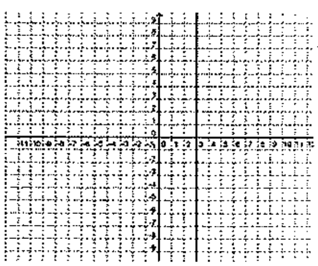

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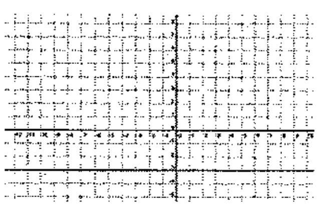
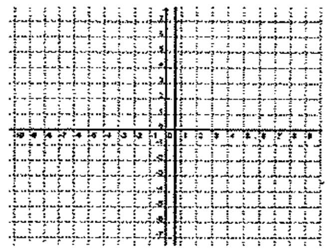


23.



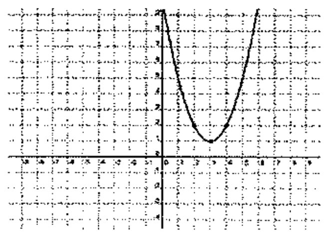
24. $3x - 5y = 10$

25. $5x + y = -3$

26. $y = -5$

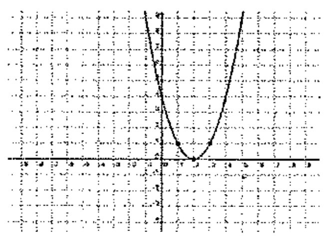
27. $x = 3$

28.

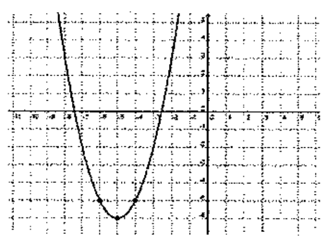


29. Answered on page.

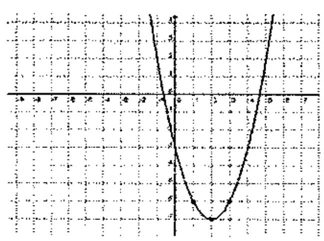
30.



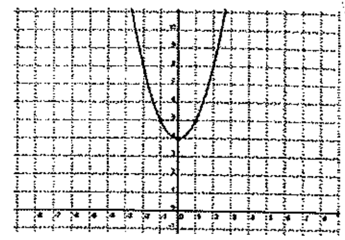
31.



32.



33.



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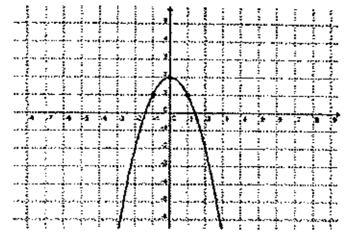

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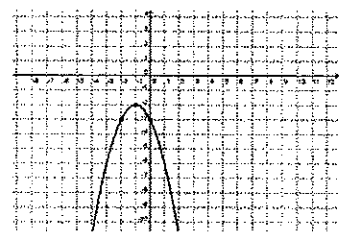
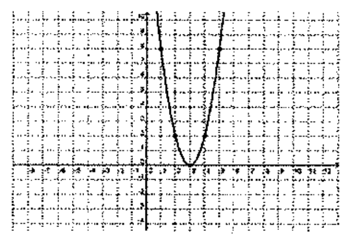
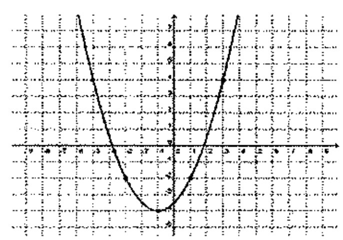


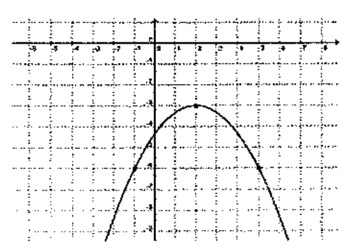
36.



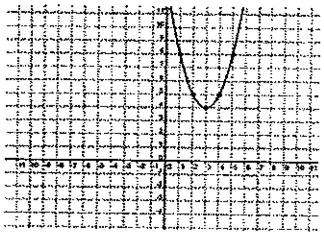
37.



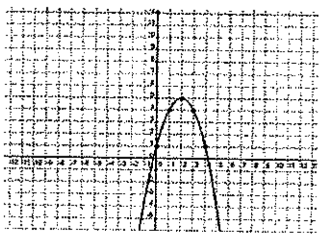
38.



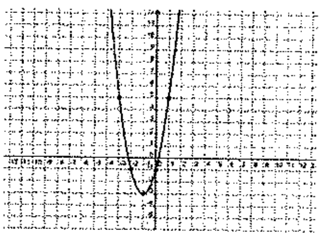
39.



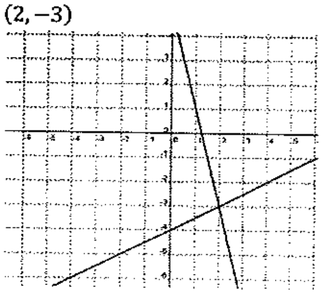
40.



41.

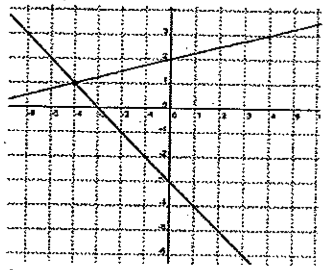


42. No. It does not satisfy both equations.
43. It means to find point (ordered pair) that satisfies all the equations of the system.
44. Answered on page.
45. yes
46. yes
47. $(2,3)$
48. $(-1,-3)$ and $(0,-1)$
49. $(-2,-1)$ and $(0,-1)$
50. Zero, one, infinite
51. Zero, one, two
52. Zero, one, two, infinite
53. $(2,-3)$

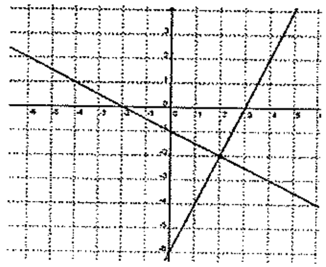


54. Answered on page.

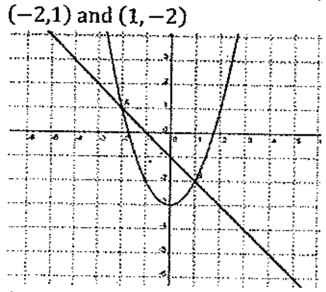
55. $(-4,1)$



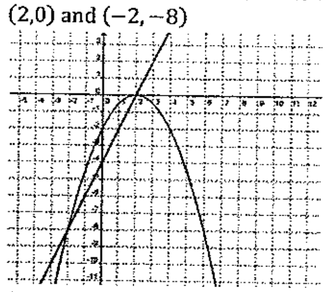
56. $(2,-2)$



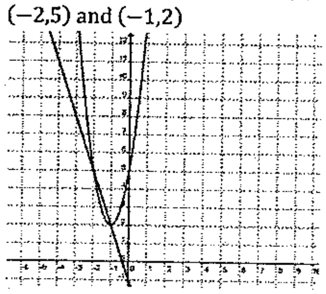
57. $(-2,1)$ and $(1,-2)$



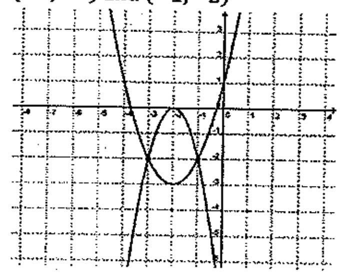
58. $(2,0)$ and $(-2,-8)$



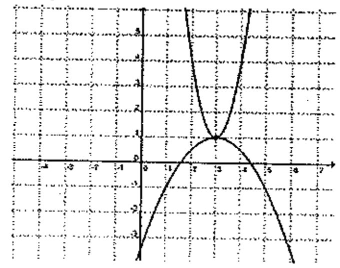
59. $(-2,5)$ and $(-1,2)$



60. $(-3,-2)$ and $(-1,-2)$



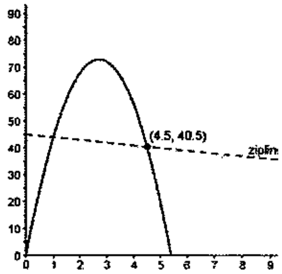
61. $(3,1)$



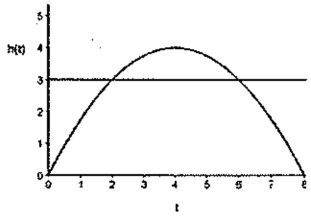
62. *No Solution*

63. Answered on page 13.
64. Answered on page 13.
65. Answered on page.
66. $(1,-1)$
67. $(0,-5)$
68. $\left(-\frac{24}{5},-\frac{21}{5}\right)$
69. Answered on page.
70. $(-1,5)$ and $(2,14)$
71. $(-4,10)$ and $(2,4)$
72. . $(2,-3)$
73. Answered on page.
74. $(2,1)$
75. Infinite solutions [same line]
76. No solution [parallel lines]
77. $(2,-1)$
78. $(-4,4)$
79. $(0,4)$ and $(2,4)$
80. No solution.
81. $(1,4)$ and $(4,7)$
82. $(0,0)$ and $(-6,-6)$
83. $(4,-6)$
84. $(1,-2)$ and $(3,-2)$
85. Answered on page.
86. $(-10,240)$ and $(3,19)$
87. $(1,8)$ and $(2,26)$
88. $(1,-1)$ and $(25,-241)$
89. $(-2,0)$ and $\left(-\frac{3}{2},\frac{5}{4}\right)$

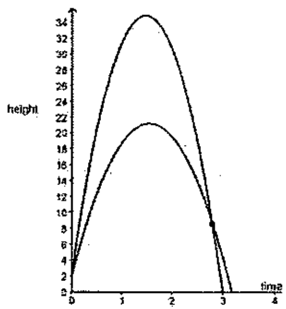
90. $(2\sqrt{2}, 70 - 2\sqrt{2})$ and $(-2\sqrt{2}, 70 + 2\sqrt{2})$
91. $x = 1.1, 5.4$
92. $x = 1.0, 9.0$
93. 3 seconds.
94. Answered on page.
95. 4.5 seconds.



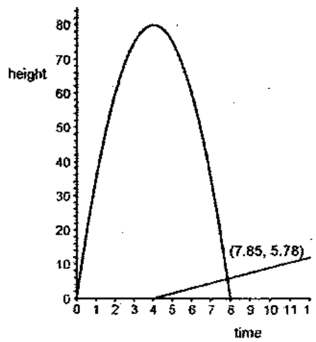
96. 2 seconds



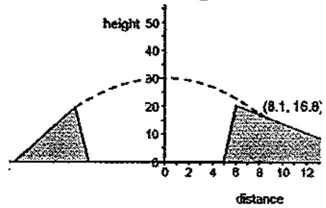
97. 2.8 seconds.



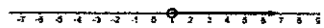
98. $10 < n < 65$
99. Approximately 7.9 seconds after launch.



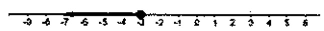
100. 16.8 feet above the ground.



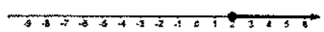
101. $x > 1$



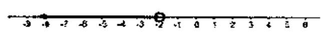
102. Answered on page.
103. $x \leq -3$



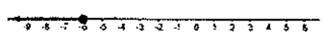
104. $x \geq 2$



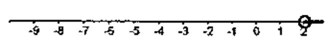
105. $x < -2$



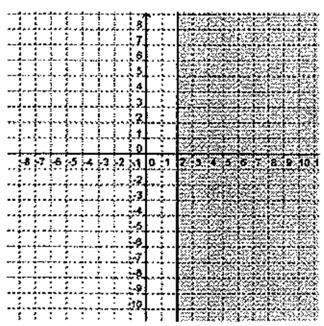
106. $x \leq -6$



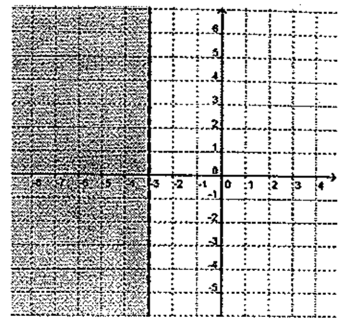
107. $x > 2$



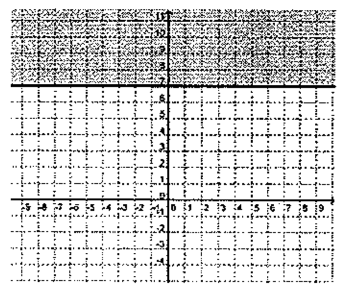
108.



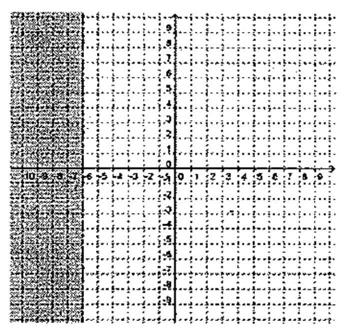
109. There are an infinite number of ordered pairs Eg. $(0,0), (2,1), (3,1), (0,-8), (-2,-9)$
110. Answered on page.
111. Note: dotted line.



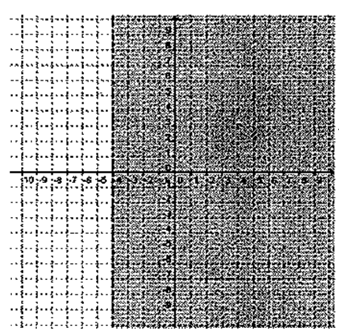
112.



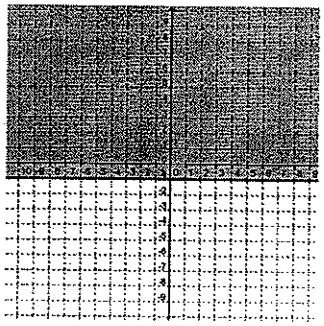
113.



114.

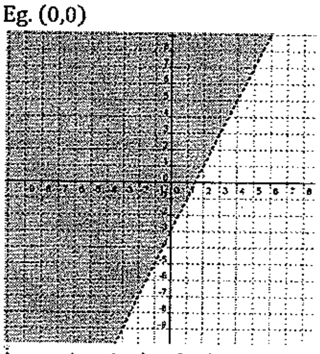


115.

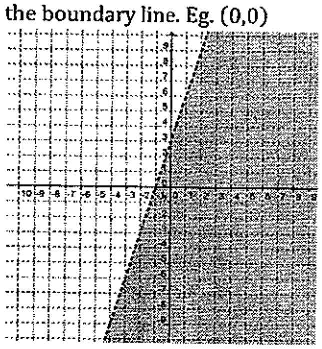


116. Answered on page.

117. Any points in the shaded region but NOT including the boundary line.
Eg. (0,0)

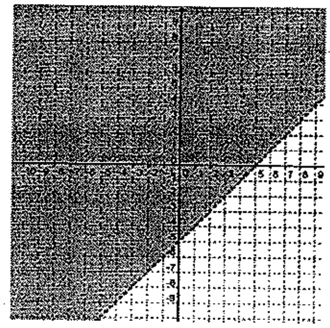


118. Any points in the shaded region but NOT including the boundary line. Eg. (0,0)

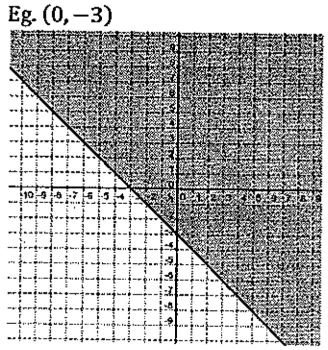


119. Any points in the shaded region but NOT including the boundary line. Eg.

(1, −2)



120. Any points in the shaded region including the boundary line.
Eg. (0, −3)

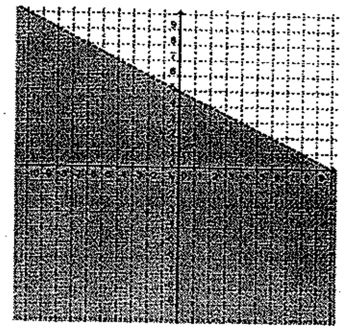


121. No. It is not possible because there are an infinite number of points that satisfy the inequality.

122. It is similar because the solution is a half-plane created by a boundary line. It is different because the boundary line on the third inequality is part of the solution but it was not in the first two.

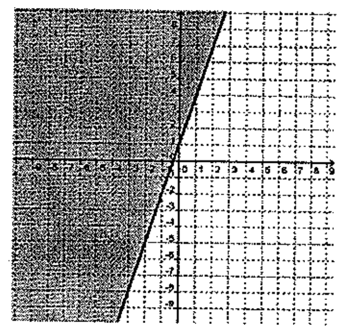
123. All the points on the boundary line make left side = right side. That is why points on the boundary line are only included in the solution if the inequality is either $\geq or \leq$.

124.

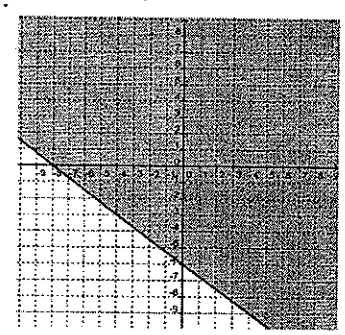


125. Answered on page.

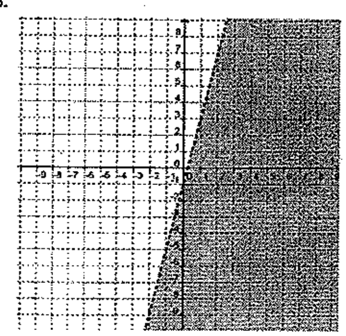
126.



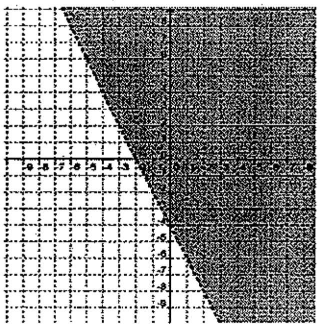
127.



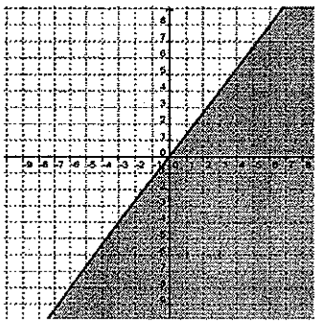
128.



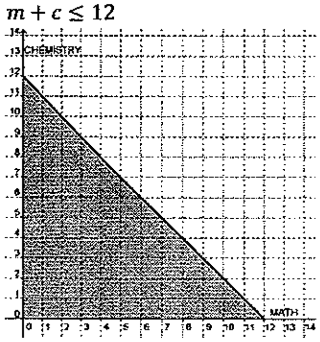
129.



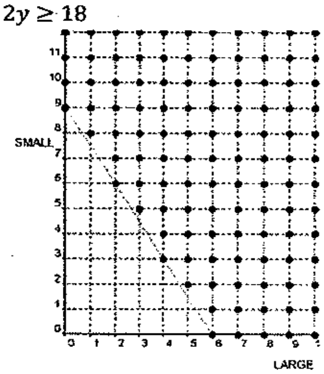
130.



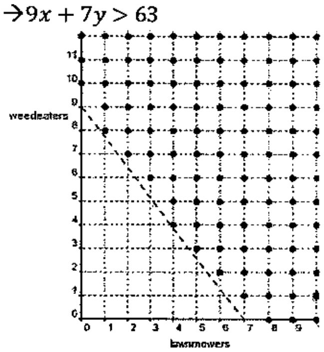
131. $y \geq 2x - 6$
132. $y < \frac{5}{4}x - 5$
133. $y > 3$
134. $y > \frac{3}{2}x + 1$
135. $x \geq -6$
136. $y \leq x + 6$
137. $n \leq 6$
138. $n \leq 12$
139. $b + g \leq 7$
140. $w + t > 250$
141. $5t + 3s \geq 90$
142. $0.10A \geq 2000$
143. $5r + 3s \leq 15$
144. Only whole roasts or steaks will be carried in the back pack. Discrete data.
145. There are 13 combinations (dots).
146. $m + c \leq 12$



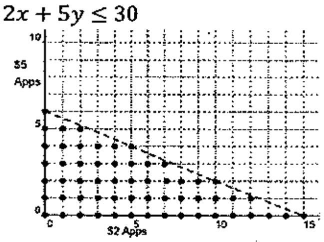
147. Answers may vary. Inequality is $2x + 3y < 18$ and data is discrete.
148. Answers may vary. Inequality is $3x + 2y > 24$ and data is continuous.
149. $15x + 10y \geq 90 \rightarrow 3x + 2y \geq 18$



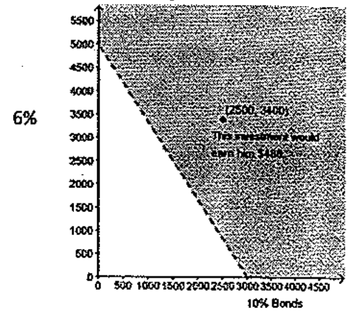
150. $45x + 35y > 315$ $\rightarrow 9x + 7y > 63$



151. $2x + 5y \leq 30$

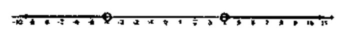


152. $0.10x + 0.06y > 300$

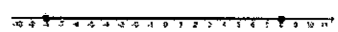


153. $x \leq -3$ or $x \geq 3$

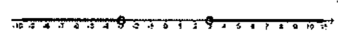
154. Answered on page.
155. $x < -4$ or $x > 4$



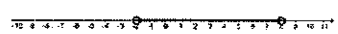
156. $-8 \leq x \leq 8$



157. $x < -3$ or $x > 3$

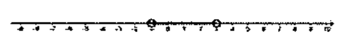


158. $-2 < x < 8$

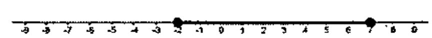


159. $x \leq -3$ or $x \geq 3$
160. Answered on page.
161. $-3 \leq x \leq 3$

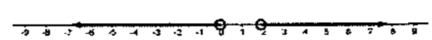
162. $-1 < x < 3$



163. $x < -2$ or $x > 3$
164. Answered on page.
165. Answered on page.
166. $-2 \leq x \leq 7$

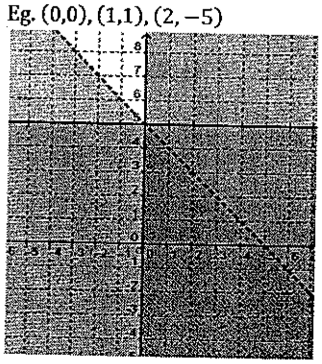


167. $x < 0$ or $x > 2$

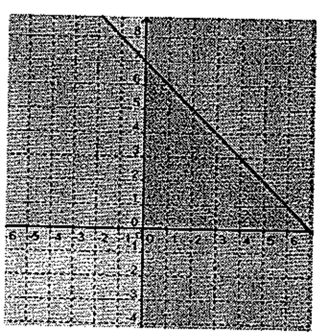


168. No solution. When $x = 3$, the function will be equal to zero but never greater than zero.
169. $-1 < x < 5$
170. $-1 \leq x \leq 6$
171. No solution.
172. $x < 0.2$ or $x > 2.3$

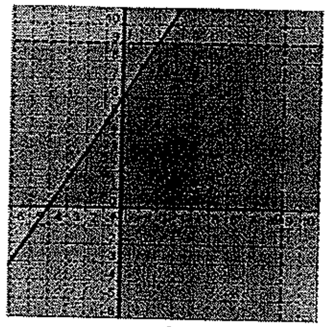
173. $0 \le x \le 2.7$
174. $x < -2.5$ or $x > 3.2$
175. $-1.8 < x < 2.2$
176. $x < -2$ or $x > 5$
177. Answered on page.
178. $-1 < x < \frac{5}{4}$
179. 4 seconds
180. Approximately 5.4 seconds.
181. $y > x^2 + 1$
182. Answered on page.
183. $y > -x^2 + 1$
184. $y \ge (x-3)^2 + 1$
or $y \ge x^2 - 6x + 10$
185. $y < (x+2)^2 - 3$
or $y < x^2 + 4x + 1$
186. $y < -2(x+2)^2 + 5$
or $y < -2x^2 - 8x - 3$
187. Any ordered pairs in the region shaded by both inequalities.
Eg. (2,0), (3,0), (4, −2)
188. $x \ge 1$ and $x + y \le 7$
189. Answered on page.
190. $x \le 1, y \ge -2, y \le x + 7$
191. $x \ge 0,\ y \ge 0, 2x + y \ge 8$
192. $x \ge 0,\ y \ge 0, 2x + 3y \le 9$
193. $y < 3,\ x + y > 3,$
$x - y < 3$
194. Eg. (0,0), (1,1), (2, −5)



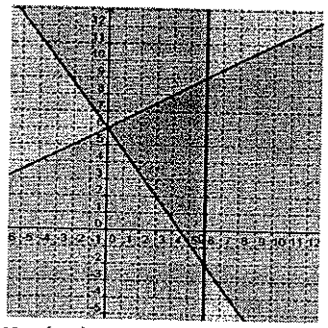
195. Answered on page.
196.



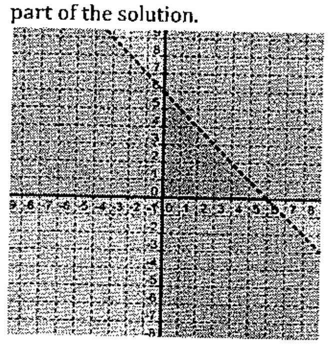
197.



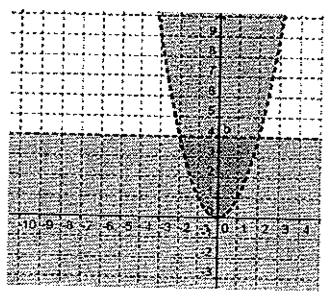
198. Area $= 33\ units^2$



199. No. (2,4) is not a solution to the system because it is on the boundary line for $y < -x + 6$ which is NOT part of the solution.

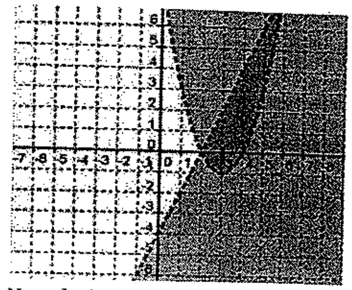


200.

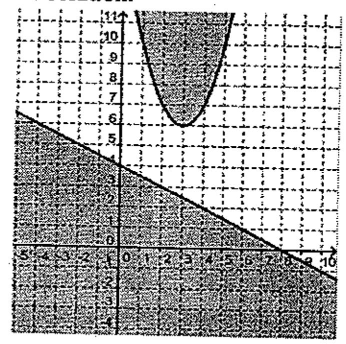


201. Answered on page.

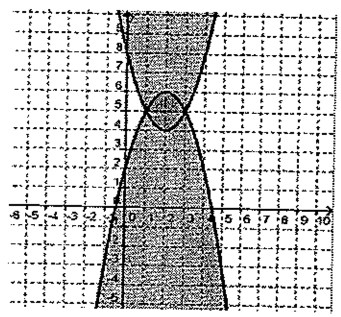
202.



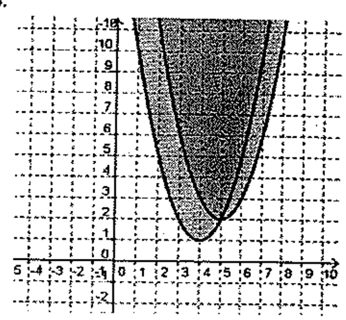
203. No solution.



204.

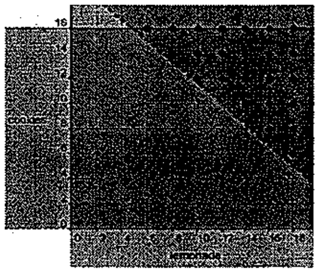


205.



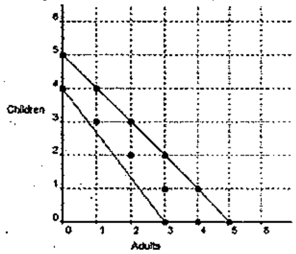
206. $y \le -0.05(x-9)^2 + 4$
$y \ge -0.025(x-9)^2 + 2$
207. Answered on page.
208. $h(x) \ge -0.04x^2 + 3.53x,$
$h(x) \le -0.05x^2 + 4.5x$
209. $y \le \frac{1}{40}x^2$ and $y \ge \frac{1}{60}x^2$
210. $x \ge 0, y \ge 0, x \le 20,$
$y \le 16, 0.50x + 0.60y \ge$

12



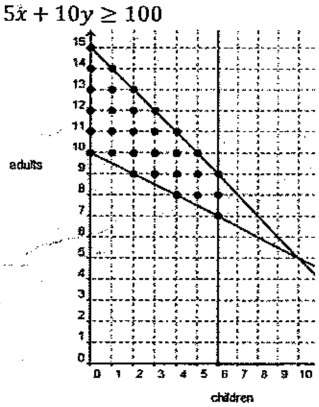
211. Answered on page.

212. $x \geq 0, y \geq 0, x + y \leq 5,$
$4x + 3y \geq 12$



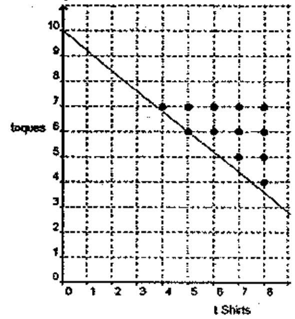
5 adults and 0 children would maximize revenue.
12 possible combinations.

213. $x \geq 0, y \geq 0, x + y \leq 15,$
$5x + 10y \geq 100$



30 possible combinations.

214. $x \geq 0, y \geq 0, x \leq 8, y \leq 7$
$12x + 15y \geq 150$



8 shirts and 7 toques will produce a maximum revenue of \$201.