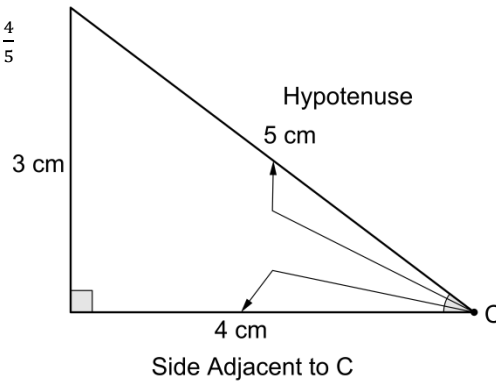## The Cosine Ratio

The cosine ratio is a ratio involving the hypotenuse and one leg (adjacent to angle) of the right triangle.

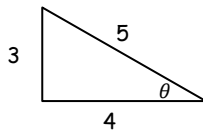From the diagram to the right we see that $\cos C = \frac{4}{5}$

This means the ratio of the adjacent leg to the hypotenuse is 4:5 or 0.80.

Hypotenuse
5 cm

3 cm

4 cm
Side Adjacent to C

C

Find the cosine ratio for the indicated angles below.

Answer as a fraction AND as a decimal to 4 places.

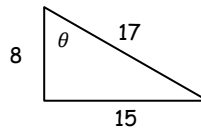| 107. Find the cosine ratio for angle θ in the triangle below.	108. Find the cosine ratio for angle C in the triangle below.	109. Find the cosine ratio for angle C in the triangle below.
5, 3, 4, θ	8, 17, 15, θ	9, 15, 12, θ
$\cos\theta = \frac{4}{5}$	$\cos\theta = \frac{8}{17}$	$\cos\theta = \frac{12}{15}$
= 0.8000	= 0.4706	= 0.8060
110. What is $\cos\theta$ for the triangle below?	111. What is $\cos\theta$ for the triangle below?	112. What is $\cos\theta$ for the triangle below?
10, 26, 24, θ	42, 70, 56, θ	24, 74, 70, θ
$\cos\theta = \frac{10}{26} = \frac{5}{13}$	$\cos\theta = \frac{42}{70} = \frac{3}{5}$	$\cos\theta = \frac{24}{74} = \frac{12}{37}$
= 0.3846	= 0.6000	= 0.3243
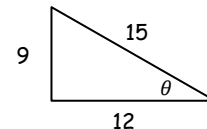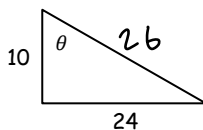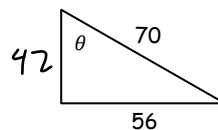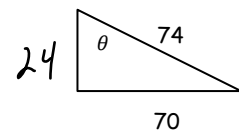
Finding Missing Sides Lengths Using the Cosine Ratio

113. Draw a diagram illustrating the cosine ratio for ∠P in right ΔPQR.

$$\cos P = \frac{PR}{PQ}$$
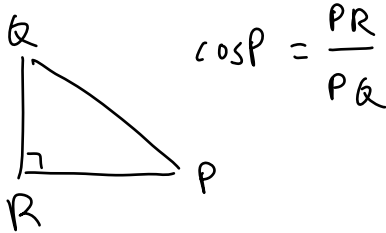
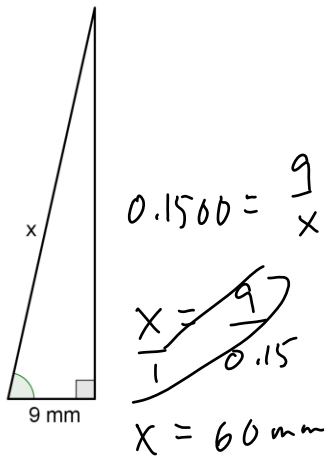**114.** The cosine ratio is a ratio of what two sides in a right triangle?

adjacent ;

hypotenuse

$$\cos P = \frac{adj}{hyp}$$

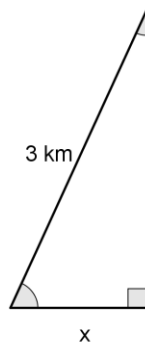**115.** Can you use the cosine ratio to find the hypotenuse of a right triangle?

yes

Use the cosine ratio to find the missing side lengths to the nearest tenth.

116. The cosine ratio for the triangle below is 0.1500. Find x.

x, 9 mm

$$0.1500 = \frac{9}{x}$$

$$x = \frac{9}{0.15}$$

$$x = \frac{9}{1}$$

$$x = 60 \, mm$$

117. The cosine ratio for the triangle below is 0.3255. Find x.

3 km, x

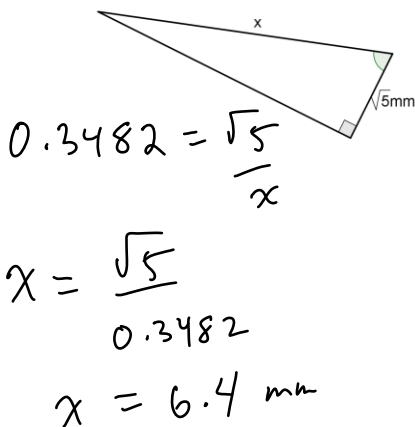$$0.3255 = \frac{x}{3}$$

$$x = 3(0.3255) = 1.0 \, km$$

118. The cosine ratio for angle Φ in a right triangle is 1.7421. Find the length of the hypotenuse if the side adjacent to Φ is 3.5 cm long.

not possible.

Can not have cosine ratio greater than 1.0

119. The cosine ratio for the triangle below is 0.3482. Find x.

x, √5mm

$$0.3482 = \frac{\sqrt{5}}{x}$$

$$x = \frac{\sqrt{5}}{0.3482}$$

$$x = 6.4 \, mm$$

120. Draw a right triangle with a cosine ratio of 0.3277. Label all side lengths.

$$0.3277 = \frac{adj}{1000}$$
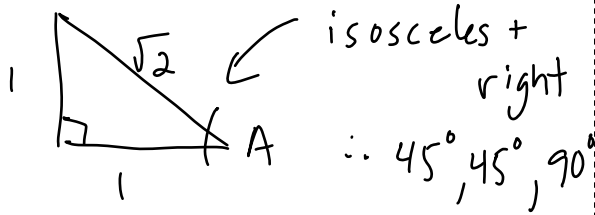
B, 944.8, 1000, C, 327.7, A
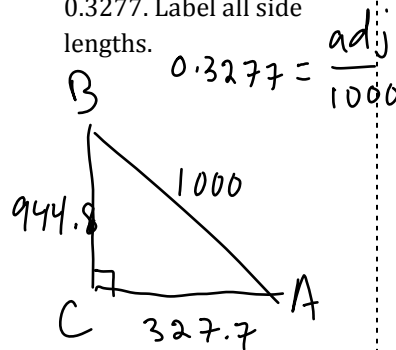
Many Answers

121. Explain why you cannot have a right triangle with a cosine ratio of 1.6713.

$$\frac{adjacent}{hypotenuse}$$

1.6713 would mean adj is bigger than hyp → NOT POSS.

122. Draw a right triangle with an acute angle that has an adjacent side equal in length to the opposite side. Find the cosine ratio for that angle.
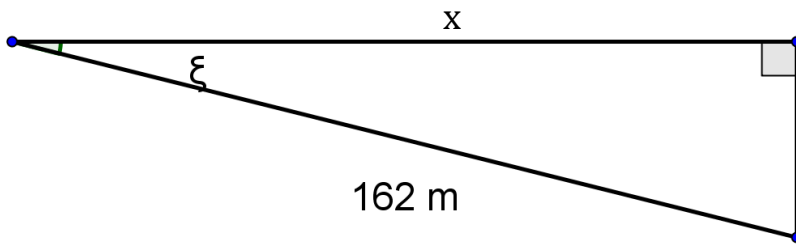
isosceles +
right
∴ 45°, 45°, 90°

$\cos A = \dfrac{1}{\sqrt{2}} = 0.7071$

123. Draw a right triangle with an acute angle that has a hypotenuse 50% longer than the adjacent side. Find the cosine ratio for that angle.
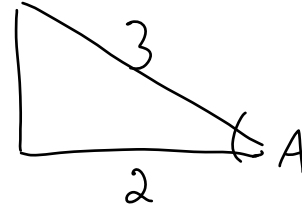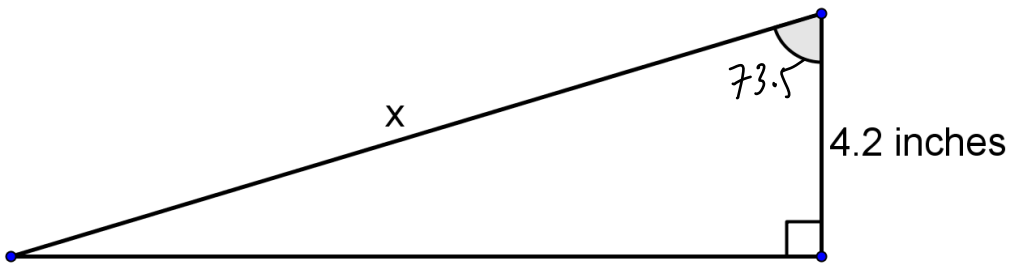
$\cos A = \dfrac{2}{3} = 0.\overline{6}$

124. Use a protractor to measure the indicated angle. Then determine the length of side x using the cosine ratio.

x

162 m

125. Use a protractor to measure the indicated angle. Then determine the length of side x using the cosine ratio.

73.5

x

4.2 inches

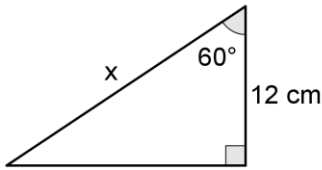$\cos 73.5 = \dfrac{4.2}{x}$

$x = \dfrac{4.2}{\cos 73.5} = 14.8''$

Use your calculator to find the following ratios to 4 decimal places, then find the indicated side length to 2 decimal places.

126. $\cos 60° =$ ___0.5000___

127. $\cos 20° =$ 0.9397

128. $\cos 42° =$ 0.7431

129. Find the length of side x.

$\cos 60 = \dfrac{12}{x}$

$\dfrac{12}{\cos 60} = x \quad x = 24.00$

130. Find the length of side x.

$\cos 20 = \dfrac{2}{x}$

$x = \dfrac{2}{\cos 20} \quad x = 2.13$

131. Find the length of side x.

$\cos 42 = \dfrac{x}{23.5}$

$23.5 \cos 42 = x$

$17.46 = x$

Find the length of the indicated side to the nearest tenth.
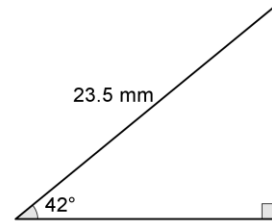
132.
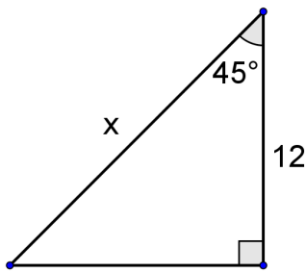
$\cos 45 = \dfrac{12}{x}$

$\dfrac{12}{\cos 45} = x \quad x = 17.6$

133.
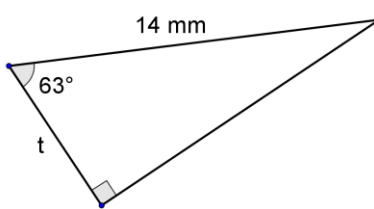
$\cos 63 = \dfrac{t}{14}$

$14 \cos 63 = t$

$6.4 = t$

134.

$\cos 45 = \dfrac{1.6}{w}$

$w = \dfrac{1.6}{\cos 45} \quad w = 2.3$

135.

$\cos 27 = \dfrac{42}{r}$

$\dfrac{42}{\cos 27} = r$

$r = 47.1$

136.

$\cos 27 = \dfrac{z}{75.4}$

$75.4 \cos 27 = z$

$67.2 = z$

137. Find the length of each leg.

$\cos 63 = \dfrac{b}{14.3}$

$b = 6.5$

$14.3^2 - 6.5^2 = a^2$

$a = 12.7$

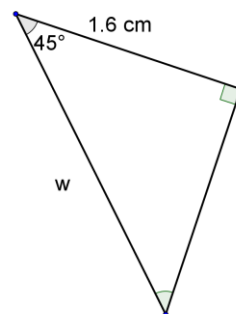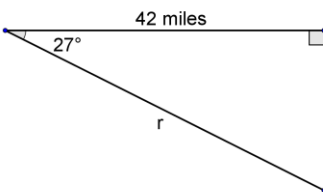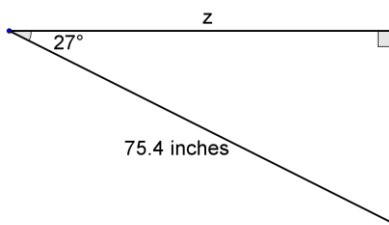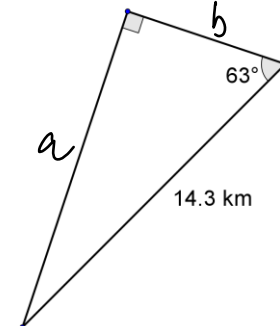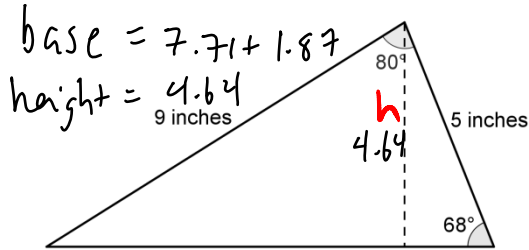Solve the following problems involving triangles.

---

138. Find the area of the triangle below.

$base = 7.71 + 1.87$

$height = 4.64$

9 inches   80°   h   5 inches   4.64   68°

$A = \dfrac{(9.58)(4.64)}{2}$

$= 22.2$

$\sin 68 = \dfrac{h}{5}$

$h = 5\sin 68$

$h = 4.64$

$9^2 - 4.64^2 = x^2$ $5^2 - 4.64^2 = y^2$

$x = 7.71$ $y = 1.87$

---

139. Find the area of a rectangle with a diagonal of 20 m if the angle between the diagonal and longer side is 25 degrees. (nearest tenth)

20   25°   x   y

$A = (8.45)(18.13)$

$= 153.2 \, m^2$

$\sin 25 = \dfrac{x}{20}$ $\cos 25 = \dfrac{y}{20}$

$x = 8.45$ $y = 18.13$

---

140. Tucker has two choices to get his ball to the hole at the 17th at Cordova Bay, go around the lake or go over it. He decides to go around the lake as shown on the diagram. How much farther does he have to hit the ball going around the lake instead of going straight over it?

(Careful, not a right triangle shown.)

108.6   60.7 a   y   56°   34   120 = x   150 yards   37°
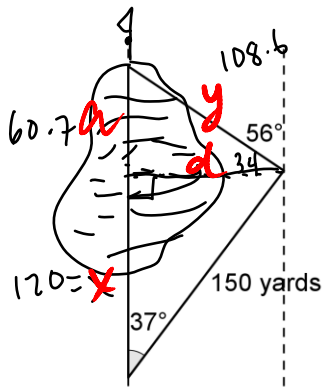
$\cos 37 = \dfrac{x}{150}$

$x = 150\cos 35$

$x = 120$

$150^2 - 120^2 = d^2$

$d = 90$

$\cos 34 = \dfrac{90}{y}$

$\dfrac{90}{\cos 34} = y$

$y = 108.6$

$(108.6)^2 - 90^2 = a^2$

$a = 60.7$

across
$120 + 60.7 = 180.7$

around
$150 + 108.6 = 258.6$

$258.6 - 180.7 = 77.9 \text{ yards}$

---

141. Find the area of the circle that is not covered by the shaded rectangle to the nearest tenth of a square unit.

x   d   12 cm   37°
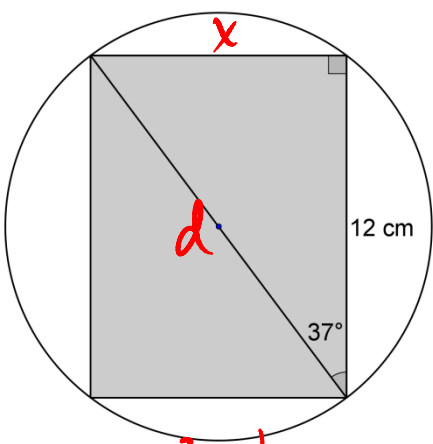
$\cos 37 = \dfrac{12}{d}$

$d = \dfrac{12}{\cos 37}$

$= 15.0$

$\therefore r = \dfrac{15.0}{2} = 7.5$

$A_c = \pi r^2$

$= \pi (7.5)^2$

$= 177$

$\sin 37 = \dfrac{x}{15.0}$ $A_r = lw$

$15.0\sin 37 = x$ $= 108.3$

$x = 9.02$

Not shaded:

$177 - 108.3$

$= 69 \, cm^2$

---

142. Find the area of the circle not covered by the shaded rectangle.

$14000 - 7023 = 6976$

$\tan 26 = \frac{x}{120}$

$x = 58.5$

120

120 cm

26°

$A_r = 58.5(120)$
$= 7020$

$\cos 26 = \frac{120}{d}$

$d = \frac{120}{\cos 26}$

$d = 133.5$

$\therefore r = 66.76$

$A_c = (66.76^2)\pi$
$= 14000$

$\therefore 14000 - 7020 = 6980$
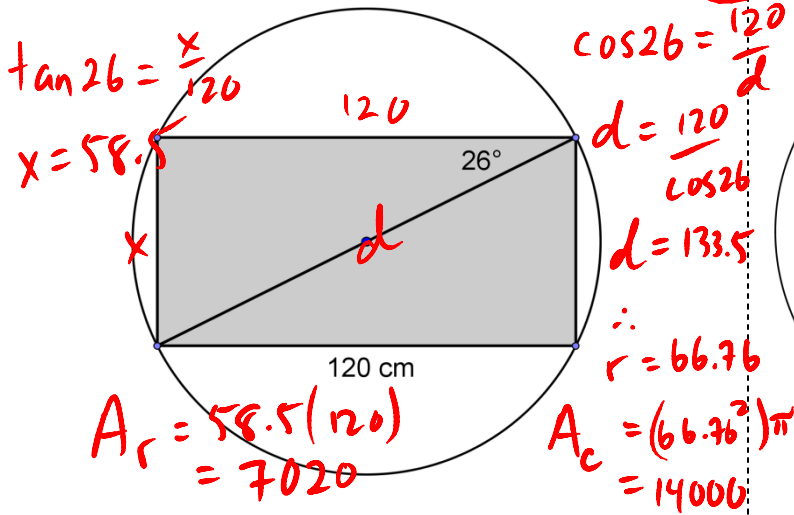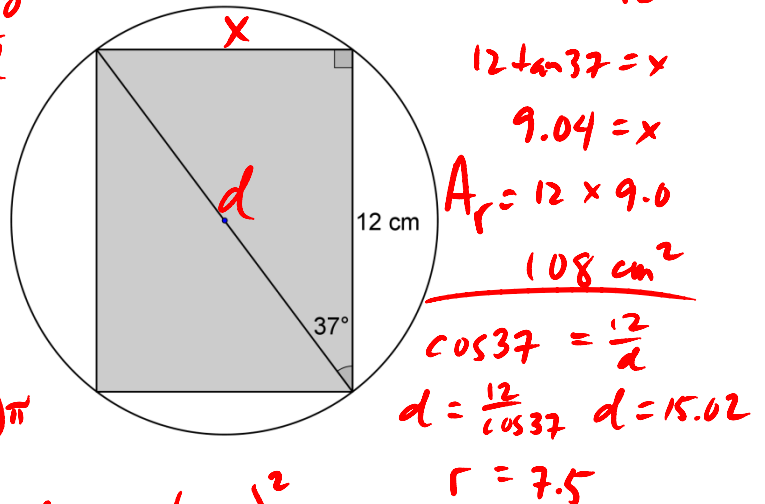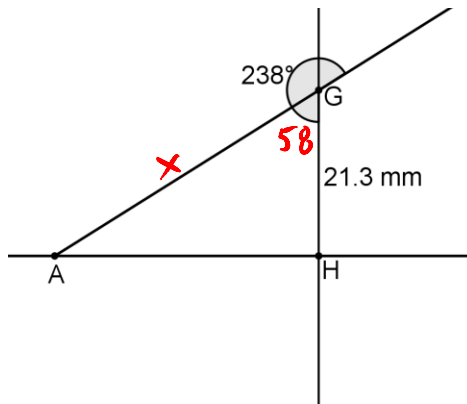
143. Find the area of the circle not covered by the shaded rectangle.

$\tan 37 = \frac{x}{12}$

$12 \tan 37 = x$

$9.04 = x$

X

12 cm

37°

$A_r = 12 \times 9.0$

108 cm^2

$\cos 37 = \frac{12}{d}$

$d = \frac{12}{\cos 37}$ $d = 15.02$

$r = 7.5$

$A_c = \pi(7.5)^2$
$= 177$

Not shaded:

$177 - 108 = 69 \text{ cm}^2$

144. Find the length of AG to the nearest tenth of a millimetre.

238°

G

58

x

21.3 mm

A   H

$\cos 58 = \frac{21.3}{x}$

$x = \frac{21.3}{\cos 58}$ $x = 40.2 \text{ mm}$

145. Find the area of rectangle WXYZ to the nearest square unit.

X

Y

4 cm

ℓ

39°   Z

W

8 cm

$\cos 39 = \frac{\ell}{8}$

$8 \cos 39 = \ell$

$\ell = 6.2$
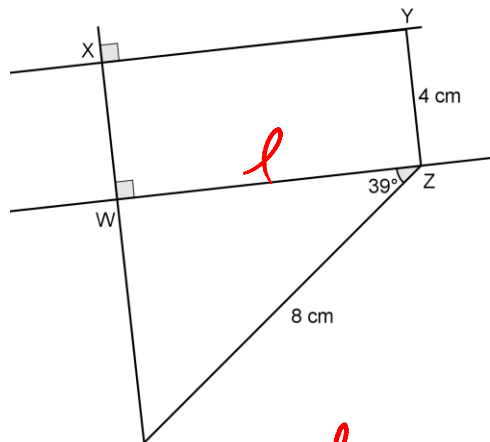
$A = 4(6.2)$

$= 24.9 \text{ cm}^2$

Finding Angles Using the Three Ratios

Recall:

The three primary trig. ratios:

Tangent Ratio: $\tan\theta = \frac{length\ of\ side\ opposite\ \theta}{length\ of\ side\ adjacent\ \theta}$

| Unless otherwise stated, calculate the measure of angles to the nearest tenth of a degree.

Eg. 42.8° |

Sine Ratio: $sin\theta = \frac{length\ of\ side\ opposite\ \theta}{length\ of\ hypotenuse}$

Cosine Ratio: $cos\theta = \frac{length\ of\ side\ adjacent\ \theta}{length\ of\ hypotenuse}$

The stored values in your calculator allow you to find angles using the ratios.

The magic of sin^{-1}, cos^{-1}, and tan^{-1}.

The "inverse trigonometric functions". These functions convert the stored ratios in your calculator to the angle.

Challenge

146. Find the measure of angle A in a right triangle if tanA = 1.000.
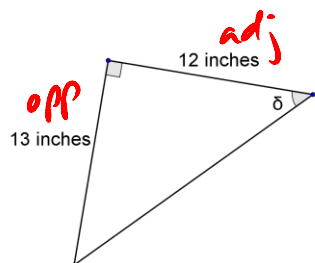
$$tan^{-1}(1.000) = 45°$$

Challenge

147. Find the measure angle B in a right triangle if sinB = 0.5000.

$$sin^{-1}(0.5000) = 30°$$

Challenge

148. What ratio would you use to find the measure of the indicated angle?

adj
12 inches
opp
δ
13 inches

$$tan = \frac{opp}{adj}$$

Challenge

149. Find the measure of the indicated angle.

12 inches
δ
13 inches

$$tan\delta = \frac{13}{12}$$

$$tan^{-1}\left(\frac{13}{12}\right)$$

$$= 47.3°$$
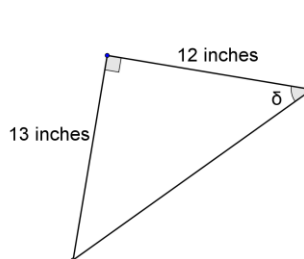
Use the Inverse functions to find the indicated angle to the nearest tenth.

---

150. Find the measure angle A in a right triangle if tanA=1.000.

use the tan⁻¹ button.
Type:
$tan^{-1}(1.000)=A$
$A = 45°$

On your calculator, find the inverse functions. Usually 2ⁿᵈ function required.

These give us the angle from the ratio!

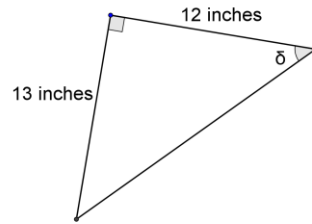**151.** Find the measure angle B in a right triangle if sinB=0.5000.

use the sin⁻¹ button.
Type:
$sin^{-1}(0.5000)=B$
$B = 30°$

152. What ratio would you use to find the measure of the indicated angle?

12 inches

δ

13 inches

use the tangent ratio:

$tan\delta = \frac{13}{12}$

Find the measure of the indicated angle.

Type: $tan^{-1}\left(\frac{13}{12}\right) = \delta$
$\delta = 47.3°$

---

153. Find angle A, if sinA=0.2654.

$Sin^{-1}(0.2654) = 15.4°$

154. Find angle B, if cosB=0.2654.

$Cos^{-1}(0.2654) = 74.6°$

155. Find angle Q, if tanQ=0.34884.

$tan^{-1}(0.34884) = 19.2°$

---

156. Find angle T, if sinT=0.7854.

$51.7°$

157. Find angle D, if cosD=1.2654.

not possible

158. Find angle U, if tanU=2.6784.

$tan^{-1}(2.6784) = 69.5°$

---

159. In a right triangle, one acute angle has sine ratio of 0.5. Find the sine ratio of the other acute angle.

$sin^{-1}(0.5) = 30°$
∴ other angle 60° → $sin 60°$
$\doteq 0.8666$

160. In a right triangle, one acute angle has cosine ratio of 0.7071. Find the sine ratio of the other acute angle.

$sin^{-1}(0.7071) = 45°$
other → 45° → 0.7071

161. In a right triangle, one acute angle has cosine ratio of 0.5. Find the tangent ratio of the other acute angle.

$cos^{-1}(0.5) = 60°$
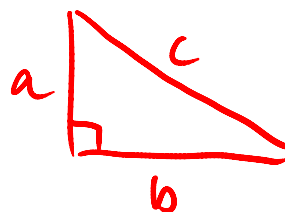other → 30° $tan 30 = 0.5774$

---

162. Which of the three trigonometric ratios (sine, cosine, tangent) can have a value greater than 1 ?

only tangent

163. Draw a right triangle and use it to explain your answer to the previous question.
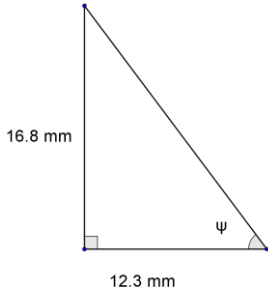
$tan A = \frac{a}{b}$

a   c

b   A

$sin A = \frac{a}{c}$
$cos A = \frac{b}{c}$

a and b are smaller than c ∴ ratio < 1.

## Working with the ratios to find angles.

Have a plan…
1. Choose the correct ratio {sine, cosine, or tangent}.
2. Fill in the known side lengths into your chosen ratio.
3. Use the "inverse trig. function" to convert ratio → angle.

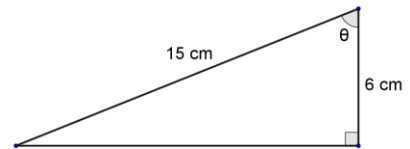**164.** What ratio do the given sides form for the indicated angle?

16.8 mm

12.3 mm

ψ

Sine   Cosine   ⟨Tangent⟩

165. What ratio do the given sides form for the indicated angle?

σ

23.7 km

18.7 km
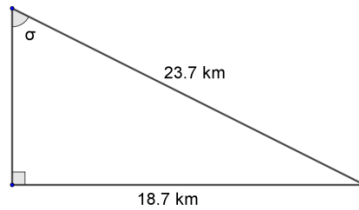
⟨Sine⟩   Cosine   Tangent

166. What ratio do the given sides form for the indicated angle?

15 cm

θ

6 cm

Sine   ⟨Cosine⟩   Tangent

---

167. Calculate the ratio above. Round to 4 decimal places.
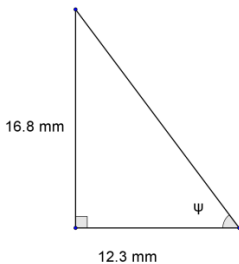
$$\frac{16.8}{12.3} = 1.3659$$

168. Calculate the ratio above. Round to 4 decimal places.

$$\frac{18.7}{23.7} = 0.7890$$

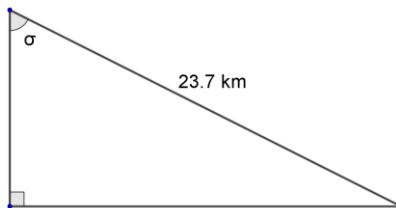**169.** Calculate the ratio above. Round to 4 decimal places.

$$\frac{6}{15} = 0.4000$$

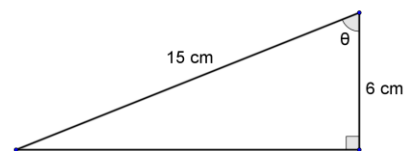**170.** Calculate the measure of angle ψ to the nearest tenth of a degree.

16.8 mm

12.3 mm

ψ

$$\tan^{-1}\left(\frac{16.8}{12.3}\right) = 53.8°$$

171. Calculate the measure of angle σ to the nearest tenth of a degree.

σ

23.7 km

18.7 km

$$\sin^{-1}\left(\frac{18.7}{23.7}\right) = 52.1°$$

172. Calculate the measure of angle θ to the nearest tenth of a degree.

15 cm

θ

6 cm

$$\cos^{-1}\left(0.4\right) = 66.4°$$
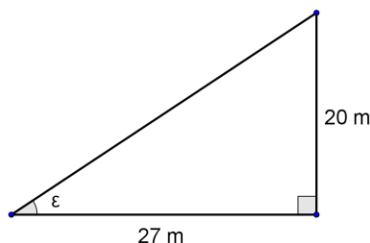

Working with the ratios to find angles.

173. What ratio do the given sides form for the indicated angle?

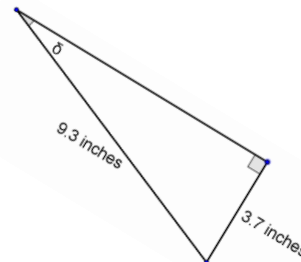
37 mm   15 mm   α
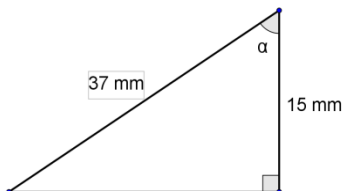
Sine   (Cosine)   Tangent

174. What ratio do the given sides form for the indicated angle?

20 m   27 m   ε

Sine   Cosine   (Tangent)

175. What ratio do the given sides form for the indicated angle?

9.3 inches   3.7 inches   δ

(Sine)   Cosine   Tangent

176. Calculate the ratio above. Round to 4 decimal places.
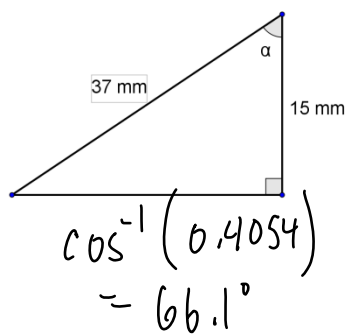
$$\frac{15}{37} = 0.4054$$

177. Calculate the ratio above. Round to 4 decimal places.
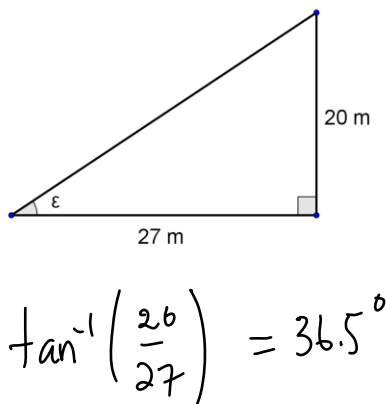
$$\frac{20}{27} = 0.7407$$

178. Calculate the ratio above. Round to 4 decimal places.

$$0.3978$$

179. Calculate the measure of angle ∝ to the nearest tenth of a degree.

37 mm   15 mm   α

$$\cos^{-1}(0.4054)$$
$$= 66.1°$$

180. Calculate the measure of angle ε to the nearest tenth of a degree.

20 m   27 m   ε

$$\tan^{-1}\left(\frac{20}{27}\right) = 36.5°$$

181. Calculate the measure of angle θ to the nearest tenth of a degree.

9.3 inches   3.7 inches

$$\sin^{-1}\left(\frac{3.7}{9.3}\right) =$$
$$23.4°$$
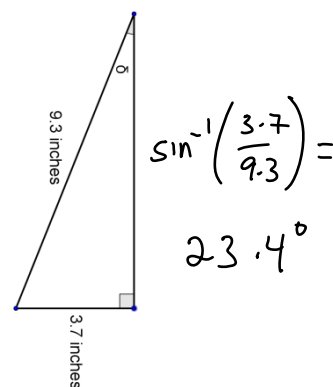
Find the measure of the indicated angle. Round answers to the nearest tenth of a degree.

182.

hyp

152 miles

Φ

104 miles adj

$\cos^{-1}\left(\dfrac{104}{152}\right)$

$= 46.8°$

183.

φ

12.3 m

10.4 m

$\tan^{-1}\left(\dfrac{10.4}{12.3}\right)$

$= 40.2°$

184.

1.75 mm

2.23 mm

θ

$\cos^{-1}\left(\dfrac{1.75}{2.23}\right)$

$= 38.3°$

185.

27.2 yards

19.3

34°

x

16 yards

θ

56°
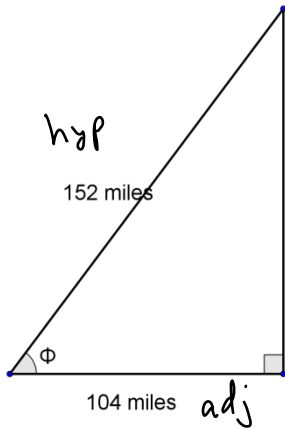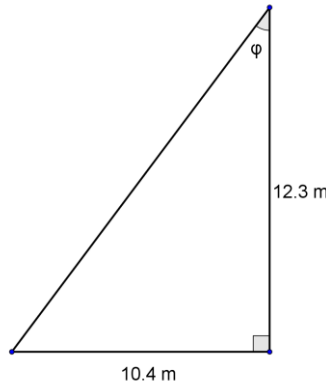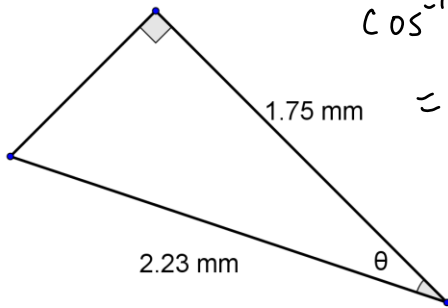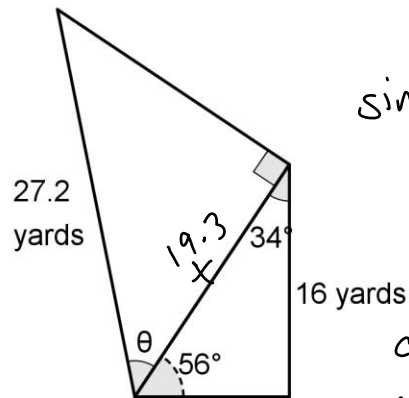
$\sin 56 = \dfrac{16}{x}$

$x = \dfrac{16}{\sin 56}$

$= 19.3$

$\cos \theta = \dfrac{19.3}{27.2}$

$\theta = \cos^{-1}\left(\dfrac{19.3}{27.2}\right)$

θ –

186. Find the measure of angle x to the nearest tenth of a degree..

123

63

θ

x
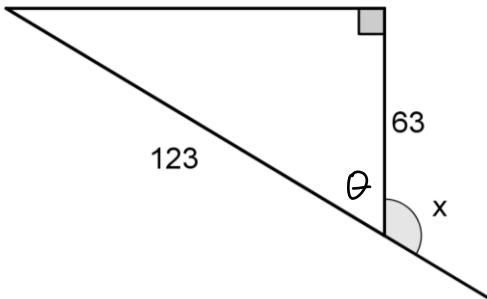
$\theta = \cos^{-1}\left(\dfrac{63}{123}\right)$

$\theta = 59.2$

$x = 180° - 59.2$

$= 120.8°$

187. Find the measure of angle θ to the nearest degree.

180°

θ

G

α

AG= 38.2 mm

GH = 21.3 mm

A

H

$\alpha = \cos^{-1}\left(\dfrac{21.3}{38.2}\right)$

$= 56.1$

$\theta = 180 + 56.1$

$= 236.1°$

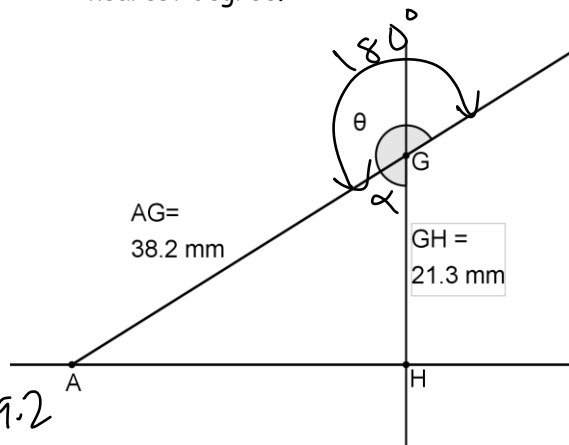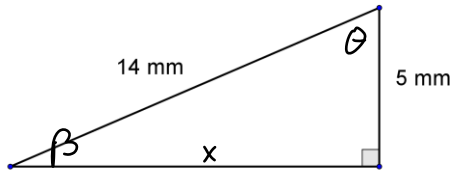Solve the following triangles. Calculate answers to the nearest tenth.

188.

14 mm 5 mm θ β x

$14^2 - 5^2 = x^2$
$x = 13.1$

$cos^{-1}\left(\frac{5}{14}\right) = \theta$
$69.1 = \theta$

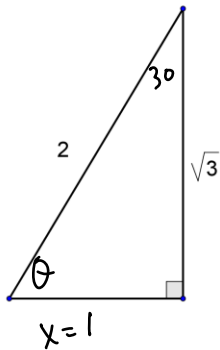$\beta = 90 - 69.1$
$= 20.9°$

189.

θ 17 m 17 m 24.0
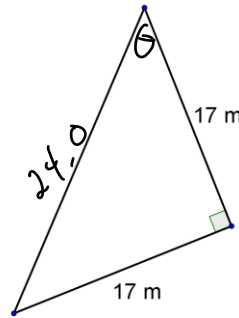
$17^2 + 17^2 = c^2$
$c = 24.04$

$tan^{-1}\left(\frac{17}{17}\right) = \theta$
$\theta = 45$

$45°, 45°, 24.0_m$

190.

30 2 $\sqrt{3}$ θ x=1

$2^2 - (\sqrt{3})^2 = x^2$
$x = 1$
$cos^{-1}\left(\frac{1}{2}\right) = \theta$
$\theta = 60°$
$\therefore$ other $\angle = 30°$

$60°, 30°, 1$

191.
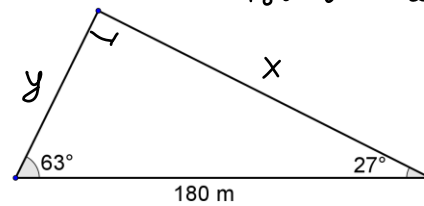
$180 - 63 - 27 = 90°$
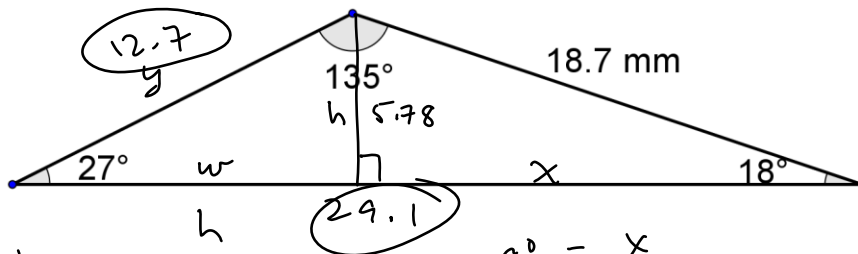
y x 63° 27° 180 m

$sin27° = \frac{y}{180}$, $y = 180sin27$
$= 81.7$

$sin63 = \frac{x}{180}$, $180sin63 = x$
$= 160.4$

192.

12.7 y 135° 18.7 mm h 5.78 27° w x 18° 29.1

$sin27 = \frac{h}{y}$
$sin27 = \frac{5.78}{y}$
$y = \frac{5.78}{sin27}$
$= 12.7$ $tan27 = \frac{5.78}{w}$
$w = \frac{5.78}{tan27} = 11.34$

$cos18° = \frac{x}{18.7}$
$18.7 cos18 = x$
$= 17.78$

$sin18 = \frac{h}{18.7}$
$18.7sin18 = h$
$h = 5.78$

$w + x = 29.1$

Find the area of the following triangles to the nearest square unit.

193.

1.6 cm
45°

$$Area = \frac{base \times height}{2}$$

Find base:

$$tan45 = \frac{opposite}{1.6}$$

$$\therefore base = 1.6 \; cm$$

$$Area = \frac{1.6 \times 1.6}{2}$$

$$Area = 1.2cm^2$$

194.

base
height
12 m
63°

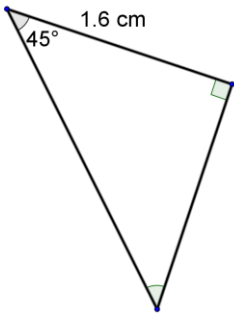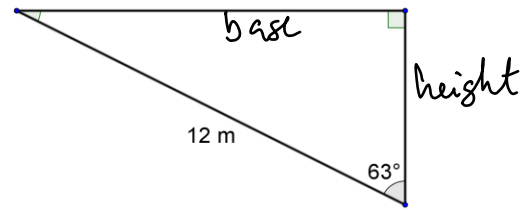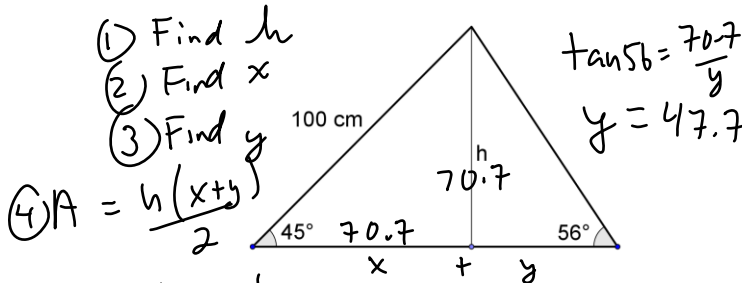$$sin63 = \frac{base}{12} \qquad cos63 = \frac{height}{12}$$

$$base = 10.69 \qquad h = 5.45$$

$$A = \frac{(10.69)(5.45)}{2}$$

$$= 29 m^2$$

195.

① Find h
② Find x
③ Find y

④ $A = \frac{h(x+y)}{2}$

100 cm
h
70.7
45° 70.7 56°
   x + y

$tan56 = \frac{70.7}{y}$

$y = 47.7$

$$sin45 = \frac{h}{100}$$

$$h = 100 sin45$$

$$h = 70.7$$

$$cos45 = \frac{x}{100}$$

$$x = 70.7$$

$$A = \frac{70.7(70.7 + 47.7)}{2}$$

$$= 4186 \; cm^2$$

196.

8 feet
a
23° 31°
b c

$$A = \frac{a(b+c)}{2}$$

$$= 34.2$$

$$= 34 \; sqft$$

$$sin31 = \frac{a}{8}$$

$$a = 8 sin31 = 4.12$$

$$tan23 = \frac{4.12}{b}$$

$$b = \frac{4.12}{tan23} = 9.73$$
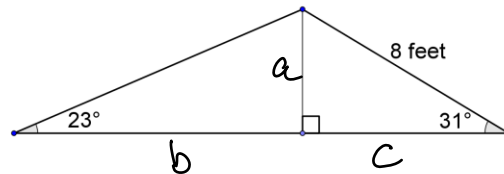
$$cos31 = \frac{c}{8} \qquad c = 8 cos31$$

$$= 6.85$$

197. A triangle has side lengths of 8 cm, 7cm and 12 cm. Find the area of the triangle if the angle between the 8 cm and 12 cm side is 34°.

8   h 7
34
   12

$$sin34 = \frac{h}{8}$$
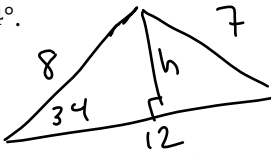
$$8sin34 = h$$

$$h = 4.47$$

$$A = \frac{12(4.47)}{2}$$

$$A = 26.8 \; cm^2$$

198. A triangle has side lengths of 10 km, 23 km and 32 km. The angle opposite the 10 km side is 9.2° Find the area of the triangle.

10   h 23
      9.2
   32

$$sin9.2 = \frac{h}{23}$$

$$h = 23 sin9.2$$

$$= 3.68$$

$$A = \frac{(3.68)(32)}{2}$$

$$= 59 \; km^2$$

Applications of trigonometry.

199. A kite stuck in a nearby tree. A child standing 25 m from the base of a tree pulls the string tight. If the tree is 30 m tall, approximately how far is the kite from the child to the nearest metre?
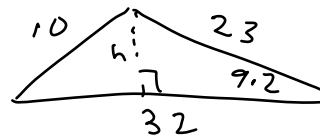
$$30^2 + 25^2 = x^2$$

$$X = 39.1$$

39 m

200. A surveyor measures the angle of elevation to the top of a building to be 23°. If the surveyor is 1345 feet from the base of the building, how tall is the building to the nearest foot?

$$\tan 23 = \frac{x}{1345}$$

$$1345 \tan 23 = x$$

$$571 = x$$

201. From the top of a 20 m cliff above a road, the angle of depression to two approaching cars is 25° and 40° respectively. How far apart are the cars to the nearest metre?

distance = x - y

$$\tan 25 = \frac{20}{x} \quad x = \frac{20}{\tan 25}$$

$$\tan 40 = \frac{20}{y} \quad y = \frac{20}{\tan 40}$$

42.8

distance is 19 m

202. Two hot air balloons float above the ocean at a height of 1000 feet. From a sailboat, the angle of elevation to one balloon is 60° and to the other balloon is 50°. How far apart are the balloons to the nearest foot?
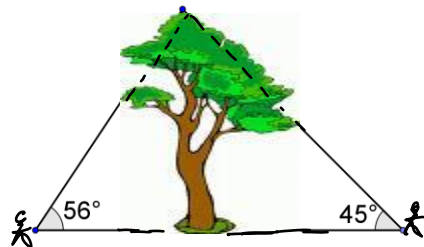
$$\tan 60 = \frac{1000}{a}$$

$$\tan 50 = \frac{1000}{b}$$

$$a = 577$$
$$b = 839$$

$$x = 262'$$

203. Two boys on opposite sides of the tree below measure the angle of elevation to the top of the tree. If the tree is 175 feet tall, how many feet apart are the boys?

56°   45°

$$\tan 56 = \frac{175}{x}$$

$$x = \frac{175}{\tan 56}$$

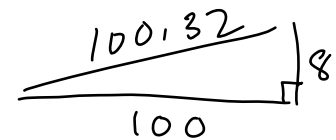$$\tan 45 = \frac{175}{y}$$

$$y = \frac{175}{\tan 45}$$

$$118.0 + 175 = 293'$$

204. Highway sign shows that the road descends at a rate of 8%. Draw a diagram that shows what this means.

8%

$$\frac{100.32}{100} \quad 8$$

If a 3 km section of straight road descends at this grade, what is the drop in elevation?

$$\frac{100.32}{8} = \frac{3}{x}$$

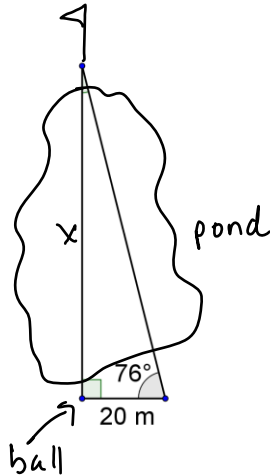$$\frac{3(8)}{100.32} = 0.24 \text{ km}$$

205. While golfing with his father-in-law, Mr. J hits a shot short of a pond. The flag (hole) is directly across the pond from his ball. He paces 20 m to the right of his ball and measures the angle back to the hole to be 76°. How far is the ball from the hole to the nearest metre?
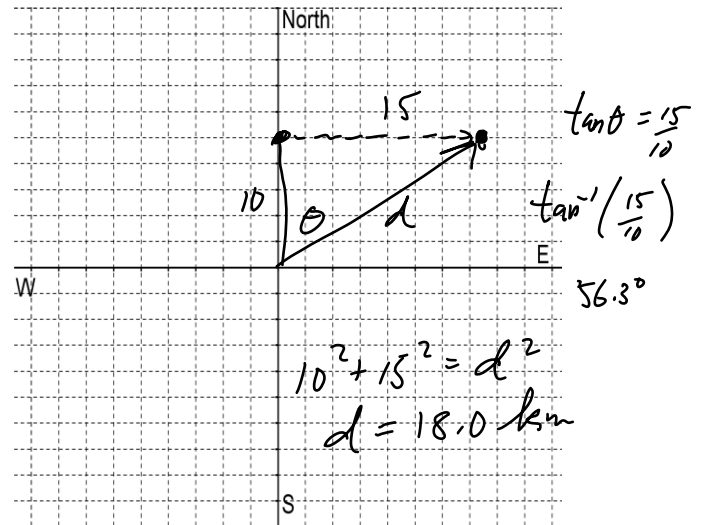
$$\tan 76 = \frac{x}{20}$$

$$x = 20 \tan 76$$

$$x = 80.2$$

$$= 80m$$

pond

x

76°

20 m

ball

206. A hiker leaves base camp travelling due north at 5 km/h. After two hours, she turns and travels east. Three hours later, she sprains her ankle. At what bearing would a rescue team need to travel to reach the injured hiker? How far away is she from base camp? (nearest tenth)

North

15

10

θ

d

W

E

S

$$\tan \theta = \frac{15}{10}$$

$$\tan^{-1}\left(\frac{15}{10}\right)$$

$$56.3°$$

$$10^2 + 15^2 = d^2$$

$$d = 18.0 \; km$$

207. A student approaches a large Sequoia tree outside the entrance to the school and wonders how tall the tree is. He paces 150 metres from the base of the tree and measures the angle of elevation to the top of the tree to be 35°. Find the height of the tree to the nearest metre.

35°

h

150m

$$\tan 35 = \frac{h}{150}$$

$$150 \tan 35 = h$$

$$h = 105 \, m$$
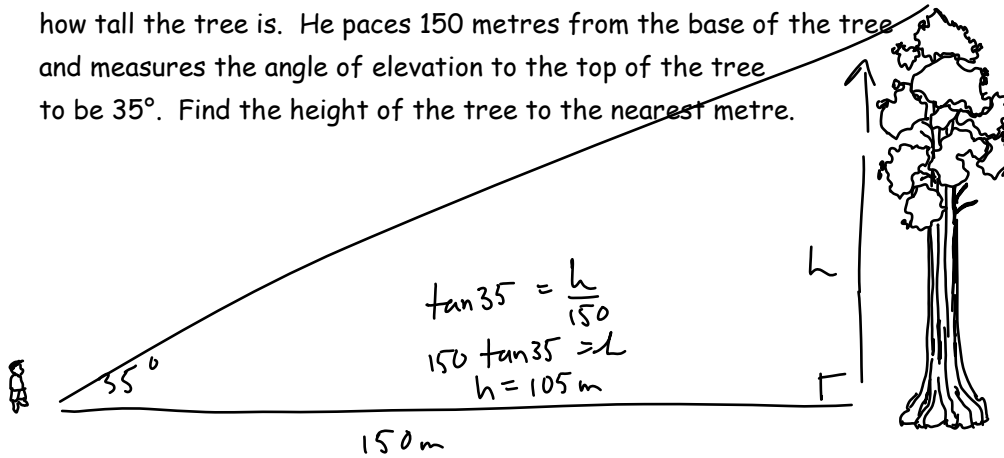
208. A homeowner wants to cut a new board to replace a decaying roof truss. He can measure the horizontal distance and the angle of inclination but needs to know how long to cut the board. The horizontal distance is 14 feet and the angle of inclination is 24°. Find the distance to the nearest tenth of a foot.

$$\cos 24 = \frac{14}{x}$$

$$x = \frac{14}{\cos 24}$$

$$x = 15.3'$$
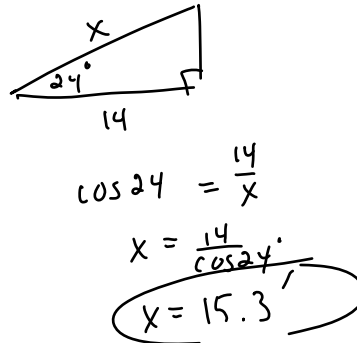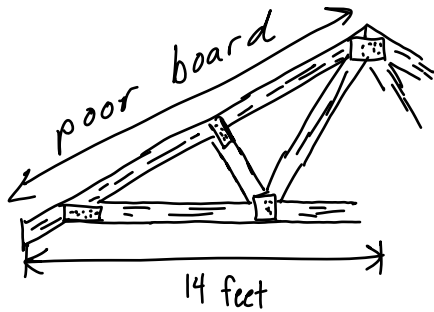
209. An engineer is constructing a Ferris wheel for a downtown park. There are 16 passenger carts and the radius of the wheel is 10 metres. How far apart are the passenger carts to the nearest hundredth of a metre?
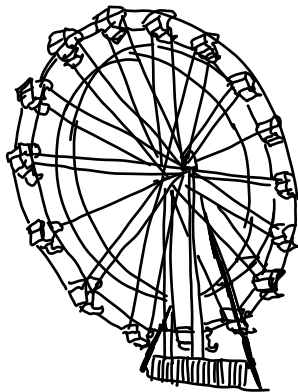
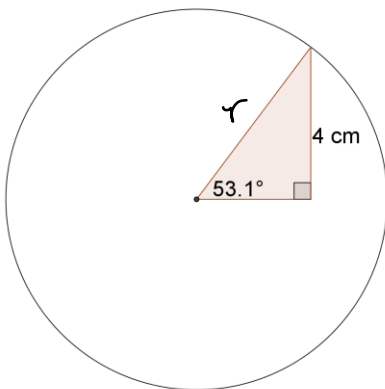① Recall, there are $360°$ in a full circle.

∴ $\frac{360}{16} = 22.5°$
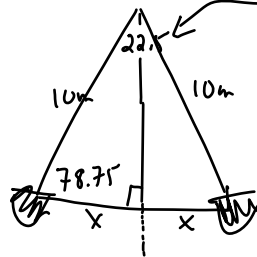
② Bisect $22.5° = 11.25°$
∴ other acute $\angle = 78.75°$

③ $\cos 78.25° = \frac{x}{10}$

$10 \cos 78.25 = x$

$x = 1.95m$

distance $= 2(1.95)$
$= 3.90m$

---

210. Find the area of the circle to the nearest square centimetre. $[A = \pi r^2]$
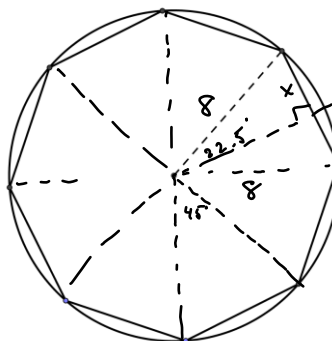
$$\sin 53.1 = \frac{4}{r}$$

$$r = \frac{4}{\sin 53.1}$$

$$r = 5.06$$

$$A = \pi (5)^2$$
$$= 25\pi$$
$$\approx 78.5 cm^2$$

211. Find the perimeter of the octagon inscribed in a circle of radius 8 cm. (Nearest cm)

$$\sin 22.5 = \frac{x}{8}$$
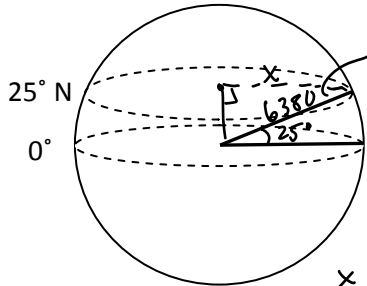
$$8 \sin 22.5 = x$$
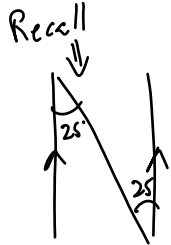
$$3.06 = x$$

side $= 2(3.06)$
$= 6.12$

Perimeter
$= 8(6.12)$
$= 48.98$
$\approx 49 cm$

212. Find the length of the 25° line of latitude. The Earth's radius is 6380 km. Answer to the nearest km.

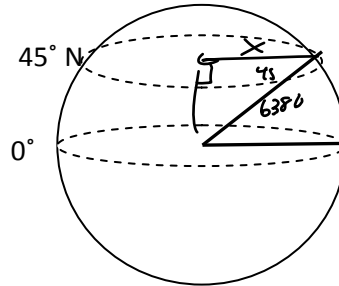I know this because the two radii are parallel

25° N

0°

Recall

$$\cos 25 = \frac{x}{6380}$$

$$x = 6380 \cos 25$$

$$= 5782$$

$$C = 2\pi r \quad = 2\pi(5782) = 33631$$

213. Find the length of the 45° line of latitude. The Earth's radius is 6380 km. Answer to the nearest km.

45° N

0°

$$\cos 45 = \frac{x}{6380}$$

$$x = 4511$$

$$C = 2\pi(4511)$$

$$= 28346$$

214. Mr. Teespré 's backyard slopes away from his house towards the beach. The instructions for his new lawnmower state that the mower should not be used if the slope is greater than 15°. Being a trigonometry specialist, he extends a level string 125 feet from the base of his house. From that point, he measures that the distance along the ground back to his house is 130 m.
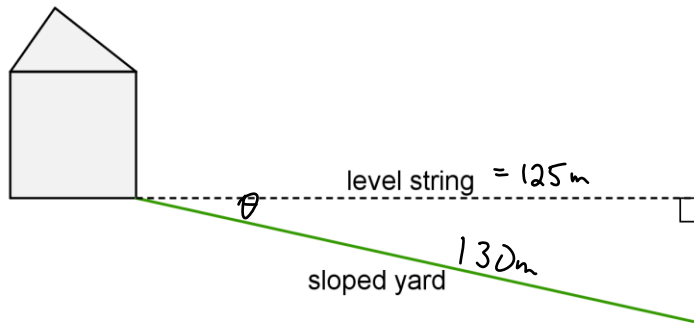
Is his yard too steep for this mower?

level string = 125 m

130 m

sloped yard

$$\cos \theta = \frac{125}{130}$$

$$\cos^{-1}\left(\frac{125}{130}\right) = \theta$$

$$15.94° = \theta$$

TOO STEEP !!

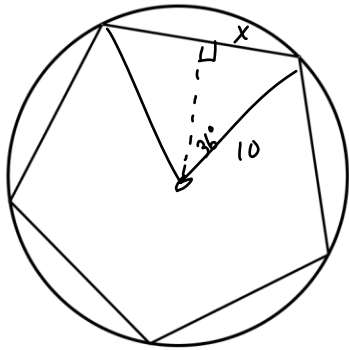**215.** A regular pentagon is inscribed in a circle of radius 10 cm. Calculate the perimeter of the pentagon. Answer to the nearest cm.

Perimeter = 10x

$\sin 36 = \frac{x}{10}$

$10\sin 36 = x$

$5.88 = x$

$x = 59\,cm$

216. A regular decagon (10 sides) is inscribed inside a circle of radius 8 cm. Find the perimeter of the decagon. Answer to the nearest cm.

Consider problem to left.

Changes → 10 sides makes the working angle 18°
→ radius is 8
→ Perimeter = 20x
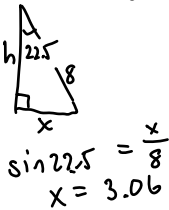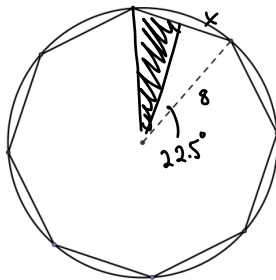

$\sin 18 = \frac{x}{8}$

$8\sin 18 = x$

$2.47 = x$

$P = 49.44$

≈ 49

217. Find the area of the octagon inscribed in a circle of radius 8 cm. Answer to the nearest cm.

① Find shaded area

② Area = 16 × shaded
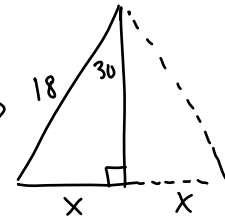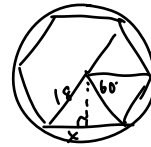

$\sin 22.5 = \frac{x}{8}$

$x = 3.06$

Find h: $a^2 + b^2 = c^2$

$8^2 - 3.06^2 = h^2$

$h = 7.4$

$A_{shaded} = \frac{(7.4)(3.06)}{2} = 11.3$

TOTAL:

$16(11.3) = 181\,cm^2$

218. A regular hexagon is inscribed in a circle with a radius 18 cm. What would be the side length of the hexagon? Answer to the nearest cm.

* side = 2x

side = 18 cm

$\sin 30 = \frac{x}{18}$

$18\sin 30 = x$

$x = 9$

219. From a point 15 m from the base of a tree, a woman found the angle of inclination to the top of the tree to be 45°. Her sister found the angle to be 18° from a point farther away from the base of the tree. How far away are the two women away from each other?

① Find tree height.

② Use tree height to find second distance.

③ Find difference.
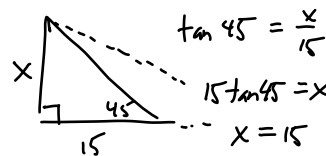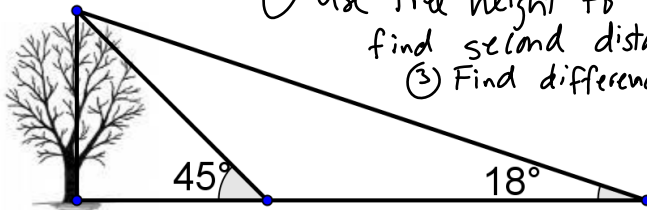
$\tan 45 = \frac{x}{15}$

$15\tan 45 = x$

$x = 15$

$\tan 18 = \frac{15}{a}$, $a = \frac{15}{\tan 18}$) $a = 46.16$

distance = $46.16 - 15 = 31.2\,m$

More word problems using right triangles:

- Draw a diagram.
- Fill in known values.
- Let a variable represent unknown(s)
- Choose an appropriate strategy to solve for the unknown(s).
- Interpret the problem.

---

220. Solve the triangle given the following.

ΔXYZ

$$x = 9\ cm$$
$$\angle Y = 90°$$
$$\angle Z = 36°$$
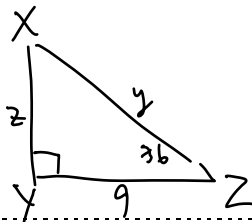
$\cos 36 = \dfrac{9}{y}$

$y = \dfrac{9}{\cos 36} = 11.1$
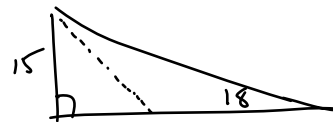
$\tan 36 = \dfrac{z}{9}$

$9 \tan 36 = z$
$= 6.5$

$\angle X =$
$180 - 90 - 36$
$= 54°$

---

221. A firefighter is walking along the river at point C when she spots two fires on the opposite river bank. She measures the angles below and paces a distance of 300 m from point C to point D. Point D is directly across the river from one of the fires. How far apart are the fires to the nearest metre?

x = 221.7   300

38   30

d

38°   30°

C   D
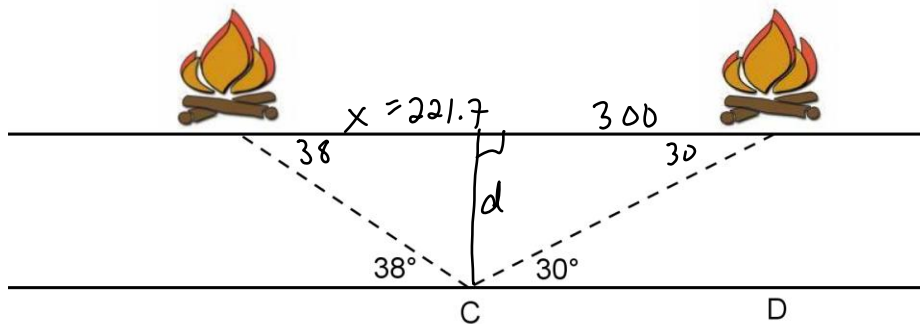
$\tan 30 = \dfrac{d}{300}$

$300 \tan 30 = d$

$173.2 = d$

$\tan 38 = \dfrac{173.2}{x}$

$x = \dfrac{173.2}{\tan 38}$

distance $= 222 + 300$
$= 522m$

---

222. Anya stands on top of a building in downtown Victoria. From her position, the angle of elevation to the top of an adjacent building is 47°. The angle of depression to the base of the building is 62°. She is told that the buildings are 45 m apart. Based on this information, what is the height of the taller building?

$$\tan 47 = \frac{x}{45} \qquad x = 45\tan 47$$

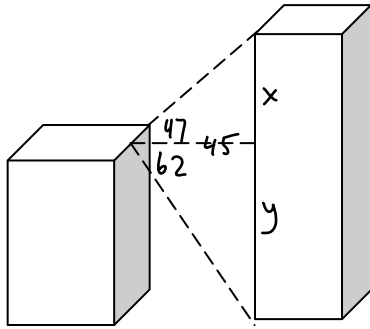$$\tan 62 = \frac{y}{45} \qquad y = 45\tan 62$$

$$45\tan 47 + 45\tan 62 = 132.8 \, m$$

223. Find the length of diagonal BG in the rectangular prism. Answer to the nearest tenth of a millimeter.

4 mm

53°

B   8 mm
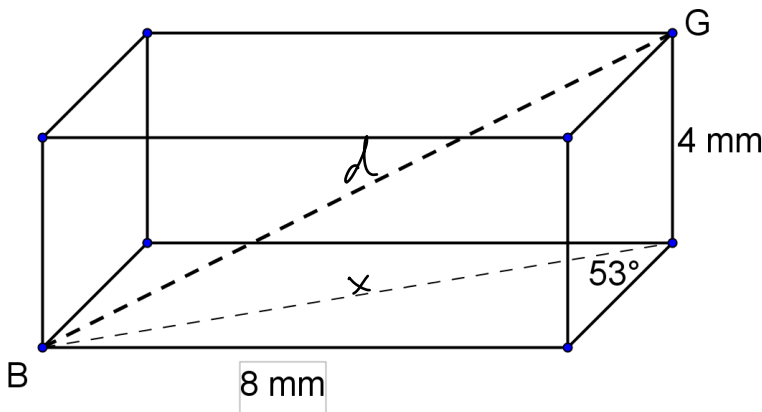
$$\sin 53 = \frac{8}{x}$$

$$x = \frac{8}{\sin 53} = 10.02$$

$$x^2 + 4^2 = d^2$$
$$10^2 + 4^2 = d^2$$
$$\sqrt{116} = d$$
$$10.8 = d$$

224. The line of sight from an inflatable boat to the top of an oil derrick is 24 degrees. If the derrick is 45 m tall, how far is the boat from its base? (nearest tenth)

45

24°

$$\tan 24 = \frac{45}{x}$$

$$x = \frac{45}{\tan 24}$$

$$x = 101.1 \, m$$

225. A pilot on a level path knows she should descend at an angle of 3 degrees to maintain comfort and safety. If she is flying at an altitude of 12 000 feet, how many miles from the runway should she begin her descent?
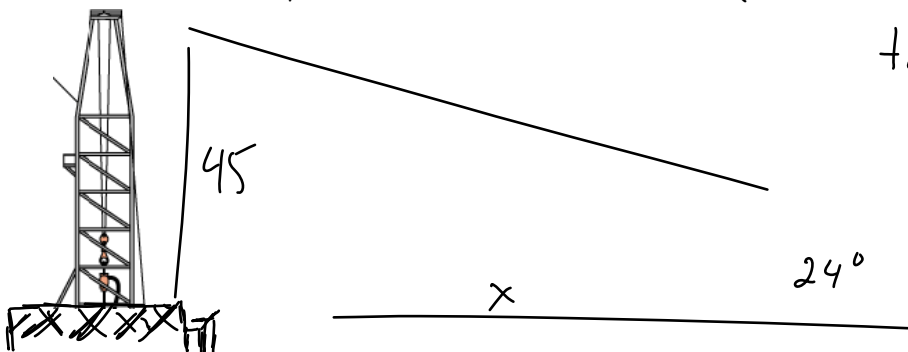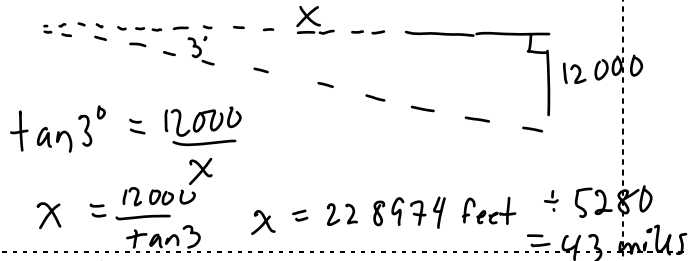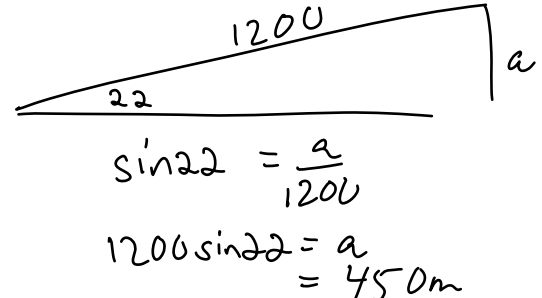
$$\tan 3° = \frac{12000}{x}$$

$$x = \frac{12000}{\tan 3} \quad x = 228974 \text{ feet} \div 5280$$
$$= 43 \text{ miles}$$

226. An aircraft ascends after takeoff at an angle of 22 degrees. What will be the altitude of the aircraft after it flies at that angle for 1200 m? (nearest metre)

$$\sin 22 = \frac{a}{1200}$$

$$1200 \sin 22 = a$$
$$= 450 m$$

227. A hamster scurries up a ramp at a speed of 1.5 m/s. The ramp is inclined at an angle of 18 degrees. How many metres above the ground will the hamster be after 30 seconds?

30 seconds → 1.5 × 30
45m

$$\sin 18 = \frac{x}{45} \quad 45 \sin 18$$
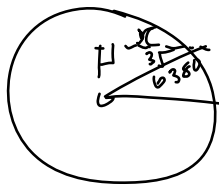$$= 14m$$
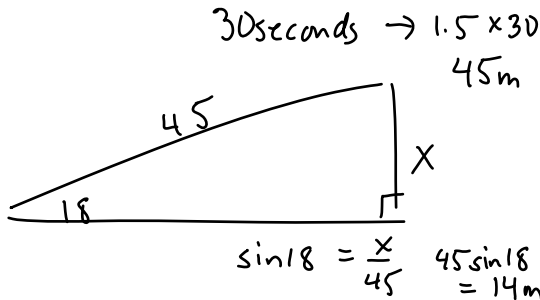
228. Anya travels down a zip line at 25 km/h. The angle of descent of the zip line is 11 degrees. How many vertical metres has she fallen after 3 minutes?

$$\frac{25 \text{ km}}{h} \times \frac{1 h}{60 \text{ min}} = \frac{0.41\bar{6} \text{ km}}{\text{min}}$$

$$\frac{0.41\bar{6} \text{ km}}{\text{min}} \times 3 \text{ min} = 1.25 \text{ km}$$

$$\sin 11 = \frac{x}{1.25}$$
$$1.25 \sin 11 = x$$
$$x = 0.239 \text{ km}$$
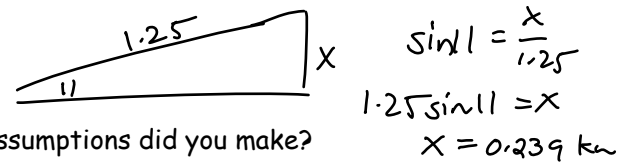
What assumptions did you make?

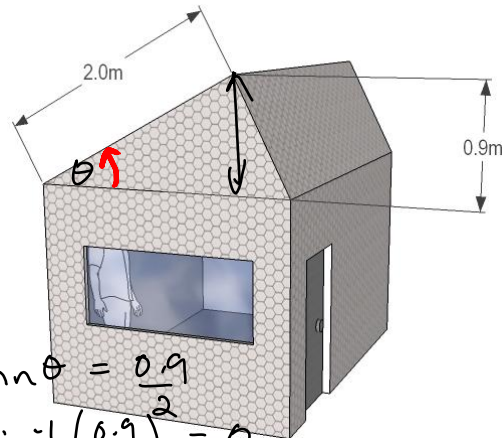229. The Earth's radius is 6380 km. Find the length of the 35° latitude to the nearest 10 km.

$$\cos 35 = \frac{x}{6380}$$

$$x = 6380 \cos 35$$
$$= 5226$$

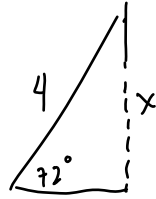$$C = 2\pi r = 32840 \text{ km}$$

230. Find the angle of inclination at the back =239 m of the roof. The "rise" of the roof is 0.9 m.

2.0m

0.9m

$$\sin \theta = \frac{0.9}{2}$$

$$\sin^{-1}\left(\frac{0.9}{2}\right) = \theta$$
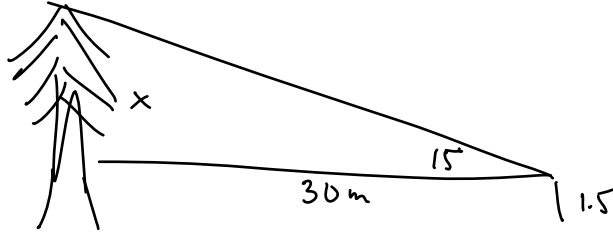
$$\theta = 26.7°$$

231. A ladder should make an angle of 72° with the ground for maximum safety. If the ladder is 4 m long, how far should it reach up the wall? (nearest tenth)

$$\sin 72 = \frac{x}{4}$$

$$4\sin 72 = x \quad = 3.8m$$

232. The angle of elevation to the top of a tree, measured on a 1.5 m transit from a distance of 30 m, is 15°. Find the height of the tree. (nearest tenth)
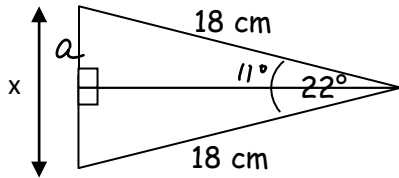
$$\tan 15 = \frac{x}{30} \qquad x = 30\tan 15$$
$$= 8.04$$
$$8.04 + 1.5 = 9.5m$$

30 m   15°   1.5
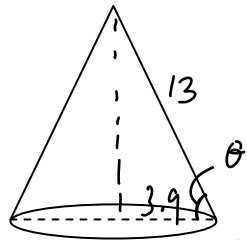
233. Find the value of 'x'.

18 cm
11° 22°
x
18 cm

$$\sin 11 = \frac{a}{18}$$

$$18\sin 11 = a$$
$$a = 3.43$$
$$x = 2a = 6.9\ cm$$

234. Mr. J has developed the ideal ice cream cone. The cone has a slant height of 13 cm and a diameter of 7.8 cm. Find the angle that the curved surface makes with the diameter.

13   θ
3.9

$$\cos \theta = \frac{3.9}{13} \quad \bigg|\bigg| \quad \cos^{-1}\left(\frac{3.9}{13}\right) = \theta$$
$$72.5 = \theta$$

235. Mr. J continues to work on his isolated surf hut. Below is two-thirds of a roof truss he wants to complete. Find the length of wood he must cut (nearest tenth) to complete the truss. The long side is 8.2 m and the short side is 6.8 m. The angle between them is 35°.

$$3.9^2 + 2.6^2 = b^2$$
$$4.7 = b$$

6.8   h 3.9   b = 4.7
35°   x   2.6
5.6
8.2
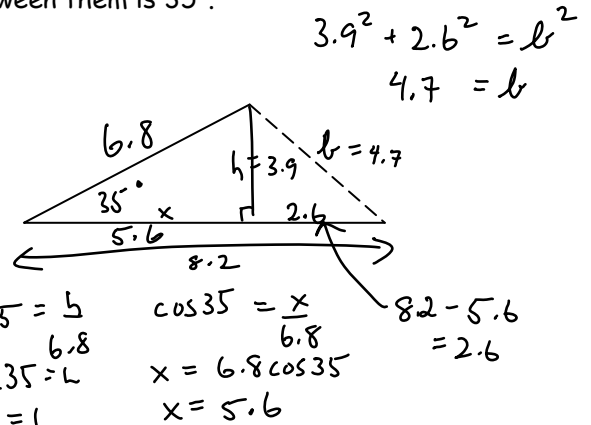
$$\sin 35 = \frac{h}{6.8}$$
$$6.8\sin 35 = h$$
$$3.9 = h$$

$$\cos 35 = \frac{x}{6.8}$$
$$x = 6.8\cos 35$$
$$x = 5.6$$

$$8.2 - 5.6$$
$$= 2.6$$

236. Both triangles (large and smaller inset) are isosceles. Find the area of the shaded trapezoid to the nearest tenth of a square unit.

26.5

10 / 53°

26.5

30

$\sin 26.5 = \frac{x}{10}$

$x = 4.46$

$\cos 26.5 = \frac{h}{10}$
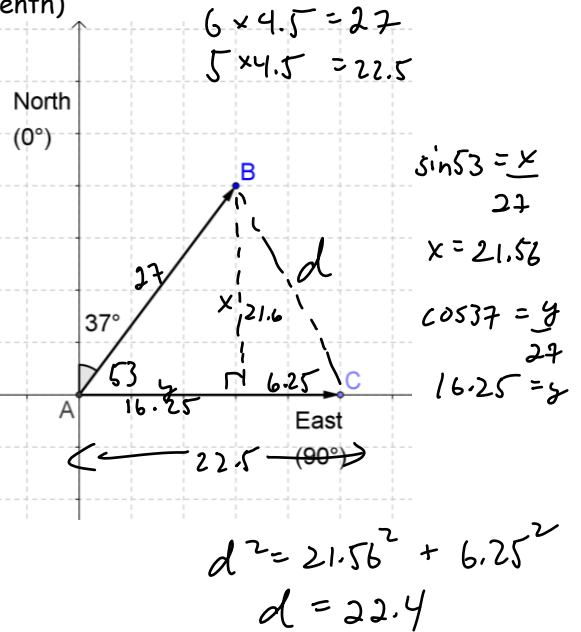
$h = 8.94$

$Area_{small} = 39.9$

30 / 26.5

a

y

$\sin 26.5 = \frac{y}{30}$

$y = 13.39$

$\cos 26.5 = \frac{a}{30}$

$a = 26.85$

$Area_{big} = \frac{(26.78)(26.85)}{2}$

$= 359.5$

$A_{big} - A_{small}$

$= 319.6$ sq units
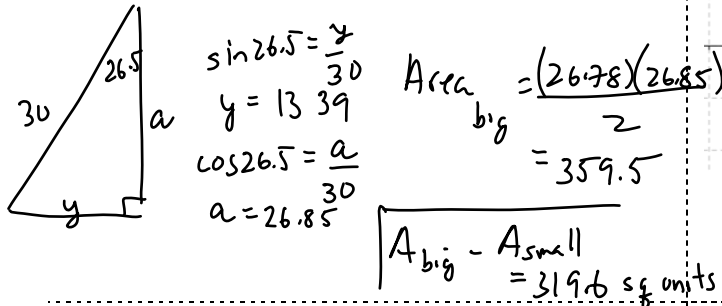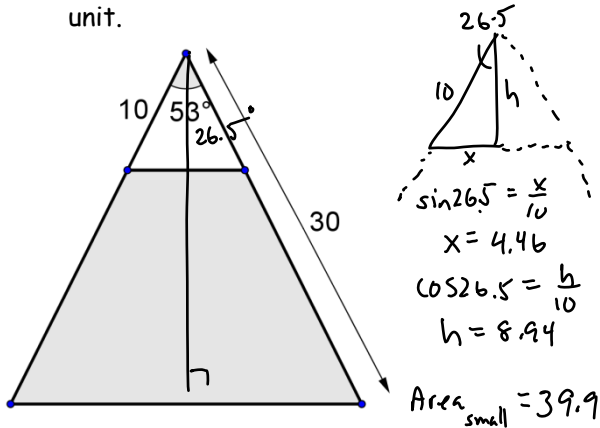
237. From a fire station in central BC, Georgia travels on a bearing of 37° at 6 km/h. Shelby leaves the station at the same time travelling due east at 5 km/h. How far apart are they after 4.5 hours? (Nearest tenth)
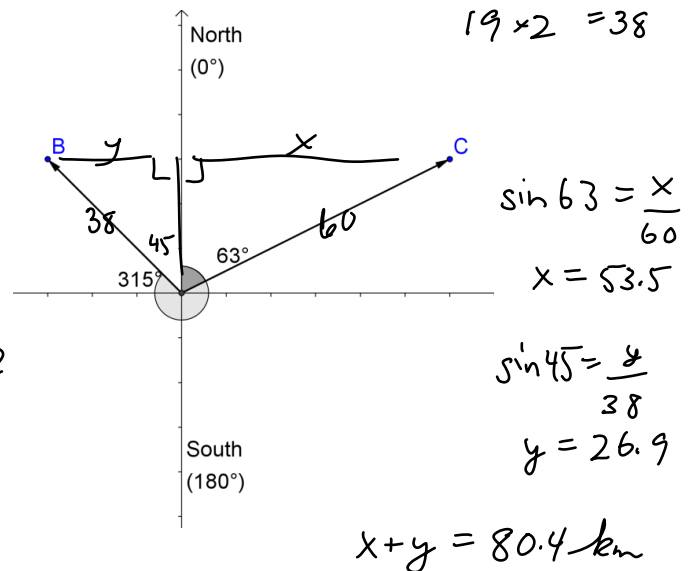
$6 \times 4.5 = 27$

$5 \times 4.5 = 22.5$

North
(0°)

B

27

d

37°

x 21.6

53

16.25

$\underset{A}{}$ 16.25 $\boxed{}$ 6.25 C

East

$\longleftarrow$ 22.5 $\longrightarrow$ (90°)

$\sin 53 = \frac{x}{27}$

$x = 21.56$

$\cos 37 = \frac{y}{27}$

$16.25 = y$

$d^2 = 21.56^2 + 6.25^2$

$d = 22.4$

238. Find the measure of angle x to the nearest tenth of a degree.

123

63

θ x
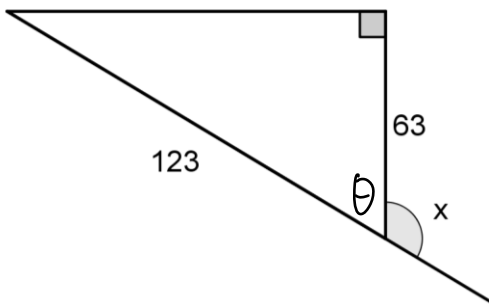
$\cos\theta = \frac{63}{123}$

$\cos^{-1}\left(\frac{63}{123}\right) = \theta$

$\theta = 59.2$

$x = 180 - 59.2$

$= 120.8°$

239. At 9:00 am, a ship leaves port traveling at 30 km/h on a bearing of 63°. At the same time, another ship leaves port on a bearing of 315° at a speed of 19 km/h. When the boats stop after two hours, how far east is the boat at point C?
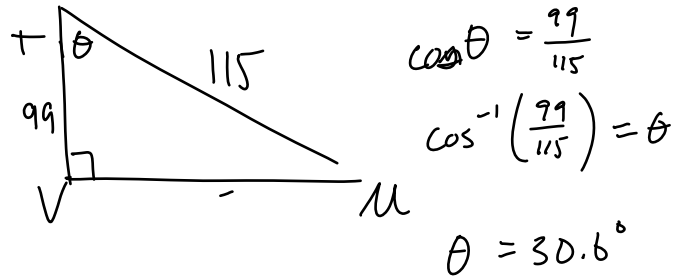
$30 \times 2 = 60$ km

$19 \times 2 = 38$

North
(0°)

B y x C

38   45   63°   60

315°

South
(180°)

$\sin 63 = \frac{x}{60}$

$x = 53.5$

$\sin 45 = \frac{y}{38}$

$y = 26.9$

$x + y = 80.4$ km

Draw an accurate diagram to answer each of the following questions.

240. In $\triangle QRS$, $\angle QSR = 90°$, $QR = 12\ cm$ and $QS = 10\ cm$. Find the measure of $\angle QRS$

$$\sin\theta = \frac{10}{12}$$

$$\sin^{-1}\left(\frac{10}{12}\right) = \theta$$

$$56.4 = \theta$$
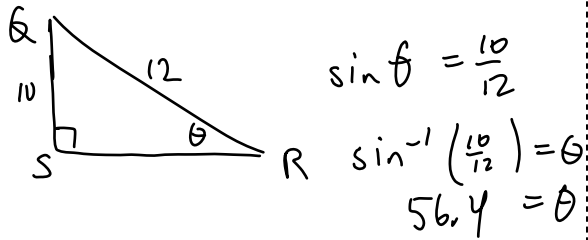
241. In $\triangle TUV$, $\angle TVU = 90°$, $TU = 115\ m$ and $TV = 99\ m$. Find the measure of $\angle UTV$

$$\cos\theta = \frac{99}{115}$$

$$\cos^{-1}\left(\frac{99}{115}\right) = \theta$$

$$\theta = 30.6°$$

242. In $\triangle DEF$, $\angle DFE = 90°$, $DE = 12\ cm$ and $\angle DEF = 30°$. Find the length of FE.

$$\cos 30 = \frac{x}{12}$$

$$12\cos 30 = x$$

$$x = 10.4$$
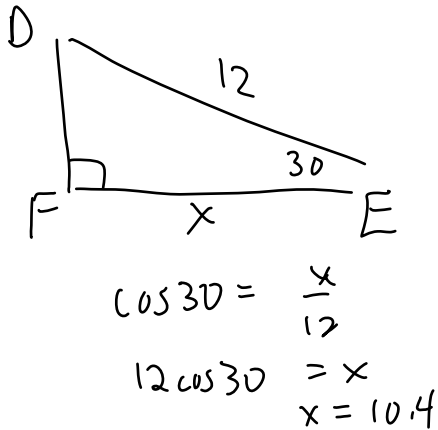
243. In $\triangle ABC$, $\angle ACB = 90°$, $BC = 5\ cm$ and $\angle ABC = 12°$. Find the length of AC.

$$\tan 12 = \frac{x}{5}$$

$$5\tan 12 = x$$

$$x = 1.06$$

$$x \approx 1.1\ cm$$