## HW Mark: $\begin{array}{lllllll}10 & 9 & 8 & 7 & 6 & \text { RE-Submit }\end{array}$

## Trigonometry

## This booklet belongs

to: $\qquad$ Period

| LESSON \# | DATE | QUESTIONS FROM <br> NOTES | Questions that I find <br> difficult |
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Your teacher has important instructions for you to write down below.
$\qquad$
$\qquad$

Trigonometry

[C] Communication [PS] Problem Solving, [CN] Connections [R] Reasoning, [ME] Mental Mathematics [T] Technology, and Estimation, [V] Visualizatio

Key Terms

| Term | Definition |
| :--- | :---: |
| Triangle |  |
| Similar Triangles |  |
| Theta |  |
| Acute angle |  |
| Obtuse angle |  |
| Right Triangle |  |
| Oblique Triangle |  |
| Legs |  |
| Hypotenuse |  |
| Trigonometry |  |
| Opposite side |  |
| Adjacent side |  |
| Sine ratio |  |
| Cosine ratio |  |
| Tangent ratio |  |
| Theta ( $\theta$ ) |  |
|  |  |
|  |  |

## Why Trigonometry?

There is an application of trigonometry that could solve each problem below.
A student approaches a large Sequoia tree outside the entrance to the school and wonders how tall the tree is.


A homeowner wants to cut a new board to replace a decaying roof truss. He can measure the horizontal distance and the angle of inclination but needs to know how long to cut the board.


An engineer is constructing a Ferris wheel for a downtown park. There are 16 passenger carts.


## Similar Triangles:

To understand what trigonometry is, we need to understand the properties of similar triangles.

Two triangles are similar if...

- They have the same angles.
- Ratios of corresponding sides are equal.


Determine if each of the following pairs of triangles are similar. Explain why or why or not.


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Find the indicated side length (nearest tenth) in the following right triangles.


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One acute angle is indicated on each of the following triangles. If possible, label each triangle with: opposite, adjacent, and hypotenuse in respect to that angle. Remember, only right triangles can be labeled this way.


Observe the three "embedded" similar triangles below. Find the missing information on the right.

17. What are the names of the three triangles?

$$
\begin{aligned}
& \triangle A B C \approx \triangle A D E \approx A G H \\
& \triangle A E \\
& G A H
\end{aligned}
$$

18. $\angle B A C=\angle$ $\qquad$ $=\angle$ $\qquad$

$$
\angle A B C=\angle A D E=\angle A G H
$$

$$
\angle B C A=\angle D E A=\angle G H A
$$

19. $\frac{B C}{A C}=\frac{3}{4}=0.7500$

$$
\begin{aligned}
& \frac{D E}{A E}=\frac{6}{8}=0.7500 \\
& \frac{G H}{A H}=\frac{9}{12}=0.7500
\end{aligned}
$$

20. Based on the work you've done, how do you know all three triangles are similar? ratios of corresponding sides are congruent (sane)
21. Earlier, we saw that similar triangles have the following similar ratios:
(using the two smaller triangles embedded above)

$$
\frac{B C}{D E}=\frac{A C}{A E}=\frac{A B}{A D}=\frac{1}{2}
$$

In the questions above we see that we could write different equivalent ratios:

$$
\frac{B C}{A C}=\frac{D E}{A E}=\frac{G H}{A G}=\frac{3}{4} \text { or } 0.7500
$$

Write two more sets of equivalent ratios that would be true for the similar right triangles above.

$$
\begin{aligned}
& \frac{B C}{A B^{A D}} \overline{D E G}=A^{3} G p r \\
& A E \\
& A C=\bar{A} H^{G} \\
& A B \bar{A} \bar{A} A^{r}
\end{aligned}
$$

## Trigonometry of Right Triangles

Since similar right triangles have equivalent ratios for corresponding angles, we can use those ratios to find unknown angles and/or side lengths.

These ratios have been calculated and stored in our calculator for many angles to help us solve problems.

We will use the three primary trigonometric ratios:

## Tangent ratio Sine ratio Cosine ratio

The Tangent Ratio

For an acute angle in a right triangle, the ratio of $\frac{\text { opposite } \angle \theta}{\text { adjacent } \angle \theta}$ is called the TANGENT RATIO

We have seen previously that in similar right triangles, the ratios of the legs of a triangle remain constant despite reducing or enlarging the triangle.

These ratios have been calculated and stored in our calculator for many angles to help us solve problems.
22. Challenge Question

Find the tangent ratio for angle $C$ in the triangle to the right.


23. Challenge Question

What is $\tan \theta$ for the triangle to the right?

$$
\tan \theta=\frac{\mathrm{opp}}{\operatorname{adj}} \quad \tan \theta=\frac{35}{12}
$$

## The Tangent Ratio

Remember, the tangent ratio is a ratio involving the "legs" of the right triangle.

TanC $=\frac{\text { opposite } \angle \theta}{\text { adjacent } \angle \theta}$

From the diagram we see that $\operatorname{Tan} C=\frac{3}{4}$


Find the tangent ratio for the indicated angles below.
Answer as a fraction AND as a decimal to 4 places.


Find the tangent ratio for the indicated angles below.
Answer as a fraction AND as a decimal to 4 places.


Skill Reminder: Solve the following equations. Answer to the nearest hundredth if necessary.

36. Challenge Question

Given that the ratio of $\frac{\text { opposite }}{\text { adjacent }}$ for $\angle A$ in $\triangle A B C$ is 0.5000 , find the length of the missing leg.


$$
\tan A=\frac{000}{a d j}
$$



Given that the tangent ratio of Angle $F$ is 0.4000 , find the length of the missing leg.

38. Challenge Question

Use your calculator to find $\tan 30^{\circ}$ (round to 4 decimal places).
Use $\tan 30^{\circ}$ to find the length of the missing leg.

$$
\tan 30^{\circ}=0.5774
$$



$$
\begin{aligned}
& w=\frac{4.3}{0.5774} \\
& w=7.4 \mathrm{~km}
\end{aligned}
$$

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Finding Missing Sides Lengths Using the Tangent Ratio


Use your calculator to find the following ratios to 4 decimal places, then solve for $x$.


Find the length of the indicated side to the nearest tenth.


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Solving Triangles:
To "solve a triangle" means to find the length of all unknown sides and measure of unknown angles.
56. Explain the steps you would take to solve the following triangle.


Solve the following triangles. Answer to tenths.


$$
\begin{aligned}
& \frac{\tan 37=}{12.57} a \\
& \frac{12.5}{\tan 37}=a \\
& a=16.6 \mathrm{in} \\
& \frac{(166)^{2}+(12.5)^{2}}{1}=c^{2} \\
& c=20.8 \mathrm{in}
\end{aligned}
$$

59. 



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60.

The dotted line is an altitude.


$$
a=\frac{3.3}{\tan 31} \quad a=5.5
$$

Solve each of the following word problems. Include a diagram in your solution.
61. From a point 220 m from the Empire State Building, a tourist measures the angle of inclination to the top to be $60^{\circ}$. Calculate the height of the building to the nearest metre..

62. A radio tower is 396 feet tall. How far from the base of the tower is a technician if the angle of inclination to the top of the tower is $27^{\circ}$ ? Answer to the nearest foot.

63. An airplane approaches a control tower. The angle of depression from the pilot to the tower is $12^{\circ}$. If the plane is flying at an altitude of 1500 m , how far is the plane from being directly above the tower (to the nearest kilometer)?

64. At 11:00 in the morning, the angle of elevation to the sun $58^{\circ}$. A tree in the school yard casts a shadow of 56 m . How tall is the tree to the nearest metre?

$$
\tan 58=\frac{x}{56}
$$


65. A student crossing to the west building casts a shadow on the path.
She is 165 cm tall and the angle to the sun is $25^{\circ}$. How long is the shadow on the path to the nearest centimetre?

$$
56 \tan 58=x
$$

$$
x=89.6 \rightarrow 90 \mathrm{~m}
$$



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## The Sine Ratio

The sine ratio is a ratio involving the hypotenuse and one leg of the right triangle.

From the diagram to the right we see that $\sin C=\frac{3}{5}$

The ratio of the opposite leg to the hypotenuse is $3: 5$ or 0.60 .


Find the sine ratio for the indicated angles below.
Answer as a fraction AND as a decimal to 4 places.

| 66. Find the sine ratio for angle $\theta$ in the triangle below. | 67. Find the sine ratio for angle $C$ in the triangle below. | 68. Find the sine ratio for angle $C$ in the triangle below. |
| :---: | :---: | :---: |
|  |  |  |
| 4 | 15 | 12 |
| $\sin \theta=\frac{3}{5}=$ | $\sin \theta=\frac{15}{17}$ | $\sin \theta=\frac{9}{15}$ |
| $=0.6000$ | $=0.8824$ | $=0.6000$ |
| 69. What is $\sin \theta$ for the triangle below? | 70. What is $\sin \theta$ for the triangle below? | 71. What is $\sin \theta$ for the triangle below? |
| $10 \gg 26$ |  | $\widehat{\theta} \quad 74$ |
| 24 | 56 | 70 |
| $\sin \theta=\frac{24}{26}$ | $\sin \theta=\frac{56}{70}$ | $\sin \theta=\frac{70}{74}=\frac{35}{37}$ |
| $=0.9231$ | $=0.8000$ | $=0.9459$ |

Finding Missing Sides Lengths Using the Sine Ratio
72. Draw a diagram
illustrating the sine ratio
for $\angle P$ in right $Q Q R$.

73. The sine ratio is a ratio of what two sides in a right triangle?

74. Can you use the sine ratio to find the hypotenuse of a right triangle?


Use the sine ratio to find the missing side lengths to the nearest tenth.
78. The sine ratio of a right triangle is 0.8000 . If the hypotenuse is 20 cm long, what are the lengths of the other two sides?

$$
\begin{aligned}
& \sin A=\frac{o p p}{h y p} \\
& 0.8000=\frac{o p o}{20}
\end{aligned}
$$

$$
20(0.8)_{0 p \theta}=0 \mathrm{PP}
$$

76. The sine ratio for the triangle below is 0.2000 . Find the length of side $y$

77. The sine ratio for the triangle below is 0.6018 . Find the length of side $t$.

$$
\begin{aligned}
& 0.6018=\frac{t}{39} \\
& t=39(0.6018)
\end{aligned}
$$

79. The sine ratio of a right triangle is 0.4500 . If the hypotenuse is 8 m long, what are the lengths of the other two sides?
$0.4560=\frac{x}{8}$
$x=8(45)^{8} /$
$x=3.6 / 8^{2}-3.6=y^{2}$
$y=71$ triangle (s 7.1004. Find the hypotenuse if the opposite side is 17 cm . not possible
80. Explain why the previous questions have no solutions.

What do you notice about the value of ratio where this happens? Interpret that kind of ratio in the sense of a right triangle.


$$
\sin A=\frac{a}{c} \Rightarrow \begin{gathered}
\text { cannot have } \\
\text { sine chat io } \\
\text { ont } \\
\text { one wise a a }>c \text {, } \\
\text { But }
\end{gathered}
$$

Use your calculator to find the following ratios to 4 decimal places. side!
83. $\sin 45^{\circ}=$ $\qquad$ 0.7071
84. $\sin 60^{\circ}=$ $\qquad$ 85. $\sin 42^{\circ}=0.6691$
86. Find the length of side $x$.

87. Find the length of side $x$.

88. Find the length of side $x$.


Find the length of the indicated side to the nearest tenth.
91.


$$
d=28.2 \mathrm{~cm}
$$


92.

96. A lighthouse attendant has a range of visibility of 24 km . A ship on the horizon passes by the lighthouse. The attendant sees the ship for a total of 120 degrees. For how many kilometers was the ship within the attendant's range of sight? (nearest tenth)


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Draw a scale diagram that would represent each of the following.
97. Draw a triangle that has a the following:
$\sin \alpha=\frac{2}{5}$, hypotenuse is 10 cm long.

98. Draw a triangle that has a the following: $\tan \beta=\frac{12}{5}$, hypotenuse is 26 cm long.


$$
13=c
$$


99. Solve the triangle.

103. While golfing with his father-inlaw, Mr. J hits a shot short of a pond. He walks 20 m to his left to a point directly across the pond from the hole. The angle between the two lines of sight is $22^{\circ}$. Find the distance from his ball to the hole to the nearest tenth of a metre.

105. Any lets out 125 feet of kite string at Clover Point. The wind pulls the kite string tight at an angle of $55^{\circ}$ to the ground. Approximate the height of the kite torture nearest foot.

What assumptions did you make?
 straight.

106. The diameter of a circular tunnel in Shanghai is 15.43 m . During a flood, a worker in the water at the side of the tunnel measured an angle to the centre $b$ to be $37^{\circ}$. Find the depth of the water 125 at its deepest point. (The water surface forms a chord across the tunnel.)

